Saml Hewett
Rl. N. College
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PHILOSOPHICAL TRANSACTIONS,

OF THE

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FOR THE YEAR MDCCXCVI.

PART I.

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ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable, that a Committee of their members should be appointed to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds

of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they receive them, are to be considered in no other light than as a matter of civility, in return for the respect shewn to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public news-papers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports, and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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APPENDIX.

Meteorological Journal kept at the Apartments of the Royal Society by Order of the President and Council.

THE President and Council of the Royal Society adjudged, for the year 1795, the Medal on Sir Godfrey Copley's Donation, to Mr. Jesse Ramsden, F. R. S. for his various inventions and improvements in the construction of the instruments for the trigonometrical measurements, carried on by the late Major General Roy, and by Lieutenant Colonel Williams and his Associates.

PHILOSOPHICAL

TRANSACTIONS.

I. *The* Croonian *Lecture on Muscular Motion. By* Everard Home, *Esq. F. R. S.*

Read November 12, 1795.

In the Croonian Lecture which I had the honour of laying before this learned Society last year, I endeavoured to prove, that the adjustment of the eye to different distances could take place independent of the crystalline lens; and when this was the case, it appeared to arise from a change in the curvature of the cornea.

I propose in the present lecture to prosecute the inquiry; and it will be found in this, as well as in the former, that I have received the most essential assistance from Mr. Ramsden, who continues to interest himself in the investigation, and has made all the optical experiments.

As this was a new mode of explaining the adjustment of the eye, and differed from the theories that have been previously formed upon the subject, it was thought right to consider it with caution, to pay attention to all the objections that could

be made to it, and to put it to the test of such experiments as appeared likely to refute or confirm our former observations.

It readily suggested itself, that if the convexity of the cornea was increased to a certain degree, it could be measured by means of an image reflected from its surface, and viewed in an achromatic microscope with a divided eye-glass micrometer.

To ascertain whether the quantity of increase of the convexity of the cornea, in the adjustment of the eye, could in this way be ascertained, the following experiments were contrived, and made by Mr. RAMSDEN.

Our former experiments had sufficiently proved the unsteadiness of the human eye; the first trials on the present occasion were therefore made upon convex mirrors, as these artificial corneas could be more readily managed, and such previous experiments would enable us to apply the same instruments with more facility to the eye itself.

Two convex mirrors, one $\frac{4}{10}$ of an inch focus, the other $\frac{5}{10}$, had their flat surfaces made rough, and blacked, to prevent an image being seen from both surfaces, which was found to be the case when this precaution was omitted. One of these mirrors was stuck upon a piece of wood directly opposite to a window, at twelve feet distance from it; a board, three feet long, and six inches broad, was placed perpendicularly against the sash of the window, and its image reflected from the mirror upon the object-glass of an achromatic microscope, with a divided eye-glass micrometer.

The two images were separated by means of the divided eye-glass, till their surface of contact, which appears like a black line, was rendered as small as possible. When this effect was produced on the images from the mirror of $\frac{4}{10}$ of an

inch focus, that mirror was removed, and the other put in its place; the contact of the two images, which before appeared like a line, had now acquired considerable breadth; corresponding exactly to the difference between the convexities of the mirrors.

Having in this way made trial of the instruments, and arranged all the necessary circumstances, the head of a person was so placed as to bring the eye into the same situation as the mirror, and made steady by the apparatus described in our former experiments. Under these circumstances the image reflected from the cornea was measured by the micrometer.

Mr. Ramsden made an experiment with this instrument upon my eye. In the first trials, when the eye was fresh, there was a perceptible change in the micrometer, but extremely small; this was not, however, seen afterwards, and the eye very soon became so much fatigued that it was necessary to desist. He found that every time the eye adapted itself to different distances, it was necessary to move the object-glass of the microscope further from, or nearer to, the cornea.

This experiment was repeated on four different days; and in each experiment, on the first trial, the result was a change in the micrometer, but in all the subsequent trials it could not be detected. We were induced to conclude, that the effect on the micrometer might arise from the head being moved forwards, as we found, in making experiments with the mirror, that this effect could be produced by such motion; but had it arisen from that cause, it should more frequently have occurred, and rather after the head and eye were tired, than on the first trials. It was supposed to arise from the action of the muscles of the head, but that should have produced a contrary

appearance. The effect produced on the micrometer, therefore, did not seem to depend upon external circumstances, but to arise from a change in the cornea; it was, however, too small to admit of any conclusions being drawn from it.

The same experiment was made upon several young persons; but we found it necessary, that whoever was the subject of the experiment should understand perfectly what was meant to be done, otherwise the conclusions could not be depended on; for if the eye does not see the near object with a very defined outline, it is not accurately adjusted to it; and the length of time they kept their eye upon the near object without making any complaint of being fatigued, was greater, we knew, from our own observation, than it was possible to do it, had the object been seen with the necessary degree of distinctness.

I have to regret that Sir Henry Englefield, who took a part in the former experiments, and whose assistance in making these would have been of material advantage, was unable to remain in town.

Finding from these experiments, that the change in the convexity of the cornea was not to be seen distinctly in the micrometer, it became an object to ascertain the degree of change which could in this way be distinctly determined.

For this purpose two mirrors were ground, and prepared in the same way as those used in the preceding experiment; their radii were exactly ascertained by measuring the tools in which they were finished off; the one was $\frac{4}{10}$ of an inch focus, the other $\frac{408}{1000}$; the difference between the size of the images reflected from their surface was just visible in the micrometer; and from their remaining fixed, the experiment could be made with every advantage; but it did not appear

probable that the same difference would have been visible had the mirror not been perfectly at rest. A smaller change could not therefore be detected in the eye; and when we consider the disadvantages under which an experiment of this nature must be made upon the human eye, from the unsteadiness of that organ, the short time it remains adjusted (a part of which is lost in bringing it within the focus of the microscope), and also from the motions of the head; it is not unreasonable to suppose that a change might take place in the cornea, to the same extent, without being distinctly seen.

To give an idea of the short time that a part can remain nicely adjusted by muscular action, I shall point out an experiment which any one may make upon himself: let him take a glass spirit level, and rest one end of it on a table, supporting the other with his hand, and endeavour to keep the air bubble in the middle; if the hand is very steady the bubble may be kept nearly in its place, but not exactly so, it will undulate, its motion corresponding with the actions of the muscles; making up for want of steadiness by short motions in contrary directions.

From these experiments the change in the curvature of the cornea could not be more than $\frac{1}{125}$ part of an inch, as any greater quantity would probably have been distinctly seen in the micrometer; this, however, is still more than was ascertained by our former experiments, which made it to exceed $\frac{1}{800}$ part of an inch.

This change in the cornea, on the first view of the subject, appeared sufficient to account for the adjustment of the eye, and when the lens is removed it probably may be sufficient; but the refractions at the cornea are so much changed by those at the

lens, as considerably to lessen their effect in fitting the eye for seeing near objects, and make this small increase of convexity inadequate to such an effect.

Finding this to be the case, it became necessary to examine the eye with attention, to see in what way the full effect was most likely to be produced. For this purpose the following experiments were made upon the human eye, to determine whether the axis of vision could be elongated by any uniform pressure applied to its coats.

The experiments were made in the following manner: an eye of a dead subject was carefully removed from the socket, before any change could be produced in consequence of death, and its different diameters were measured by a pair of calliper compasses. As soon as these were determined, a hole was made in the centre of the optic nerve, and a pipe fixed into it, through which air could be thrown into the cavity of the eye, so as to distend its coats. While distended in a moderate degree, by compressing with the hand a small bladder, containing air and quicksilver, attached to the pipe, the same diameters were measured again, and compared with those which were taken while in the natural state.

These experiments were made by Mr. MUTTLEBURY and Mr. WILLIAMS, two very intelligent and diligent students in surgery, who were filling situations that gave opportunity of making such experiments. They measured the diameters in these two states, and marked them on paper, without ascertaining their difference, so that there could be no fallacy in the measurement from any preconceived opinion; and I have every reason to believe there was none from inattention.

		Transverse diameter.	Axis from optic nerve.	Axis of vision.
		Twentieth parts of an inch.	Twentieth parts of an inch.	Twentieth parts of an inch.
The eye of a boy 6 years old, 45 minutes after death - -	Natural state	17 $\frac{1}{2}$	17 $\frac{1}{2}$	17 $\frac{1}{2}$
	Distended state	17 $\frac{1}{4}$+	17 $\frac{1}{4}$+	18
The eye of a man 25 years old, 1 hour after death - -	Natural state	17 $\frac{3}{4}$	17 $\frac{3}{4}$	17
	Distended state	17 $\frac{1}{2}$	17 $\frac{1}{2}$	17 $\frac{1}{2}$
The eye of a man 50 years old, 20 minutes after death - -	Natural state	19	19	18 $\frac{1}{2}$
	Distended state	19	19	18 $\frac{1}{2}$

From these experiments it appears, that the diameters of the eye do not always bear the same proportion; sometimes the transverse diameter is the longest, in other eyes it is of the same length as the axis of vision; but when the coats are distended, the transverse diameter is diminished, and the axis of vision is lengthened.

This change, however, does not take place at all ages, for at 50 it was not met with.

In these experiments the pressure was made in the most unfavourable way for producing the greatest degree of elongation in the axis of vision; it was, however, the least exceptionable mode for ascertaining that such an effect could take place; when the pressure is made laterally and from without, the elongation must be still greater; and the action of the

straight muscles is the most advantageous that could be imagined for that purpose.

This lateral pressure will not only elongate the eye, and increase the convexity of the cornea, but it will produce an effect upon the crystalline lens and ciliary processes, pushing them forward in the same proportion as the cornea is stretched. This is necessary for two reasons; *viz.* to preserve the cavity containing the aqueous humour always of the same size, and to keep the cornea and lens at the same distance from each other. The ciliary processes, as they form a complete septum between the vitreous and aqueous humours, must be moved forward, together with the lens, when the cornea is rendered more convex, and when the cornea recovers itself they are thrown back into their former situation. In order to effect this with the nicety that is required, the ciliary processes are probably possessed of a muscular power.

That the ciliary processes are muscular is a very generally received opinion, and in the course of this lecture I shall adduce some facts in favour of it; they will also tend to confirm the opinion of these processes being a sling, in which the lens is suspended, and rendered capable of a small degree of motion.

The result of this inquiry, which has not been confined to the support of any particular theory, but carried on with the sole view of discovering the truth, appears to be, that the adjustment of the eye is produced by three different changes in that organ; an increase of curvature in the cornea, an elongation of the axis of vision, and a motion of the crystalline lens. These changes in a great measure depend upon the contraction of the four straight muscles of the eye.

Mr. RAMSDEN has been good enough to make a computa-

tion, by which the degree of adjustment produced by each of these changes may be ascertained. This he has promised to render more correct; and also to institute a series of experiments by which the effects of the motion of the lens may be more accurately determined. From Mr. RAMSDEN's computation, the increase of curvature of the cornea appears capable of producing one-third of the effect; and the change of place of the lens, and elongation of the axis of vision, sufficiently account for the other two-thirds of the quantity of adjustment necessary to make up the whole.

Having explained the mode by which the axis of vision can be elongated, and the convexity of the cornea increased, in the human eye, for the purpose of its adjustment, I was desirous of applying these observations to the eyes of other animals, that I might see whether their different structures would admit of the necessary changes, for producing an adjustment to different distances in the same way.

As many animals are known to have their vision distinct at very different distances, it appeared that much information might be gained by examining the structure of the eyes of those whose range of vision varies most from that of the human eye.

Quadrupeds in general must have their eyes fitted to see very near objects, as many of them collect their food with their mouths, in which action the objects are brought very close to the eye. Birds are under the same circumstances in a still greater degree with respect to their food; but from their mode of life, they also require the power of seeing objects at a great distance. Fishes, from the nature of the medium in which they live, must have some other mode of adjusting the eye,

than that of a change in the cornea, as that substance is possessed of the same refractive power with the surrounding fluid.

To avoid confusion in so extensive a field of inquiry, I shall separately consider the peculiarities in the eyes of these different classes of animals, so far as they appear to be concerned in producing the adjustment to different distances.

Quadrupeds have three modes of procuring their food; one by their fore-paws only, which they use like hands, as all the monkey tribe; the second, by their fore-paws and mouths, as the lion, and cat tribe; the third, by the mouth only, as all ruminating animals. These three different modes require the food being brought to different distances from the eye; and it is curious, that the muscles of the eye are different in all the three tribes.

In the monkey tribe, the muscles of the eye are exactly the same as in the human. In the lion tribe, they are double in number, and the four intermediate muscles are lost in the sclerotic coat, at a greater distance from the cornea than the others. In the ruminating tribe, there are four muscles, as in the human eye; but there is also a muscle surrounding the eyeball, attached to the bottom of the orbit, round the hole through which the optic nerve passes, and lost upon the sclerotic coat immediately before the broadest diameter of the globe of the eye; the upper portion of this muscle is rather the longest, its insertion being nearly in a circular line at right angles to the axis of vision, but not to the axis of the eye from the entrance of the optic nerve.

In quadrupeds in general, the ball of the eye is broader in proportion to its depth, than in the human subject; in the bull

the proportion is $1\frac{10}{16}$ inch to $1\frac{6}{16}$. The cornea is larger and more prominent; its real thickness is hardly to be determined, since, as well as that of the human eye, it readily imbibes moisture immediately after death. When dried, it is thinner than the sclerotic coat in the same state. In ruminating animals, it appears externally of an oval form; it is not, however, really so, the cornea itself being circular, as in other animals; but a portion of it is rendered opaque, by a membrane which covers its external surface, and produces an oval appearance. This circular form of cornea is necessary, that when it is stretched it may form a regular curve.

The ciliary processes, as in the human eye, are connected with the choroide coat; but they are larger, and are united at their origin with the iris.

This structure of the eye in quadrupeds, so far as it differs from that of the human eye, appears calculated to increase the power of adjusting it to see near objects, and from the mode of life which these animals pursue, such additional powers appear necesssary to enable them with ease to procure their food.

Birds in general procure their food by means of their beak; and the distance between the eye and the point of the beak is so small, that they must have a power of seeing very near objects. From living in air, and moving through it with great velocity, they require for their own defence, as well as to assist them in procuring food, a power of seeing at great distances.

That birds of prey see objects distinctly at a great distance appears to be proved by the following observations. In the year 1778, Mr. BABER and several other gentlemen were upon a hunting party in the island of Cassimbusar in Bengal, about

15 miles north of the city of Marshedabad; they killed a wild hog of an uncommon size, and left it upon the ground near their tent. About an hour after it was killed they were walking near the spot where it lay; the sky was perfectly clear, not a cloud to be seen, and a dark spot in the air at a great distance attracted their notice; it appeared gradually to increase in size, and moved directly towards them: as it advanced it proved to be a vulture, flying in a direct line to the dead animal, on which it alighted, and began to feed voraciously. In less than an hour, 70 other vultures came in all directions, some horizontally, but most of them from the upper regions of the air, in which a few minutes before nothing could be seen. Mr. BABER was so much struck with the circumstance at the moment, that he said to his friends, MILTON's poetical description of the vulture, being lured to its prey by the smell, would not apply to what they had just seen.

VOLNEY, in his travels through Egypt, mentions a circumstance somewhat similar. He says, "the conspicuous situation of Aleppo brings numbers of birds thither, and affords the curious a very singular amusement: if you go after dinner on the terraces of the houses, and make a motion as if throwing bread, numerous flocks of birds will fly instantly around you, though at first you cannot discover one; but they are floating aloft in the air, and descend in a moment to seize in their flight the morsels of bread which the inhabitants frequently amuse themselves with throwing to them."* This account of VOLNEY is confirmed by my friend Dr. RUSSEL, who has furnished me with an additional fact upon this sub-

* VOLNEY, English Translation, Vol. II. chap. 27, page 154.

ject. Dr. Russel says, that the relation of Volney is true; and that it is the amusement of the inhabitants, or rather of the Europeans, to allure birds by throwing up pieces of bread from the flat tops of the houses; these birds, to the best of his recollection, are the common gull *(larus canus* Linn.), which appear only at certain seasons.

But a fact more to the purpose of the present inquiry, is what Dr. Russel remembers often to have heard asserted by the European sportsmen at Aleppo, and indeed sometimes observed himself; namely, that in the most serene weather, when not a speck could be seen in the sky, nor any object discovered in the horizon of an extensive plain, a dog or other animal killed by accident, or shot, and left behind by the sportsmen as they traversed the country, in the space of a few minutes was surrounded by birds, before invisible, either of the vulture tribe or the sea eagles (*ossifragus* Linn.). Whether these birds by vision were directed to their prey, or allured by scent, he would not undertake to pronounce, but the phænomenon occasioned wonder; and the more so, as there was not time for putrefaction to take place, which might be supposed to diffuse scent to a great distance.

The eyes of birds are larger in proportion than those of any other animal, the eye of a thrush being equal to that of a rabbit. They are also broader in proportion to their depth than in the quadruped; and the cornea is more prominent.

The cornea is very thin when examined immediately after death, and is at that time more elastic than afterwards. In the goose, it was stretched so as to be elongated $\frac{2}{20}$ of an inch, but in an hour afterwards it had become thicker, and less elastic. The cornea is not united to the sclerotic coat by the

terminating of one abruptly in the other; but the two edges are bevilled off, and laid over each other for nearly one-tenth of an inch in the eye of the goose, and more where the eye is larger. In the recent state, the thin edge of the cornea is readily torn off from the inner surface of the sclerotic coat to which it adheres, so as to show this mode of union. This circumstance was known to HALLER, and is particularly described in his works.

There is a bony rim surrounding the basis of the cornea in the eyes of birds, which is peculiar to this class of animals. It is made up of a number of different parts, very commonly 13 in number; some of these are lapped over each other, but some have an irregular union, one part passing before, and the other behind the bony scale next to it. This bony circle, thus made up, is not equally broad in its different parts; it is broadest where it covers the upper and outer part of the eye, and narrowest where it covers the cornea towards the inner canthus.

This bony rim does not give an origin to the cornea, as might appear to a superficial observer, but is a bony hoop laid over the junction between the sclerotic coat and cornea; and as the thin edge of the cornea passes within the sclerotic coat, the principal attachment of the bony rim must be to that coat. The bony rim is adapted to the surface upon which it lies; the greatest part of its breadth is firmly connected to the sclerotic coat; and where the cornea projects, the anterior edge of the rim is turned forwards to correspond with that projection; here the scales are extremely thin, they terminate in a fine edge, and admit of being forced a little asunder, to adapt them to the stretched state of the cornea; but no such effect

can be produced upon the posterior part of the rim, the different parts being too firmly connected to admit of any separation.

The structure of this bony rim differs in different birds. In the goose and turkey the scales are thin and weak; in the cassuary they are thicker; and in the eagle they are very strong. In the owl, they put on a very different appearance; they are 15 in number, $\frac{6}{10}$ of an inch long, and instead of being lapped over one another, as in other birds, they are united by indented sutures; each portion is broadest next the sclerotic coat, and narrowest towards the cornea, giving the bony rim a conical form.*

This structure in the owl's eye differs from that in other birds, the anterior edge not admitting of being dilated to correspond with the change of figure in the cornea; this purpose in the owl is answered by a circular elastic ligament firmly attached to the anterior edge of the bony rim, and lying upon the outside of the basis of the cornea; there is a similar ligament in other birds, but less conspicuous.

This bony rim in the eyes of birds is particularly noticed by Haller; specimens of it, whole and in separate parts, are preserved in Mr. Hunter's collection; it has been also described by Mr. Smith, in a paper read before this Society: I shall, therefore, not dwell longer upon its structure, as it is not to my present purpose to take further notice of it than to explain its use respecting the adjustment of the eye, the subject of the present lecture.

The straight muscles of the eye in birds arise from the bottom of the bony orbit, as in the quadruped, and are firmly

* See the annexed plate.

attached to the posterior edge of the bony rim just described; they are four in number.

The ciliary processes are larger and longer in birds, than in other animals whose eyes are of the same size; they are evidently continued from the choroide coat, and adhere firmly to the capsula of the crystalline lens.

In the eyes of birds there is a substance which is peculiar to that class of animals, called the marsupium. It is a process composed of a corrugated vascular membrane attached to the centre of the retina, where the optic nerve terminates. Its origin is in a straight line, extending from the termination of the optic nerve to the lower part of the eye; in the turkey $\frac{5}{20}$ of an inch in length, and connected with the bottom of the eye by an elastic ligament about $\frac{1}{40}$ of an inch thick. The number of folds of which it is composed varies in different birds, from 5 to 15, or more; they are all of the same length, which in the turkey is about $\frac{4}{20}$ of an inch; they are covered with the nigrum pigmentum, and are attached anteriorly to the capsula of the crystalline lens, either immediately, as in the goose, or by intermediate membrane, as in the turkey.*

The structure of the marsupium is very similar to that of the ciliary processes, but stronger in all its parts, and like them it has a connection with the crystalline lens.

The connection between the marsupium and lens, in a natural state of the parts, is from its transparency invisible; but in the goose and cassuary, where the marsupium extends to the capsula of the lens, if the parts are coagulated in spirits, it becomes very apparent, and in these birds such a connection is generally allowed. In other birds, it is doubted by some, and

* *Vide* plate.

denied by others, who have written upon the subject. HALLER has taken some pains upon this point: he found, that by pulling the marsupium the motion was communicated to the lens, but he was unable to make out the mode of union; and all his attempts to coagulate the cells of the vitreous humour were unsuccessful; he says, no spirits can produce such a change. I have found, however, that, after the eye has remained a few days in rectified spirits, the medium between the marsupium and lens is coagulated, and rendered visible. By this means I have detected it in the turkey's eye; it is connected to the whole anterior extremity of the marsupium, extends to the capsula of the lens, aud appears to be about one half the length of the marsupium itself.

The union has been supposed to be extremely weak, because after death it readily gives way; this, however, is by no means the case, for when it is coagulated in rectified spirits, it is not easily torn; and the reason of its giving way in the dead eye, is probably from dissolution readily taking place when surrounded by moisture.

The anterior edge of the marsupium in some birds is narrower than its base, as in the cassuary; in others, it is of the same extent, as in the turkey; and in all, I believe, it is an uniform line; but when it is separated from the lens the folds contract irregularly, and appear of different lengths. In the eagle the marsupium is uncommonly strong.

From the similarity of structure in the marsupium and ciliary processes, as also their connection with the crystalline lens, I was desirous of ascertaining whether the marsupium was possessed of any muscular power, as this would determine

the same point with respect to the ciliary processes, and might lead to an explanation of the use of both these parts.

With this view I made the following experiments. The marsupium and crystalline lens of a goose's eye were exposed immediately after death; and the lens was pushed forwards, by which means the marsupium was elongated, and measured $\frac{5}{20}$ of an inch; upon taking off the pressure, it again contracted to $\frac{7}{40}$; this was repeated several times. The parts were then left, till it was supposed that all remains of life were gone, and the same experiment was repeated. In the stretched state it measured as before, $\frac{5}{20}$ of an inch, but in the contracted state, $\frac{4}{20}$; this change arose from the elasticity of the ligament connecting the marsupium to the bottom of the eye; and therefore the contraction of $\frac{3}{40}$, which was now lost, must have arisen from some other cause.

The result of this experiment favours the idea, that the marsupium possesses a muscular power, but in matters where we are so liable to be deceived, it seemed not a sufficient proof; I therefore made several other experiments, but they were all liable to some objections; the following, however, appears satisfactory, and shows that there is a power of contraction in the marsupium independent of elasticity.

The crystalline lens of a turkey's eye was extracted, and immediately afterwards the turkey was killed, by wounding the spinal marrow; the two eyes were taken out, and put into spirits.* In the one, the marsupium had nothing to pre-

* In the act of dying, the muscles are found to contract to their utmost, where there is no resistance to prevent such action; this is also found to take place in the greatest degree, when the animal is killed by any violence committed upon the brain, or spinal marrow.

vent its contracting to the utmost; while in the other, the lens being in its natural situation, could not allow of any unusual contraction. Some days after, the two eyes were examined; in the perfect eye the marsupium measured $\frac{4}{20}$ of an inch, and the different folds of it were semitransparent; in the imperfect eye the marsupium measured $\frac{3}{20}$ of an inch, and the folds were much more opaque. Here, then, was a difference of $\frac{1}{20}$ of an inch in the length of the two marsupia; which could arise from no other cause than the one having contracted so much more than the other, which contraction we must consider as muscular.

HALLER denies the marsupium to be muscular, because there is no such appearance in its structure. My own opinions upon the structure of muscles have been already explained to this learned Society; and I have lately met with an observation in LYONET's dissection of a caterpillar which tends to confirm them. He says, the muscles of the caterpillar are, in their natural state, transparent as jelly, and have vessels passing through their substance in every direction, which afford to the eye of the observer in the microscope the most beautiful appearance of a congeries of vessels.*

The peculiarities in the bird's eye are such as tend to facilitate both the lengthening of the axis of vision, and increasing the convexity of the cornea.

* "Les muscles des chenilles, dans leur état naturel, ils sont mous, ils prêtent extrêmement, ils ont la transparence d'une gelée, ils sont d'un gris bleuâtre, et les bronches argentées, ou vaisseaux aëriens, qu'on voit alors distinctement ramper par dessus, et penetrer dans toute leur substance, offrent à la loupe un spectacle qu'on ne se lasse point d'admirer."—*Traité Anatomique de la Chenille, par* PIERRE LYONET, chap. 6, page 92.

The bony rim, to which the muscles are attached, confines the effect of their pressure to the broadest part of the eye; and as their action throws forwards the cornea, the anterior edge of the bony rim yields, to adapt itself to that change; the ciliary processes are long, to admit of the lens being moved forwards, and by their action bring it back to its place; by these means the eyes of birds are adjusted to see very near objects with more facility than the eyes of other animals.

As the eyes of birds are likewise to be adjusted to see very distant objects, the marsupium is placed behind the crystalline lens, to draw it backwards, and when it acts, part of the pressure from behind being removed, the cornea is rendered flatter; and the anterior edge of the bony rim is adapted to it, in this state, by the contraction of the annular elastic ligament.

It may be said, that to see with parallel rays no such great change is necessary; it must, however, be considered that where vision is to be very distinct, a certain nicety of adjustment becomes necessary, and the action of the marsupium is probably intended for that purpose.

In the bird (although not immediately connected with the present subject) there is one of the most beautiful illustrations of the combination of muscular and elastic substances. This is employed for the motion of the membrana nictitans, and as it shews that such a combination is adopted wherever it can be used with advantage, and is provided as a defence for the organ in which I am endeavouring to explain such a combination, I cannot avoid taking notice of it. The membrana nictitans is composed of an elastic membrane, which is connected by means of a tendon, with two muscles situated

upon the posterior part of the eyeball; the action of these muscles brings the membrane over the cornea, and the instant they cease to contract, the elasticity of the membrane draws it back again.

The eyes of fishes have several peculiarities, and in many respects their structure differs from that which is observed in the quadruped and bird.

The muscles of the eye, that correspond to the straight muscles in the quadruped, are four in number, they are, however, differently placed; they do not surround the eyeball; but two of them are on that side of the orbit next to the nose of the fish, the other two on the opposite side; their attachment to the eye is close to the edge of the cornea; they do not, however, pass round the eyeball towards the posterior part, as in other animals, but are connected with the bones of the head at some distance from the eye on each side; so that they cannot at all compress the eye laterally, they can only pull it backwards by the combined effect of their action.

The bottom of the orbit on which the eyeball rests, is solid, and adapted to it, there being no fat interposed between them as in other animals; and where the eye is removed to a great distance from the skull, and that cannot be the case, there is a strong cartilage projecting from the skull to the bottom of the eye, and that end of it next to the eye is concave, and fitted to the portion of the eyeball directly opposite the cornea, just above the entrance of the optic nerve. This is considered as a fixed point upon which the eye moves, but it will also, from the situation of the muscles, allow the eye to be forced back upon it, and the whole eye to be flattened.

The shape of the eye differs considerably in different fishes,

but in all of them the transverse diameter is the longest. In the haddock, the proportion is $\frac{11}{10}$ths to $\frac{8}{10}$ths of an inch, and in some fishes it differs much more.

The size of the eye does not correspond with that of the fish; the salmon's eye being smaller than the haddock's.

The sclerotic coat is in some fishes membranous;* in some partly bone,† in others entirely so,‡ but in general the posterior part is membranous, although the lateral parts are bone.§

The cornea is in general flat, not always circular in its shape, is very thin, made up of laminæ, and does not lose its transparency in spirits, appearing like talc.|| In others it is more convex, as in fish of prey; this appears to adapt it to the spherical crystalline lens, which in them lies directly behind it.** The tunica conjunctiva forms the anterior layer of the cornea,†† and in some fishes is quite detached.

In the eel there is a transparent horny convex covering, at some distance before the eye, to defend it from external accidents. This covering, to an eye fitted to see in air, would entirely take off the effects arising from change of figure in the cornea; but in water, where no such change could be attended with advantage, such a covering is employed as an external defence.

In the eyes of fishes, the ciliary processes are entirely wanting. The crystalline lens is spherical, and imbedded in the vitreous humour, which is inclosed in cells of a stronger texture than in other animals.

The iris does not admit of motion; this is taken notice of by

* Haddock. † Sword-fish. ‡ Devil fish. § Mackerel.
|| Sword-fish. ** Pike. †† Haddock.

HALLER; and the reason probably is, that the light in water is never too strong for the eye to bear.

There is a muscle situated between the retina and the sclerotic coat, which is, I believe, common to all fish. This muscle is particularly described by HALLER; and its use is stated to be that of bringing the retina nearer the crystalline lens, for the purpose of seeing objects at a greater distance. Mr. HUNTER called it the choroide muscle, and has preserved several preparations of it.

This muscle has a tendinous centre round the optic nerve, at which part it is attached to the sclerotic coat; the muscular fibres are short, and go off from the central tendon in all directions; the shape of the muscle is nearly that of a horseshoe; anteriorly it is attached to the choroide coat, and by means of that to the sclerotic. Its action tends evidently to bring the retina forwards; and in general, the optic nerve in fishes makes a bend where it enters the eye, to admit of this motion without the nerve being stretched.

In those fishes that have the sclerotic coat completely covered with bone, the whole adjustment to great distances must be produced by the action of the choroide muscle; but in the others, which are by far the greater number, this effect will be much assisted by the action of the straight muscles pulling the eyeball against the socket, and compressing the posterior part; which, as it is the only membranous part in many fishes, would appear to be formed so for that purpose.

In fishes, the eye in its natural easy state appears to be adjusted to near objects, requiring some change to adapt it to see distant ones; in this respect differing entirely from the bird, the quadruped, and the human.

As the change which the eye is to undergo is different, so are the parts which produce it. The cornea, in many fishes, is neither circular, prominent, nor elastic, and the ciliary processes are wanting. The straight muscles pass off in different directions, to prevent the eye from being pressed upon laterally; the coats of the eye at that part are bony, in some fishes, to prevent the same effect; and the bottom of the orbit, which in other animals is filled with fat and loose cellular membrane, has no such covering, but is a hard concave surface, to give resistance, and assist in flattening the eye.

From the preceding observations, deduced from the structure of the eye in different animals, it appears that there are two modes of adjusting the eye, one for seeing in air, the other for seeing in water; and it is probably the want of this knowledge that has misled former inquirers, by confining their researches to the discovery of some one principle common to the eyes of all animals.

The crystalline lens, as the most conspicuous part, engrossed their whole attention, and they did not think any of the others capable of giving material assistance in producing so curious an effect.

The ciliary processes, from their connection with the lens, were by some believed capable of bringing it forwards; by others they were supposed to contract, and by that action elongate the eye, and remove the lens further from the retina: but these processes could never bring the lens forwards, unless the cornea was also moved forwards; for the lens and processes forming a complete septum, the aqueous humour would prevent the lens from making any advance in that direction: and the processes themselves are neither strong enough in

their muscular power, nor sufficiently attached to the coats of the eye, to alter its form by their contraction. In birds likewise, the bony rim renders this impossible.

That the axis of vision is really lengthened, and the lens moved forwards, for the purpose of adjusting the eye to see near objects, is rendered highly probable, since all the facts I have been able to collect seem to point out these changes; nor can the action of the external muscles increase the curvature of the cornea without producing them.

If the axis of vision being lengthened was believed by some physiologists to produce the whole adjustment of the eye to see near objects; if the crystalline lens being moved forwards was supposed by others to do the same thing; and if the cornea being rendered more convex appeared at the first view equally to account for it; all the three, when combined for that purpose, must undoubtedly be considered as sufficient to produce the effect.

EXPLANATION OF THE PLATE. (Tab. I.)

Fig. 1. A side view of the cornea of the eye of a goose, to show the bony rim, and elastic annular ligament, in their natural situation; *a* the bony rim; *b* the elastic ligament.

Fig. 2. A view of the same parts, in the eye of the great horned owl, to show the difference of structure; taken from a dried preparation in Mr. HUNTER's collection.*

* Since this lecture was read before the Royal Society, Sir JOSEPH BANKS has put into my hands a paper upon the anatomical structure of the eye, in which there is a plate, containing four views of the bony rim in the owl's eye. The parts they re-

Fig. 3. The marsupium in the eye of the turkey, attached to the bottom of the eye, and connected by a transparent membranous union with the crystalline lens; made visible by coagulation in rectified spirits.

Fig. 4. The marsupium in the eye of the emeu, from New South Wales, with a portion of the membrane that connects it to the lens; the marsupium is drawn together at that end next the lens, giving it the appearance of a purse, from which it probably got the name marsupium.

Fig. 5. and 6. Two views of the crystalline lens of the eye of a goose, to show the attachment of the marsupium to the lens.

These different drawings are of the natural size of the parts they represent.

present are exactly similar to those shown in the second figure; and had the paper been published in this country, would have rendered it unnecessary.

The paper is intituled *Esposizione Anatomica delle parti relative all' Encefalo degli Uccelli, del Sig.* Vincenzo Malacarne; it is published in the Italian Transactions, called *Memorie di Matematica e Fisica della Società Italiana,* Tomo VII. Verona, 1794.

Fig: 1.

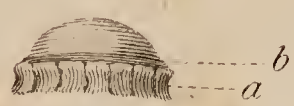


Fig: 2.

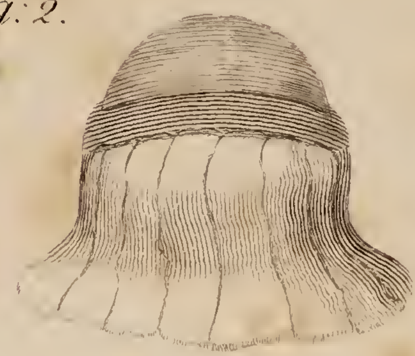


Fig: 3.

Fig: 4.

Fig: 5.

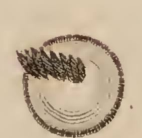


Fig: 6.

Basire del. et Sc.

II. *Some Particulars in the Anatomy of a Whale. By Mr.* John Abernethy. *Communicated by* Everard Home, *Esq. F. R. S.*

Read November 26, 1795.

There are some particulars in the anatomy of the whale, which, I believe, have either entirely escaped observation, or have not been as yet communicated to the public. The parts which in the whale correspond in situation and office with the mesenteric glands of other animals, differ considerably from those glands in structure. These peculiarities are not only curious in themselves, but are illustrative of circumstances, hitherto esteemed obscure, in the anatomy and œconomy of the lymphatic glands in general. I therefore take the liberty of submitting the following account of them to the inspection of this learned Society.

The animal, from which the parts that I am going to describe were taken, was a male, of the genus named by Linnæus balæna.

Being desirous of making an anatomical preparation, to shew the distribution of the mesenteric vessels and lacteals of the whale, I procured for this purpose a broad portion of the mesentery with the annexed intestine; and proceeded in the first place to inject the blood vessels. The mesentery had been cut from the animal as close to the spine as possible:

had a less portion been taken away, the parts which I am about to describe would have been left with the body, for they are situated upon the origin of the blood vessels belonging to the intestines; and this, perhaps, is the reason why they have not been observed before.

When I threw a red-coloured waxen injection into the mesenteric artery, I saw it meandering in the ramifications of that vessel; but at the same time I observed it collecting in several separate heaps, about the root of the mesentery, which soon increased to the size of eggs. At the time, I imagined that the vessels had been ruptured, and that the injection in consequence had become extravasated; but I was conscious that no improper degree of force had been used in propelling the injection.

I next threw some yellow injection into the vein, when similar phænomena occurred; the branches of the vein were filled, but at the same time the masses of wax near the root of the mesentery were increased by a further effusion of the injection. These lumps had now acquired a spherical form, and some of them were of the size of an orange.

After the injection had become cold, I cut into the mesentery, in order to remove these balls of wax; when I found that they were contained in bags, in which I also observed a slimy and bloody-coloured fluid. On the inner surface of these bags a great number of small arteries and veins terminated; from the mouths of which the injection had poured into their cavities. There were seven of these bags in that piece of mesentery which I had to examine; but I am not able to determine what number belonged to the animal; for I do not know whether the portion of mesentery that I possessed

was complete. Having removed the injection from these bags, I observed on the inside of them a soft whitish substance, apparently containing a plexus of lacteal vessels. This substance entered the bags at that part of them which was nearest to the intestines, and went out at the part next to the spine. I now poured some quicksilver into those lacteals which appeared to lead to this soft substance: the quicksilver soon entered the vessels which were contained in it, and thus its nature was ascertained. A number of lacteals having entered one of these bags, were observed to communicate with each other, then again to separate, and form other vessels, which went out of the bag. It was some time before the quicksilver passed through the plexus of vessels contained in the first bag; but after having pervaded it, it passed on to a second bag, in which was concealed a similar plexus of lacteals. The quicksilver permeated these last vessels with much greater facility than it did the former, and quickly ran out of the large lacteals which were divided at the origin of the mesentery. Besides those absorbents which passed through the bags in the manner described, there were great numbers of others, which terminated by open orifices in every part of them. When quicksilver was poured into any of the lacteals, which were found near the sides of the bags, it immediately ran in a stream into their cavities. I introduced about a dozen bristles through as many lacteals, into different parts of two of these bags. These were doubtless few, in comparison to the whole number which terminated in them, but as the mesentery was fat, and the vessels were small, more could not easily be passed.

I afterwards stuffed two of the bags with horse-hair, dried them, and preserved them as an anatomical preparation. In

this state great numbers of arteries and veins, but chiefly of the former vessels, are seen terminating on their inside, in the same indistinct manner as the foramina Thebesii appear when the cavities of the heart are laid open: the bristles also render visible the termination of a certain number of lacteals. I examined the sides of these bags, which were moderately thick and firm; but I did not see any thing which, from its appearance, I could call a muscular structure.

From the circumstances that have been related, it appears, that in the whale there are two ways by which the chyle can pass from the intestines into the thoracic duct; one of these is through those lacteals, which pour the absorbed chyle into bags, in which it receives an addition of animal fluids. The other passage for the chyle is through those lacteals which form a plexus on the inside of the bags; through these vessels it passes with some difficulty, on account of their communications with each other; and it is conveyed by them to the thoracic duct, in the same state that it was when first imbibed from the intestines. The lacteals, which pour the chyle into the bags, are similar to those which terminate in the cells of the mesenteric glands of other animals: there is also an analogy between the distribution of the lacteals on the inside of these bags, and that which we sometimes observe on the outside of the lymphatic glands in general. In either case, a certain number of the vasa inferentia, as they are termed, communicate with one another, and with other vessels, named vasa efferentia.

By this communication, the progress of the fluids contained in these vessels is in some degree checked; which impediment increases the effusion into the cavities of the gland made by

the other lacteals: but should these cavities be obstructed, from disease, or other causes, an increased determination of fluids into the communicating absorbents must happen, which would overcome the resistance produced by their mutual inosculations, and the contents of the vessels would be driven forwards towards the trunk of the system. In the whale, as in other animals, we find that the impediment, occasioned by this communication of lacteals, is greatest in the first glands at which they arrive after having left the intestines.

The ready termination of so many arteries in the mesenteric glands of the whale, makes it appear probable, that there is a copious secretion of fluids mixed with the absorbed chyle; and, as I have before observed, a slimy bloody-coloured fluid was found in them. As the orifices of the veins were open, it appears probable that the contents of the bags might pass in some degree into those vessels.

The eminent anatomists, Albinus, Meckel, Hewson, and Wrisberg, were of opinion, that the lymphatic glands were not cellular, but were composed of convoluted absorbing vessels. This notion seems, however, to have been gradually declining.

Mr. Cruikshank has of late publicly maintained a contrary opinion; and has shewn, that the cells of these glands have transverse communications with each other; which it is not likely they would have, if they were only the sections of convoluted vessels. Some additional observations have occurred to me, confirming this opinion, and which, as I believe they have not been publicly noticed by others, I beg leave to relate to this Society. I have injected the lymphatic glands of the groin and axilla of horses, with wax, and afterwards destroyed

the animal substance, by immersing them in muriatic acid. In some of these glands the wax appeared in very small portions, and irregularly conjoined; which is a convincing proof, that it had acquired this irregular form from having been impelled into numerous minute cells. But in several instances, I found one solid lump of wax, after the destruction of the animal substance: and it appears to me sufficiently clear, that the glands which were filled in this manner, were formed internally of one cavity, and were not, as is commonly the case, composed of many minute cells. I have also filled glands of this structure, in the mesentery of an horse, with quicksilver: I have then dried them, cut open the bags, and introduced a bristle into them through the vas inferens. And in the human mesentery, after having injected the artery, I have filled a bag resembling a gland, with quicksilver; which being opened, a mixture of injection and quicksilver was found in its cavity.

That the lymphatic glands in most animals are cellular, may not, perhaps, be hereafter doubted: that they are sometimes mere bags, analogy and actual observation induce me to believe. It might be said, that in those instances which I have related, the cells were burst, or that the glands were diseased: to which I can only reply, that there was no appearance to lead me to such a conclusion.

If, then, the lymphatic glands are either cellular, or receptacles resembling bags for the absorbed fluids, we are naturally led to inquire, what advantage arises from this temporary effusion of the contents of the absorbents. That there is a considerable quantity of fluids poured forth from the arteries of the whale, to mix with the absorbed chyle, is very

evident; nor can it be doubted that the same thing happens in other animals; for the cells of the lymphatic glands are easily inflated, and injected from the arteries.

The ready communication of these bags with the veins of the whale, induced me to examine whether I could ascertain any thing similar in other animals. Air impelled into the lymphatic glands, however, seldom gets into the veins; sometimes indeed veins are injected from these glands; but when this has occurred to me, I have observed an absorbent arising from the gland, and terminating in the adjacent vein.

These remarks, perhaps, may not be very important; such, however, is the nature of the subject, that all the knowledge we have hitherto obtained of the absorbing vessels has been acquired by fragments, and all our future acquisitions must be made in the same manner: I have wished, therefore, by offering these observations, to contribute my mite to the general stock of our knowledge of this subject.

III. *An Account of the late Discovery of Native Gold in* Ireland. *In a Letter from* John Lloyd, *Esq. F. R. S. to Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read November 19, 1795.

Cronbane Lodge, near Rathdrum,
the 4th of November, 1795.

DEAR SIR,

THE late very important mineralogical discovery in Ireland, and a desire I had long entertained of visiting the celebrated copper mine at this place, together with the opportunity that presented itself, of making my tour in company with our friend Mr. MILLS, who is one of the proprietors, as well as sole director of the mine, determined me to seize this moment for my excursion; and yesterday Mr. MILLS and I visited the spot, where so much pure gold has been of late taken up, being distant about 5 miles from this place.

About 7 miles westward of Arklow, in the county of Wicklow, there is a very high hill, perhaps 6 or 700 yards above the sea, called Croughan Kinshēlly, one of whose NE abutments, or buttresses, is called Balinnagore, to which the ascent may be made in half or three quarters of an hour. Should you have JACOB NEVILL's map of the county of Wicklow, published in 1760, at hand, by casting your eye on the river Ovo, which runs by Arklow, at about 4 miles above the latter place, you

will perceive the conflux of two considerable streams, and of a third about half a mile higher up, close to a bridge. By tracing this last to its source, you will come to a place, set down in the map Ballinvally; this is a ravine between two others, that run down the side of the hill into a semicircular, or more properly, semi-elliptical valley, which extends in breadth from one summit to the other of the boundary of the valley, and across the valley three-quarters of a mile, or somewhat less. The hollow side of the hill forms the termination of the valley, and down which run the three ravines abovementioned. At their junction, the brook assumes the name of Ballinasloge; at this place the descent is not very rapid, and so continues a hanging level for about a quarter of a mile, or somewhat more, when the valley grows narrower, and the sides of the brook become steeper; and it should seem, that some rocky bars across the course of the brook have formed the gravelly beds, above, over, and through which the stream flows, and in which the gold is found. The bed of the brook, and the adjacent banks of gravel, on each side, for near a quarter of a mile in length, and for 20 or 30 yards in breadth, have been entirely stirred and washed by the peasants of the country, who amounted to many hundreds, at work at a time, whilst they were permitted to search for the metal.

A gentleman, who saw them at work, told me, he counted above 300 women at one time, besides great numbers of men and children.

The stream runs down to the NE from the hill, which seems to consist of a mass of shistus and quartz; for on examination of the principal ravine, which is now washed clean by the late heavy rains, the bottom consisted of shistus, intersected at

different distances, and in various places, by veins of quartz, and of which substances the gravelly beds at the bottom, where the gold is found, seem to consist.

Large tumblers of quartz are thickly scattered over the surface of the top of the hill, under a turbary of considerable thickness, upon the removal of which these tumblers appear.

I shall not take up your time in attempting to give a minute geological description of this part of the country, as I have prevailed with Mr. Mills (who from his minute examinations, and practical knowledge, is so conversant with the mineralogy of this county), to undertake that task, which I am persuaded he will perform to your satisfaction.

The gold has been found in masses of all sizes, from those of small grains to that of a piece of the weight of 5 ounces, which beautiful specimen is intended for the cabinet of a nobleman, adored in this country, and not less respected by his friends in England, and which, I dare to say, you will shortly have an opportunity of seeing in London. One piece of 22 ounces has been taken up, and which, I am told, is to be presented to his Majesty.

In our visit to this extraordinary place, we were most hospitably entertained by Mr. Graham, of Ballycoage, whose house is not more than a mile from the gold mine: from him and his brothers I learnt, that about 25 years ago, or more, one Dunaghoo, a schoolmaster, resident near the place, used frequently to entertain them with accounts of the richness of the valley in gold; and that this man used to go in the night, and break of day, to search for the treasure; and these gentlemen, with their schoolfellows, used to watch the old man in his excursions to the hill, to frighten him, deeming him to be

deranged in his intellects; however, the idea of this treasure did at last actually derange him.

JOHN BYRNE told me, that about 11 or 12 years ago, when he was a boy, he was fishing in this brook, and found a piece of gold, of a quarter of an ounce, which was sold in Dublin; but that, upon one of his brothers telling him it must have been dropped into the brook by accident, he gave over all thoughts of searching for more. CHARLES TOOLE, a miner at Cronbane, tells me, he heard of this discovery at the time, but gave no credit to it, as he never found any gold, and lives very near the place. I am credibly informed too, that a goldsmith in Dublin has, every year, for 11 or 12 years, bought 4 or 5 ounces of gold, brought constantly by the same person, but not JOHN BYRNE.

Thus, Sir, you have all I could learn respecting this important event; which is at your service to lay before the Royal Society, should you not have been furnished with an account from an abler pen.

I am, &c.

JOHN LLOYD.

P.S. I am told the name of the brook, where the gold is found, is, in Irish, *Aughatinavought*.

IV. *A mineralogical Account of the Native Gold lately discovered in* Ireland. *In a Letter from* Abraham Mills, *Esq. to Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read December 17, 1795.

Cronebane Copper Mines, near Rathdrum,
SIR, Nov. 21, 1795.

THE extraordinary circumstance of native gold being found in this vicinity, early excited my attention, and led me to seize the first opportunity that presented itself, after my late arrival here, to inspect the place where the discovery was made.

I went thither on Tuesday, the 3d of this month, with Mr. LLOYD, of Havodŷnos, and Mr. WEAVER. The former having given you some account of the circumstances which attended the original discovery, and, since he left me, a favourable day having enabled me to take a second view of the adjacent country, I shall now attempt to describe the general appearance, and add such further information as has come to my knowledge.

The workings which the peasantry recently undertook, are on the north-east side of the mountain Croughan Kinshēlly, within the barony of Arklow, and county of Wicklow, on the lands of the Earl of CARYSFORT, wherein the Earl of ORMOND claims a right to the minerals, in consequence (as I have been informed), of a grant in the reign of King HENRY the Second,

by Prince JOHN, during his command of his father's forces in Ireland; which grant was renewed and confirmed by Queen ELIZABETH, and again by King CHARLES the Second.

The summit of the mountain is the boundary between the counties of Wicklow and Wexford; seven English miles west from Arklow, ten to the south-westward of Rathdrum, and six south-westerly from Cronebane mines; by estimation about six hundred yards above the level of the sea. It extends W by N and E by S, and stretches away to the north-eastward, to Ballycoage, where shafts have formerly been sunk, and some copper and magnetic iron ore has been found; and thence to the NE there extends a tract of mineral country, eight miles in length, running through the lands of Ballymurtagh, Ballygahan, Tigrony, Cronebane, Connery, and Kilmacoe, in all which veins of copper ore are found; and terminating at the slate quarry at Balnabarny.

On the highest part of the mountain are bare rocks, being a variety of argillite,* whose joints range NNE and SSW, hade to the SSW, and in one part include a rib of quartz, three inches wide, which follows the direction of the strata. Around the rocks, for some distance, is sound ground, covered with heath; descending to the eastward, there is springy ground, abounding with coarse grass; and below that, a very extensive bog, in which the turf is from four to nine feet thick, and beneath it, in the substratum of clay, are many angular fragments of quartz, containing chlorite, and ferruginous earth. Below the turbary the ground falls with a quick descent, and three ravines are observed. The central one, which is the most considerable, has been worn by torrents, which derive their source from the bog; the others are formed lower down

* KIRWAN, Edit. 1794, p. 234.

the mountain by springs, which uniting with the former, below their junction the gold has been found. The smaller have not water sufficient to wash away the incumbent clay, so as to lay bare the substratum ; and their beds only contain gravel, consisting of quartz with chlorite, and other substances of which the mountain consists. The great ravine presents a more interesting aspect ; the water in its descent has, in a very short distance from the bog, entirely carried off the clay, and considerably worn down the substrata of rock, which it has laid open to inspection.

Descending along the bed of the great ravine, whose general course is to the eastward, a yellow argillaceous shistus is first seen ; the laminæ are much shattered, are very thin, have a slight hade to the SSW, and range ESE and WNW. Included within the shist, is a vein of compact barren quartz, about three feet wide, ranging NE and SW ; below this is another vein, about nine inches wide, having the same range as the former, and hading to the northward, consisting of quartz, including ferruginous earth. Lower down, is a vein of a compact aggregate substance, apparently compounded of quartz, ochraceous earth, chert, minute particles of mica, and some little argillite, of unknown breadth, ranging E and W, hading fast to the southward, and including strings of quartz, from one to two inches thick, the quartz containing ferruginous earth. The yellow argillaceous shistus is again seen with its former hade and range ; and then, adjacent to a quartz vein, is laminated blue argillaceous shistus, ranging NE and SW, and hading SE ; which is afterwards seen varying its range and hade, running ENE and WSW, and hading NNW ; lower down, the blue shist is observed more compact, though still

laminated. The ground, less steep, becomes springy, is inclosed, and the ravine, shallower, has deposited a considerable quantity of clay, sand, and gravel. Following the course of the ravine, or, as it may now more properly be called, the brook, arrive at the road which leads to Arklow; here is a ford, and the brook has the Irish name of *Aughatinavought* (the river that drowned the old man); hence it descends to the Aughrim river, just above its confluence with that from Rathdrum, which, after their junction, take the general name of the Ovo, that discharging itself into the sea near the town of Arklow, forms an harbour for vessels of small burthen.

The lands of Ballinvally are to the southward, and the lands of Ballinagore to the northward, of the ford, where the blue shistus rock, whose joints are nearly vertical, is seen ranging ENE and WSW, including small strings of quartz, which contain ferruginous earth. The same kind of earth is also seen in the quartz, contained in a vein from ten to twelve inches wide, ranging ENE and WSW, and hading to the southward, which has been laid open in forming the Arklow road.

Here the valley is from twenty to thirty yards in width, and is covered with substances washed down from the mountain, which on the sides have accumulated to the depth of about twelve feet. A thin stratum of vegetable soil lies uppermost; then clay, mingled with fine sand, composed of small particles of quartz, mica, and shist; beneath which the same substances are larger, and constitute a bed of gravel, that also contains nodules of fine grained iron stone, which produces 50 *per cent.* of crude iron: incumbent on the rock are large tumblers of quartz, a variety of argillite and shistus; many pieces of the quartz are perfectly pure, others are attached to the shistus,

others contain chlorite, pyrites, mica, and ferruginous earth; and the arsenical cubical pyrites frequently occurs, imbedded in the blue shistus. In this mass of matter, before the workings began, the brook had formed its channel down to the surface of the rock, and between six and seven feet wide, but in times of floods extended itself entirely over the valley.

Researches have been made for the gold, amidst the sand and gravel along the run of the brook, for near half a mile in length; but it is only about one hundred and fifty yards above, and about two hundred yards below the ford, that the trials have been attended with much success: within that space, the valley is tolerably level, and the banks of the brook have not more than five feet of sand and gravel above the rock; added to this, it takes a small turn to the southward, and, consequently, the rude surfaces of the shistus rock in some degree cross its course, and form natural impediments to the particles of gold being carried further down the stream, which still lower has a more rapid descent; besides, the rude manner in which the country people worked, seldom enabled them to penetrate to the rock, in those places where the sand and gravel were of any material depth. Their method was, to turn the course of the water wherever they deemed necessary, and then, with any instruments they could procure, to dig holes down to the rock, and by washing, in bowls and sieves, the sand and gravel they threw out, to separate the particles of gold which it contained; and from the slovenly and hasty way in which their operations were performed, much gold most probably escaped their search; and that indeed actually appears to have been the case, for since the late rains washed the clay and gravel which had been thrown up, gold has been

found lying on the surface. The situation of the place, and the constant command of water, do, however, very clearly point out the great facility with which the gold might be separated from the trash, by adopting the mode of working practised at the best managed tin stream works in the county of Cornwall; that is, entirely to remove (by machinery) the whole cover off the rock, and then wash it in proper buddles and sieves. And by thus continuing the operations, constantly advancing in the ravine towards the mountain, as long as gold should be found, the vein that forms its matrix might probably be laid bare.

The discovery was made public, and the workings began, early in the month of September last, and continued till the 15th of October, when a party of the Kildare militia arrived, and took possession by order of government; and the great concourse of people, who were busily engaged in endeavouring to procure a share of the treasure, immediately desisted from their labour, and peaceably retired.

Calculations have been made, that during the foregoing period, gold to the amount of three thousand pounds Irish sterling was sold to various persons; the average price was three pounds fifteen shillings *per* ounce; hence eight hundred ounces appear to have been collected within the short space of six weeks.

The gold is of a bright yellow colour, perfectly malleable; the specific gravity of an apparently clean piece 19,000. A specimen, assayed here by Mr. WEAVER, in the moist way, produced from 24 grains, $22\frac{58}{101}$ grains of pure gold, and $1\frac{43}{101}$ of silver. Some of the gold is intimately blended with, and adherent to quartz; some (it is said) was found united to the

fine grained iron stone, but the major part was entirely free from the matrix; every piece more or less rounded on the edges, of various weights, forms, and sizes, from the most minute particle up to 2*oz.* 17*dwt.*; only two pieces are known to have been found of superior weight, and one of those is 5, and the other 22 ounces.

I much regret not having been present when the work was going on, that I might have seen the gold as found, before prepared for sale by breaking off any extraneous matter that adhered; for in that state, a proper attention to the substances with which it was united, and a subsequent diligent inspection of the several veins that range through the mountain, might assist towards the discovery of that from whence it was detached.

I shall shortly return to England; and on my arrival, will send specimens of the gold, and of the different substances of the mountain, to be deposited (if you think proper) in the collection of the Royal Society.

And am, with great respect, &c.

ABRAHAM MILLS.

The bearings are all taken by the compass, without allowing for the variation.

BESIDES these accounts of the gold found in Ireland, the following information has been received on that subject.

William Molesworth, Esq. of Dublin, in a letter to Richard Molesworth, Esq. F. R. S. writes, that he weighed the largest piece of gold in his balance, both in air and water;

North

SKETCH of the GOLD MINE five Miles N.W. from Arklow in the County of WICKLOW.

South

Boyer sc.

that its weight was 20*oz.* 2*dwt.* 21*gr.* and its specific gravity, to that of sterling gold, as 12 to 18. Also that RICHARD KIRWAN, Esq. F. R. S. found the specific gravity of another specimen to be as 13 to 18. Hence, as the gold was worth £ 4 an ounce, Mr. WILLIAM MOLESWORTH concludes, that the specimens are full of pores and cavities, which increase their bulk, and that there are some extraneous substances, such as dirt or clay, contained in those cavities.

This opinion was discovered to be well founded, by cutting through some of the small lumps.

STANESBY ALCHORNE, Esq. his Majesty's Assay-master at the Tower of London, assayed two specimens of this native gold. The first appeared to contain, in 24 carats,

$21\frac{6}{8}$ of fine gold;
$1\frac{7}{8}$ of fine silver;
$\frac{3}{8}$ of alloy, which seemed to be copper tinged with a little iron.

The second specimen differed only in holding $21\frac{5}{8}$ instead of $21\frac{6}{8}$ of fine gold.

Major JOHN BROWN, of the royal engineers, transmitted to the Right Hon. THOMAS PELHAM a sketch of the spot where the gold was found, which Mr. PELHAM has obligingly permitted to be engraved, for the use of the Royal Society. See Tab. II.

C. B.

V. *The Construction and Analysis of geometrical Propositions, determining the Positions assumed by homogeneal Bodies which float freely, and at rest, on a Fluid's Surface; also determining the Stability of Ships, and of other floating Bodies. By* George Atwood, *Esq. F. R. S.*

Read February 18, 1796.

To investigate the positions assumed by homogeneal bodies which float freely, and at rest, on a fluid's surface, it is necessary, in the first place, to form a just conception of the several principles on which those positions depend.

The proportion of the immersed part to the whole magnitude of a floating body* will always be obtained, from having given the specific gravity of the solid in respect to that of the fluid; since it is a known law of hydrostatics, that the immersed part of the solid is to the whole magnitude, in the proportion of those specific gravities. But a solid may be immersed in a fluid numberless different ways, so that the part immersed shall be to the whole magnitude in the given proportion of the specific gravities, and yet the solid shall not rest permanently in any of these positions. The reasons are obvious. The floating body is impelled downward by its weight, acting in the direction of a vertical line, which passes through the centre of gravity; the pressure of the fluid, by

* In these pages the floating bodies are always understood to be homogeneal, unless the contrary be mentioned.

which the solid is supported, acts upward, in the direction of a vertical line (usually called the line of support), which passes through the centre of gravity of the part immersed: unless, therefore, these two lines are coincident, so that the two centres of gravity shall be in the same vertical line, it is evident that the solid thus impelled, must revolve on an axis until it finds a position in which the equilibrium of floating will be permanent.

From these observations it appears, that to ascertain the positions in which a solid body floats permanently on the surface of a fluid, it is requisite that the specific gravity of the floating body should be known, in order to fix the proportion of the part immersed to the whole: secondly, it is necessary to determine, by geometrical or analytical methods, in what positions the solid can be placed on the surface of the fluid, so that the centre of gravity of the floating body, and that of the part immersed, may be situated in the same vertical line, while a given proportion of the whole volume is immersed under the fluid's surface.

These particulars having been determined, evidently reduce the statement of the problem into a narrow compass; but they are not alone sufficient to limit it: for although it has been shewn that a body cannot float permanently on a fluid unless the two centres of gravity, that have been mentioned, are situated in the same vertical line, it does not follow that, whenever those centres of gravity are so situated, the solid will float permanently in that position:* consistently

* Admitting any proposition to be true, the converse of the proposition may be either true generally, or with exceptions. To distinguish the cases in which it is true from those in which it fails, requires a separate demonstration or investigation.

with this observation, positions may be assigned, in which a solid is immersed in a fluid to the true depth according to its specific gravity, and the centre of gravity of the solid and that of the part immersed are in the same vertical line, yet the solid does not rest in any of these positions, but assumes some other in which it will continue permanently to float. To make this evident, a very obvious instance may be referred to. Suppose a cylinder, the specific gravity of which is to that of a fluid on which it floats as 3 to 4; and let the axis of the cylinder be to the diameter of the base as 2 to 1: if this cylinder is placed on the fluid with its axis vertical, it will sink to a depth equal to a diameter and a half of the base; and as long as the axis is sustained in a vertical position by external force, the centre of gravity of the solid, and the centre of the immersed part, will be situated in the same vertical line: but the solid will not float permanently in that position; for as soon as external support is removed, it falls from its upright position, and remains floating with the axis horizontal. If the axis of the cylinder is made only $\frac{1}{2}$ instead of twice the diameter of the base, the solid being placed with its axis vertical, will sink to the depth of $\frac{3}{8}$ of a diameter, and will float permanently in that position. Even if the axis should be placed not exactly coincident with the vertical, but in a direction somewhat inclined to that line, the solid will change its position until it settles permanently with the axis perpendicular to the horizon.

The cylinder here instanced is caused either to float permanently with its axis vertical, or to overset, according to the different proportions between the length of the axis and the diameter of the base: although an exact estimate of the effects

produced by altering these proportions, cannot be obtained except by mathematical investigation (a subject to be considered in some of the following pages), yet a general idea of the causes by which so remarkable a difference is occasioned in the floating position of the two cylinders, will appear obvious by attending to the changes which take place in the position of the line of support, while the solid is inclined from the upright through a small angle. For whenever the line of support, in the direction of which the force of the fluid's pressure acts, does not pass through the centre of gravity of the floating body, that force must generate a motion of rotation round an horizontal axis which passes through the centre of gravity of the solid; and must cause an elevation of those parts of the solid which are on the same side of the axis of motion with the line of support, and consequently must depress those parts which are situated on the contrary side of that axis. Admitting, therefore, that the solid is adjusted with its centre of gravity and the centre of the immersed part precisely to the same vertical line, and that a small inclination takes place round the axis of motion; it will depend on the position of the line of support, whether that inclination shall be counteracted, so as to restore the solid to its upright position, or shall be augmented; in which latter case the solid oversets. If the nature of the figure should be such as causes the line of support to be moved toward those parts which are immersed by the inclination, that inclination will be counteracted, because the pressure of the fluid generates angular motion in a direction contrary to that in which the solid is inclined; but if the figure is such as causes the line of support to be moved toward those parts of the solid which

are elevated by the inclination, the force of the fluid's pressure must continually augment the inclination ; or, in other words, will cause the solid to overset, or change its position, until it settles in some other, in which the equilibrium is permanent.

We observe, therefore, that a solid floats permanently in a given position, only because the smallest inclination from that position creates a force by which the inclination is immediately counteracted, and the solid becomes restored to its upright position ; and consequently, since the inclination is counteracted while of evanescent magnitude, no sensible deviation from the upright can take place : in cases of instability, the solid oversets, although placed on a fluid with the centre of gravity of the solid and that of the part immersed in the same vertical line, because the smallest deviation or inclination from that position creates a force by which the inclination is augmented. And since various causes concur in preventing the two centres from remaining adjusted to the vertical with a precision absolutely mathematical, it follows that the least or evanescent inclination here mentioned must necessarily subsist, and being continually augmented by the fluid's pressure, must become a sensible rotation, by which the solid oversets from its upright position.

In either case, that is, whether the solid floats permanently, or oversets, if it is placed on the surface of a fluid, so that the centre of gravity of the solid and the centre of gravity of the part immersed shall be in the same vertical line, the solid is said to be in a position of equilibrium : and from the preceding observations it appears, that there are three species of equilibrium in which a solid may be situated when the two centres of gravity just mentioned are in the same vertical line.

1st.* The equilibrium of stability, in which the solid floats permanently in a given position.

2dly. The equilibrium of instability, in which case the solid, although its centre of gravity and that of the part immersed are in the same vertical line, spontaneously oversets, unless sustained by external force. This kind of equilibrium is similar to that which subsists when a needle, or other sharp-pointed body, is placed vertically on a smooth horizontal surface.

3dly. The third species, being a limit between the two former, is called the equilibrium of indifference, or the insensible equilibrium, in which the solid rests on the fluid indifferent to motion, without tendency to right itself when inclined, or to incline itself further.

These different kinds of equilibrium may perhaps be more clearly perceived, by referring to the instance in which a cylinder was supposed to be placed on the surface of a fluid with the axis vertical. If the axis is assumed double the diameter of the base, the solid oversets, the equilibrium of position being that of instability: but if the length of the axis is only half the diameter of the base, the solid floats permanently with the axis vertical. It seems evident, therefore, that there must be some intermediate proportion between the cylinder's axis and the diameter of the base, greater than 1 to 2, and less than 2 to 1, which will correspond to the case intermediate, where stability ceases, and instability begins: this is the precise proportion when the equilibrium is of the species called the equilibrium of indifference, or the insensible equilibrium.

When a solid body floats permanently on the surface of a

* EULER. *Théorie complette de la Construction et Manœuvre des Vaisseaux,* chap. iv.

fluid, and external force is applied to incline it from its position, the resistance opposed to this inclination is termed the stability of floating. It is obvious to every one's experience, that some floating bodies are more easily inclined from their quiescent position than others; that, after having been inclined, some will return to their original situation with more force and celerity than others; a difference particularly observable in ships at sea, in some of which a given impulse of the wind will cause a much greater inclination from the perpendicular than in others. As this property of opposing resistance to heeling or pitching, when regulated to its due quantity and proportion, has been deemed of material consequence in the construction of vessels, several eminent mathematicians have been induced to investigate rules, by which the stability of ships may be inferred, independently of any reference to trial, from knowing their weights and dimensions only. It must, however, be acknowledged, that the theorems which have been given on this subject, in the works of Mons. BOUGUER,* EULER,† FRED. CHAPMAN,‡ and other writers, for determining the stability of ships, are founded on a supposition that the inclinations from their quiescent positions are evanescent, or, in a practical sense, very small. But as ships at sea are known to heel through angles of 10°, 20°, or even 30°, a doubt may arise how far the rules demonstrated on the express condition, that the angles of inclination are of evanescent magnitude, should be admitted as practically applicable in cases where the inclinations are so great.

* BOUGUER. Liv. i. sec. iii. chap. iv.

† EULER. *Théorie complette de la Construction et Manœuvre des Vaisseaux,* chap. iv. and chap. v.

‡ *Traité de la Construction des Vaisseaux par* FRED. CHAPMAN, chap. ii. p. 17.

To put this matter in a clear point of view, let a case be assumed. Suppose two vessels to be of the same weight and dimensions in every respect, except that the sides of one of these vessels shall project more than those of the other, the projections commencing from the line coincident with the water's surface. According to the theorems of Bouguer and other writers, the stability will be the same in both ships, which is in fact true, on the supposition that their inclinations from the perpendicular are extremely small angles: but when the ships heel to 15° or 20°, the stabilities of the two vessels must evidently be very different. Even supposing the stability of a ship A to be greater than that of a ship B, when the angles of heeling are very small, it may happen in cases easily supposable that when both ships are heeled to a considerable angle of inclination, the stability of the ship B shall exceed that of the ship A. Admitting, therefore, that the theory of statics can be applied with any effect to the practice of naval architecture, it seems to be necessary that the rules investigated for determining the stability of vessels should be extended to those cases in which the angles of inclination are of any magnitude likely to occur in the practice of navigation.

When a solid is placed on the surface of a lighter fluid, at the proper depth corresponding to the relative gravities, it cannot change its position by the combined actions of its weight and the fluid's pressure, except by revolving on some horizontal axis which passes through the centre of gravity. Various axes may be drawn through the centre of gravity of a floating body in a direction parallel to the horizon: but since the motion of the solid respecting one axis only, can be the subject of the same investigation (except in extreme cases

not to be considered in this place), the figure of the floating body, and the particular object of inquiry, must determine to which of these axes the motion of the solid is to be referred, when it changes its position: thus, suppose a square beam of timber, the specific gravity of which is to that of water as 1 to 2, should be placed on the surface of that fluid with one of the flat surfaces parallel to the horizon (the length being assumed considerably greater than the breadth), no motion of rotation can take place round the transverse axis, by which the extremities of the beam would be elevated or depressed: but the solid will spontaneously revolve in this instance round the longer axis, changing its position until it settles with an angle upward.

In like manner, if the same solid should be placed horizontally on the surface of the water with an angle upward, it will not spontaneously change its position; but if one extremity of the beam should be forcibly elevated, and the other depressed, so as to incline the longer axis to the horizon, as soon as all external force is removed, the beam will revolve on a transverse horizontal axis, passing through the centre of gravity, and perpendicular to the longer axis, until it settles in such a position as to leave the longer axis horizontal. These are instances in which the figure of the body, and the particular nature of the case, determine the axis round which the solid revolves, while it changes its situation on a fluid's surface; this axis is called, for the sake of distinction, the axis of motion.

The axis of motion, round which the solid revolves, having been determined, and the specific gravity being known, it appears from the preceding observations, that the positions of permanent floating will be obtained, first by finding the

several positions of equilibrium through which the solid may be conceived to pass, while it revolves round the axis of motion; and secondly, by determining in which of those positions the equilibrium is permanent, and in which of them it is momentary and unstable.

In proceeding to investigate the principles which are the objects of the present inquiry, it will be convenient in the first instance to consider the floating body to be some homogeneal solid of regular figure, and uniform shape and dimensions, in respect of the axis of motion throughout. If such a solid is supposed to be cut through by vertical planes in a direction perpendicular to the axis of motion, the sections of these planes with the solid will be areas precisely equal and similar. Let EDHF (Tab. III. fig. 1.) represent the vertical section of such a solid, which passes through the centre of gravity G in a direction perpendicular to the axis of motion. The solid floats on the surface of a fluid IABL; consequently ADHB represents the part immersed under the fluid's surface; O is the centre of gravity of the part immersed, and the line GOC is assumed perpendicular to the horizontal line AB. We are in the next place to suppose that this solid is inclined round the axis of motion from its former position through an angle KGS (fig. 2.);* so that the line KC which was before ver-

* When this inclination takes place, the centre of gravity G, through which the axis of motion passes, is not necessarily fixed, but must evidently in most cases change its place, since the total volume immersed before the inclination is always equal to that which is immersed after the inclination; and from this cause such change of place ensues: but the motion of the axis, and of the point G, is wholly independent of the reasoning in this and the subsequent constructions and investigations; the object of which is to ascertain the angular motion round the said axis, and the several consequences thereof, and is no ways connected with the motion of the axis itself. This note

tical, may be now transferred to the position SL, which is inclined to the vertical line KC at the angle KGS: moreover the line AB, which was before horizontal, is transferred so as to coincide with the line IN, being inclined to its former position in the angle NXP, which is equal to KGS: and consequently the whole space ADHB, becomes transferred so as to coincide with the space IRMN, and the volume immersed under the fluid's surface is WRMNP. If in the line SL, GE is taken equal to GO; it is evident that in consequence of the inclination, the point O, which is the centre of gravity of the space ADHB, will be transferred to the point E, which is the centre of gravity of the equal space IRMN; and the pressure of the fluid would act on the solid in the direction of a vertical line passing through the point E, if the space IRMN was the volume immersed under the fluid's surface; but in consequence of the inclination of the solid through the angle KGS, the volume NXP, which was before above the fluid's surface, will now become immersed under it; and the volume IWX, which was before under the surface, will become elevated above it. It is evident, that on both these accounts, that is, both by the addition of the volume NXP, and the abstraction of the volume IWX, the centre of gravity E of the space IRMN will be transferred towards those parts of the solid which have become more immersed under the fluid in consequence of the inclination.

Suppose the centre of gravity of the volume immersed, WRMP, to be situated at the point Q: through Q draw

is here inserted in preference to adapting the construction so as to express the alteration in the position of the axis, which would only have the effect of embarrassing the construction with useless lines.

QS parallel to GO; through E draw EY perpendicular to SQ; and through G draw zGZ perpendicular to SQ. Then, since the point Q is the centre of gravity of the part immersed, the pressure of the fluid will act in the direction of the vertical line QS, with a force equal to the body's weight, and by the principles of mechanics will have precisely the same effect to turn the solid round its axis as if the same force was applied immediately at the point Z, acting in the same direction QS. Since, therefore, the effect of the fluid's pressure acting in the direction of a vertical line which passes through the centre of gravity Q, no way depends on the absolute position of that point, but on the perpendicular distance GZ, between the two vertical lines GO and SQ only, in proceeding to ascertain, by geometrical construction, the several positions which bodies assume on a fluid's surface, and their stability of floating, the determination of the absolute position of the point Q, or centre of gravity of the immersed part, will not be necessary; the perpendicular distance GZ between the two vertical lines which pass through the centres of gravity of the solid, and of the part immersed, being sufficient for obtaining all the results that are required.

The part immersed, before the inclination of the solid took place, is ADHB; when the solid has been inclined through the angle KGS, the part immersed is WRMP, which is the volume IRMN diminished by the space IWX, and augmented by the space NXP. But since the volume immersed under the fluid's surface must always be of the same magnitude while the solid's weight continues unaltered, it follows, that whatever additional space is immersed under the surface in consequence of the inclination, an equal space must be ele-

vated above it; consequently, whatever may be the position of the point of intersection X, the volume IXW must be equal to the volume PXN. Suppose *a* to be the centre of gravity of the space IXW, and let *d* be the centre of gravity of the space NXP; then, the part immersed WRMP, is equal to the space IRMN, diminished by the space IWX, considered as concentered in the point *a*, and increased by the equal space NXP, concentered in the point *d;* consequently the centre of gravity Q of the space WRMP will be at such a distance from E, the centre of gravity of the space IRMN, as corresponds to the alteration occasioned by removing the volume IWX, concentered in the point *a*, to the point *d*. These are the data from which the perpendicular distance GZ, of the two vertical lines KO, SQ, passing through the centres of gravity G and O, is to be obtained in the manner following: through the centres of gravity *a* and *b*, draw the lines *ab*, *dc*, perpendicular to the horizontal line AB; through E draw the indefinite line EY parallel to AB, and in the line EY, take a part ET, so that ET shall be to the line *bc* as the volume IWX, or its equal NXP, is to the whole volume immersed, WRMP or ADHB: through the point T thus found, draw the line FTS parallel to the vertical line GO; the centre of gravity Q, of the immersed part, will be somewhere in the line FS; and because ER is to EG, as the sine of the given angle of inclination is to radius, the line GO = EG being supposed given, the line ER will therefore be known, and being subtracted from the line ET before found, will leave RT or GZ the perpendicular distance between the two vertical lines, which it was required to determine by geometrical construction, and which has been accordingly determined.

The demonstration of this construction is founded on an obvious and elementary principle of mechanics.—It is this.—The common centre of gravity of any system of bodies (considered as heavy points or centres of gravity), being given in position, if one of these bodies should be moved from its place, the corresponding motion of the common centre of gravity, estimated in any given direction, will be to the motion of the aforesaid body, estimated in the same direction, as the weight of the body moved is to the weight of the whole system. To apply this proposition. The volume IRMN (fig. 2.) may be assumed as a system of bodies, of which the common centre of gravity is E. One of the bodies composing this system, namely, the volume I W X, concentered in the point *a*, is transferred in consequence of the inclination of the solid through the angle S G K from the point *a* to the point *d*, in which the equal volume N X P is concentered: this will have the effect of moving the common centre of gravity of the system E. But it is required to find how much the position of this centre E has been changed in the direction EY parallel to A B, which is the given direction stated in the proposition. The motion of the centre of gravity *a*, from *a* to *d*, estimated in the given horizontal direction, is *bc*: then, according to the mechanical proposition, as the volume W R M P or A D H B is to the volume I W X or N X P, so is the line *bc* to ET, the corresponding motion of the centre of gravity E estimated in the given horizontal direction; consequently if a line FTS is drawn through the point T parallel to the vertical line GO, the centre of gravity of the immersed part Q must be situated somewhere in the line FTS: subtracting from ET the line E R (which is the sine of the given

angle of inclination EGO when EO is the radius), there will remain the line RT or GZ, which is therefore the distance between the vertical lines GO, SZT, passing through the centres of gravity G and Q, as determined by the construction.

Let the whole volume of the immersed part of the solid be denoted by the letter V; suppose the space NXP, or volume immersed in consequence of the inclination, to be A; make GO $= d$; and the sine of the angle of inclination KGS (to radius 1) $= s$; also make $bc = b$. Then since by the proposition; as b : ET : : V : A, it appears that ET $= \frac{b \times A}{V}$;

And since as ER : EG = GO : : so is s : 1, we obtain ER $= ds$;

Wherefore RT = ET — ER $= \frac{bA}{V} - ds =$ GZ.

This result is founded on a supposition that the figure of the floating solid is uniform in respect of the axis of motion; if the solid should be of an irregular form, the construction and demonstration will be precisely the same as in the preceding case, the following particulars being attended to; the volume, or space immersed in consequence of the inclination, will no longer be represented by the area NXP, but must be obtained by a calculation founded on the shape and dimensions of the said volume; moreover the centres of gravity of the volumes PXN, IXW, will not now correspond with the centres of gravity of the areas PXN, IXW, and must therefore be obtained from the known rules, or from methods of approximation by which the position of the centre of gravity is determined in solid bodies.

The angle of inclination KGS is given by the supposition, and the solid contents of the equal volumes denoted by IXW, NXP, with the distance bc of the centres of gravity a and d,

estimated in the direction of the horizontal line AB, having been determined, let the volume NXP be put $= A$; and $bc = b$; the other quantities signifying as before; the perpendicular distance $GZ = \frac{bA}{V} - ds$, will be known. It is to be observed, that this proposition in general is equally applicable to heterogeneal bodies as to those which are homogeneal.

By this proposition the stability of vessels, and other bodies floating on a fluid's surface, at any angle of inclination, from a given position of equilibrium, is obtained. For the measure of the stability is precisely a force equal to the fluid's pressure; that is, equal to the vessel's weight,* applied perpendicularly at the distance GZ from the axis of motion, to incline the solid round that axis.

From the same proposition, the different positions assumed by bodies which float freely on a fluid's surface, may be ascertained; in some cases most easily by geometrical construction; in others, by analytical investigation. It has been already observed, that to ascertain the various positions in which a body will float permanently on the surface of a fluid, it is necessary first, to have given the ratio of the specific gravities, in order to fix the proportion of the part immersed to the whole; and secondly, the several positions are to be ascertained in which the solid may rest on the surface of a fluid, so that the centres of gravity of the solid and of the part immersed may be in the same vertical line. The general expression for the line RT (fig. 2.) or GZ, is $GZ = \frac{bA}{V} - ds$; by putting this quantity $\frac{bA}{V} - ds = 0$, an equation arises, from which one or more values of s will be obtained $=$ the sine of the angle through which the solid has been inclined from a position of

* The weight of a vessel implies the weight of the ship and lading.

equilibrium, when the line $GZ = o$; that is, when the two centres of gravity, G and Q, are again situated in the same vertical line; or in other words, when the solid is again in a position of equilibrium. By this method of proceeding, the several positions of equilibrium may be determined; it only, therefore, remains to discover in which of these positions the equilibrium is permanent, and in which of them it is momentary and unstable. This circumstance will depend on the species of equilibrium in which the solid is originally placed previously to the inclination, which, for the sake of more clearly stating the principles of stability, may be supposed known, although the rules for ascertaining this point have not yet been considered, but will appear in the pages which next follow. Assuming then the species of equilibrium, in which a solid is originally placed on the surface of a fluid, to be known, let that equilibrium be supposed permanent, or the equilibrium of stability; and let the solid be conceived to be inclined round the axis of motion, through a given angle A, till it becomes situated again in a position of equilibrium; in which case the centres of gravity of the solid, and of the part immersed, will again be in the same vertical line. Since during this inclination, the fluid's pressure acts with a force proportional to the line RT or GZ, (fig. 2.) to diminish the angular distance from the original position of equilibrium, it follows that the same force must act on the solid, so as to augment the inclination, or angular distance from the second position of equilibrium, in which the solid is situated after it has revolved through the entire angle A, or any part thereof, from its original situation; from which observations it is evident, that the second position of equilibrium must be that of instability:* and by the same

* It appears from the observations in page 49, that whenever a solid floats in a posi-

mode of argument it is shewn, that if the original position of equilibrium be that of instability, when the solid by revolving on its axis has become situated in a second position of equilibrium, it will float permanently, that is with stability, in that second position. And in general, when a floating solid revolves round a given horizontal axis, and passes through several positions of equilibrium, those of stability and instability are alternate, no position of either species following immediately a position of the same species. In order, therefore, to find what position a solid will assume after it has overset from any situation of unstable equilibrium, it is only necessary to ascertain the angle of inclination from the given situation through which the solid must revolve on the axis of motion, so that the distance GZ (fig. 2.) between the two vertical lines which pass through the centre of gravity of the solid and the centre of gravity of the part immersed may become evanescent. It is necessary in the next place, to determine whether any position of equilibrium originally given is that of stability or instability. This point will be ascertained by having recourse to the general value which has been investigated, for expressing the distance between the two vertical lines GO, ST (fig. 2.); or $GZ = \frac{Ab}{V} - ds$. In the line ER take any point t, and through t draw qtz parallel to GO. As long as $\frac{bA}{V} = ET$ is greater than $ds = ER$, the point Z, and the line of support QZ, will be between the axis and those parts of the solid which are immersed by the inclination,

tion of permanent equilibrium, and is deflected from that position through a small angle, the force of the fluid's pressure causes the solid to revolve round its axis in a direction contrary to the inclination; and if the equilibrium is unstable, the same force acts to increase the said inclination; this latter case corresponds to that of the equilibrium in which the solid is situated after it has revolved through the angle *A*.

the consequence of which is an equilibrium of stability; and whenever $\frac{bA}{V} = ET$ is less than $ds = ER$, the point q, and the line of support qz, will be on the contrary side of the axis, causing an equilibrium of instability to take place.* The equation, therefore, $GZ = \frac{bA}{V} - ds$, applied to any particular case, will always decide whether the equilibrium in which a solid is placed on the surface of a fluid is stable and permanent, or whether it is only momentary and unstable, provided the value of s, or the sine of the angle of inclination from the given position of equilibrium, be assumed evanescent; since the solid either continues to float permanently, or will overset, according to circumstances which take place while it is inclined from its position of equilibrium through the smallest angle. The application of the condition just mentioned will cause the general expression to assume a form suited to this particular case, which is in the next place to be attended to.

Referring to (fig. 2.), ADHB represents a vertical section of a floating body, passing in a direction perpendicular to the axis of motion; suppose another section to be drawn parallel to the former, and extremely near to it; these two planes will comprehend between them a small portion of the solid; and since according to the conditions of the case, the angle of inclination KGS, or NXB, is evanescent, the sine of this angle (which has been denoted by the letter s) will also become evanescent; and since the space or volume immersed in consequence of the inclination, that is NXP, is equal to the volume elevated above the surface IXW, and the angles NXP, IXW, are vertical; the point of intersection of the lines IN and AB, that is, the point X will bisect the line AB, and the

* Page 49, and page 50.

points P, B, and N, will coincide; on which account the evanescent area NXP will be $= \frac{\overline{XB}^2 \times s}{2} = \frac{\overline{AB}^2 \times s}{8}$; and if z is put to represent a line drawn through the middle of the solid, on a level with the fluid's surface, and parallel to the longer axis, the evanescent portion of the solid intercepted between the two adjacent planes, will be $\frac{\overline{AB}^2 \times s}{8} \times \dot{z}$: the perpendicular distance of the centre of gravity of this evanescent solid from the point X, is $\frac{1}{3}$ AB. But it is required in the present instance to assign the distance from the horizontal line passing through the point X, of the centre of gravity of the entire volume immersed by the inclination, that is, the common centre of gravity of all the evanescent solids $\frac{\overline{AB}^2 \times s \dot{z}}{8}$ corresponding to the entire length z. This distance may be obtained from the known rule of mechanics, which is, by multiplying each evanescent solid, considered as concentered in its centre of gravity, into the distance of that centre from the given line, and dividing the sum of the products by the sum of the solids; the result will be, the distance of the common centre of gravity from the horizontal line passing through the point X parallel to the axis; and since the evanescent solid corresponding to the small lineal increment $\dot{z}$ is $\frac{\overline{AB}^2 \times s \times \dot{z}}{8}$, and the distance of its centre of gravity from the point X $= \frac{2XB}{3}$ or $\frac{AB}{3}$, the product arising from multiplying the solid into the distance of its centre of gravity, from the given horizontal line passing through X, will be $\frac{\overline{AB}^3 \times s \times \dot{z}}{24}$; and the sum of all those products corresponding

to the whole length of the line z will be $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{24}$; and therefore the distance of the common centre of gravity of the volume immersed in consequence of the inclination from the horizontal line passing through the point X, is $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{24 A}$; in like manner the distance of the common centre of gravity of the volume, elevated above the surface by the inclination of the given plane, appears to be $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{24 A}$; and consequently the distance between the two centres of gravity measured on the horizontal line, or bc (fig. 2.) $= \frac{\text{fluent of } \overline{AB}^3 \times s . \times \dot{z}}{12 A}$: this value being substituted for b in the equation $GZ = \frac{bA}{V} - ds$, we obtain the following result, *i.e.* $GZ = \frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{12 V} - ds$, which is a general expression for ascertaining whether a solid, when placed on the surface of a fluid in a given position, will float permanently, or overset, the sine of the angle of inclination or s being assumed evanescent; for, when $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{12 V}$ is greater than ds, the line of support QZ (fig. 2.) will be situated between the axis of motion, and the parts of the solid which are immersed by the inclination, in which case the solid will float permanently; and when $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{12 V}$ is less than ds, the line of support passing through the point z will be on the contrary side of the axis, and according to the preceding determination (page 64) the solid will in this case overset.

Since, when the fluent of $\frac{\overline{AB}^3 s \dot{z}}{12 V}$ (fig. 2.) is greater than ds,

the solid floats permanently; and when ds is greater than $\frac{\text{fluent of } \overline{AB}^3 \, s \dot{z}}{12\,V}$, the equilibrium is that of instability; it follows that whenever $\frac{\text{fluent of } \overline{AB}^3 \, s \dot{z}}{12\,V} = ds$, by resolving the equation $\frac{\text{fluent of } \overline{AB}^3 \, \dot{z}}{12\,V} = d$, one or more limits are obtained (depending on the dimensions and specific gravity of the solid), separating the cases in which the solid floats with stability from those in which the equilibrium is momentary and unstable. The limits here obtained evidently correspond to that species of equilibrium which has been denominated insensible, or the equilibrium of indifference.

When the floating body is of uniform figure and dimensions, respecting the axis of motion, the expression here given for determining the stability or instability of floating will not involve any fluxional quantities, for in this case all the vertical sections which pass through the solid in a direction perpendicular to the axis are equal, and consequently the portions of those sections immersed under the fluid's surface are also equal; if, therefore, the area of any one of these sections immersed under the fluid's surface be denoted by the letter D, the solid contents or volume immersed, corresponding to the length of the line z, will be $D\,z$; wherefore, in the preceding expression $GZ = \frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{12\,V} - ds$, we have by substitution $V = D\,z$, and since AB is a constant or invariable quantity by the supposition, $\frac{\text{fluent of } AB^3 \, s \dot{z}}{12\,D\,z} = \frac{AB^3 \, s z}{12\,D\,z} = \frac{AB^3 \times s}{12\,D}$: finally, therefore, in the case under consideration, we obtain $GZ = \frac{\overline{AB}^3 \times s}{12\,D} - ds$.

In the subsequent pages, cases occur in which each of the preceding expressions are employed, not only to ascertain the laws of permanent and unstable equilibrium, but in developing other properties relating to the subject.

EFCD (fig. 3.) represents a vertical section of an oblong solid or parallelopiped, placed on the surface of a fluid IABK, with one of the flat surfaces upward, or the line CE or FD vertical: this solid is moveable round an horizontal axis, which passes through the centre of gravity G, perpendicular to the plane ECDF. Let it be required to determine the limits, depending on the dimensions and specific gravity of the solid, which separate the cases in which the solid will float permanently, from those in which it will overset; through the centre of gravity G draw the line SGL parallel to CE or DF: let the height of the solid $CE = c$; let the base $CD = a$; also let the specific gravity of the solid be to that of the fluid on which it floats in the proportion of n to 1, or as SN to SL; so that when it is placed on the fluid with the line SL vertical, it may sink to the depth SN; let O be the centre of gravity of the part immersed: suppose the solid to be placed on the surface of the fluid with the line SL vertical; then, since SN is the depth to which the solid sinks in the fluid, and SN is to SL as n to 1, it follows that $SN = nc$; and consequently $GO = \frac{c}{2} - \frac{nc}{2}$; the area immersed $ABCD = acn$; wherefore, to ascertain the perpendicular distance between the two verticals which pass through the centres of gravity of the solid and of the part immersed, when the solid is inclined through a very small angle, of which the sine is $= s$ to radius 1, re-

ferring to the general expression * $GZ = \frac{\overline{AB}^3 \times s}{12 D} - ds$, we obtain the following values $AB = a$, $D = acn$, $d = \frac{c - nc}{2}$, and therefore $GZ = \frac{a^3 s}{12\,acn} - \frac{s \times \overline{c - nc}}{2}$: by making the distance $GZ = 0$, we obtain an equation expressing the relation of the dimensions and specific gravity of the solid, when the equilibrium becomes insensible, that is, when the centres of gravity of the solid and of the part immersed remain in the same vertical line, however the value of s or the sine of the inclination from the upright position may be altered, provided it is always very small; making, therefore, $\frac{a^3 s}{12\,acn} = \frac{s \times \overline{c - cn}}{2}$, we have $6c^2 n^2 - 6c^2 n = - a^2$ and $n^2 - n = - \frac{a^2 s}{6c^2}$, which gives $n = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{a^2}{6c^2}}$, or $n = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{a^2}{6c^2}}$: from whence the following inference is obtained, *i.e.* in all cases whenever $\frac{a^2}{6c^2}$ is less than $\frac{1}{4}$, that is, whenever the height of the solid c bears to the base a a greater proportion than that of $\sqrt{2}$ to $\sqrt{3}$, two values may be assigned to the specific gravity of the solid, each of which will cause it to float in the insensible equilibrium: thus, suppose the height c to be to the base a in the proportion of equality: to ascertain the two limiting specific gravities, by referring to the preceding solution, and making $c = a$, we obtain $n = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{6}}$, or $n = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{6}}$, that is $n = \frac{1}{2} - .28868 = .21132$,

$$\text{or } n = \tfrac{1}{2} + .28868 = .78868.$$

* When the angle KGS in fig. 2. is evanescent, the line GZ vanishes: this being the case represented by fig. 3, the point Z coincides with the point G.

From the equation $GZ = \frac{a^3 s}{12\,acn} - \frac{s \times \overline{c - cn}}{2}$ it is inferred, that when the specific gravity of the solid is of very small value in respect to that of the fluid, because $\frac{a^3 s}{12\,acn}$ must in this case be necessarily greater than $\frac{s \times \overline{c - cn}}{2}$ the solid will float permanently with the line SL vertical, that is, with the flat surface EF parallel to the horizon. Secondly, the specific gravity .21183 causing the solid to float in the insensible equilibrium, is the limit at which the solid ceases to float with stability; if therefore the specific gravity is increased beyond .21183, and the solid is placed on the fluid with the flat surface upward, the equilibrium thus formed will be that of instability, from which the solid will be deflected into some other position in which the equilibrium is permanent. While the specific gravity is augmented from .211 to .788, the instability increases at first, but admits of a maximum, which is found by putting the least increment of the quantity $\frac{a^3 s}{12\,acn} - \frac{s \times \overline{c - cn}}{2} = 0$, considering n as a variable quantity, and making $a = c$; in which case n appears to be equal to $\frac{1}{\sqrt{6}}$. If the value of the specific gravity is increased beyond $\frac{1}{\sqrt{6}}$, the instability becomes less, and at last vanishes when the specific gravity is at its second limit $= .78868$: whatever value is given to the specific gravity between .78868 and 1, the solid will float permanently with the line SL vertical, or with its flat surface horizontal.

These cases arise from assuming the height of the parallelopiped SL, in a greater proportion to its base CD than that of

$\sqrt{2}$ to $\sqrt{3}$; and from the same solution it appears, that if the height bears a less proportion to the base than that of $\sqrt{2}$ to $\sqrt{3}$, no value can be given to the specific gravity, which will cause the stability to vanish, because the quantity $\sqrt{\frac{1}{4} - \frac{a^2}{6c^2}}$ becomes impossible; in which case the solid placed with the surface EF horizontal, must in all cases continue to float permanently in that position, whatever may be the specific gravity, always supposed to be less than that of the fluid.

(Fig. 4.) Similar determinations may be obtained from the same theorem respecting the equilibrium of the solid, when placed on a fluid with a plane angle upward, that is, with the diagonal line EGC vertical. Let EDCF represent a vertical section of a square parallelopiped floating on the surface of a fluid IABK: making the side DC $= a$, the line GC $= \frac{a}{\sqrt{2}}$, suppose that the specific gravity of the solid is to the specific gravity of the fluid as n to 1, and that the solid sinks in the fluid to the depth HC; let G be the centre of gravity of the solid, and O the centre of gravity of the part immersed; then the area ABC is to the area DEFC as n to 1; wherefore the space ABC $= \overline{HB}^2 = a^2 n$, and HB $=$ HC $= a \times \sqrt{n}$; AB $= 2a\sqrt{n}$; OC $= \frac{2a\sqrt{n}}{3}$ and GO $= \frac{a}{\sqrt{2}} - \frac{2a\sqrt{n}}{3} = \frac{a \times \overline{3 - \sqrt{8 \times n}}}{\sqrt{2} \times 3}$.

Referring to the quantity expressing the perpendicular distance between the two vertical lines passing through the centre of gravity of the solid, and the centre of gravity of the part immersed, when the angles of inclination from the

position of equilibrium, are very small, that is, $GZ = \frac{\overline{AB}^3 \times s}{12\,D} - ds$, and applying this equation to the case under consideration, we obtain the following values; $AB^3 = 8a^3 \times n^{\frac{3}{2}}$; $D = a^2 n$; $d = \frac{a \times \overline{3 - \sqrt{8n}}}{\sqrt{2} \times 3}$: making therefore $\frac{AB^3 s}{12\,D} = ds$, in order to obtain the limit, separating the cases of stability and instability of floating; or, which is the same thing, making $\frac{8a^3 n^{\frac{3}{2}}}{12a^2 n} = \frac{a \times \overline{3 - \sqrt{8n}}}{\sqrt{2} \times 3}$, the following equation arises, $\frac{2\sqrt{n}}{3} = \frac{3 - \sqrt{8n}}{\sqrt{2} \times 3}$, or $n = \frac{9}{32} = .28123 =$ the specific gravity, which will cause the solid to float in the insensible equilibrium, and is therefore the limit separating the specific gravities which cause the solid to float with stability from those which produce the equilibrium of instability. It is collected from the general equation $GZ = \frac{\overline{AB}^3 s}{12D} - ds$, or $GZ = \frac{8a^3 n^{\frac{3}{2}} s}{12a^2 n} - \frac{a \times \overline{3 - \sqrt{8n}}}{\sqrt{2} \times 3}$; that when the specific gravity (n) is evanescent or very small, the solid will overset when placed on the fluid with an angle upward, because in this case the quantity $\frac{8a\, n^{\frac{3}{2}} s}{12a^2 n}$ must necessarily be less than $\frac{a \times \overline{3 - \sqrt{8n}}}{\sqrt{2} \times 3}$, or ds. When the specific gravity of the solid is to that of the fluid in the proportion of 9 to 32, the solid floats in the insensible equilibrium; if therefore the specific gravity of the solid should be to that of the fluid in a less proportion than that of 9 to 32, the solid will overset; but if the specific gravity of the solid exceeds that limit when placed on the fluid with the angle upward, or diagonal line EC vertical, it will float permanently in that position.

Respecting this determination it seems remarkable, that there should be only one value of specific gravity, as a limit between the stability and instability of floating; whereas there were two specific gravities, each of which was a limit in the case when the solid was placed on the fluid with a flat surface upward. This difficulty admits of very satisfactory explanation; when the flat surface is placed upward, the conditions on which the solution is founded are not at all altered, to whatever depth the solid may sink: but in the present case, when the solid is placed on the fluid with a plane angle upward, the conditions on which the solution has been investigated imply, that as the specific gravity is increased, the section of the solid formed by the fluid's surface shall continually increase also; and on that ground the result justly gives one limit only between the stability and instability of floating; but since in reality the section of the solid by the fluid's surface increases only until the specific gravity becomes one half of that of the fluid, and afterwards decreases, it is evident, that if there should be another limit corresponding to the case when the specific gravity is greater than one-half, it must be discovered by a separate investigation. Let, therefore, the square parallelopiped EDCF (fig. 5.) of which the specific gravity is greater than $\frac{1}{2}$, that of the fluid being 1, be placed on the fluid with the diagonal line EC vertical: IABK represents the surface of the fluid, and HC the depth to which the solid sinks; G is the centre of gravity of the solid, and O the centre of gravity of the part immersed. If one of the sides DE is made $= a$, and the specific gravity put $= n$, then the area ABDCFA $= a^2 n$; and the area EAB $= a^2 - a^2 n = \overline{EH}^2$; wherefore EH $= a \times \sqrt{1 - n} =$ AH; AB $= 2a \times$

$\sqrt{1-n}$; and GH $= a \times \overline{\frac{1}{\sqrt{2}} - \sqrt{1-n}}$: let P represent the centre of gravity of the area AEB; then by the properties of the centre of gravity the following equation arises: GH $\times$ area EDCF $=$ area ABDCFA $\times$ OH $-$ area AEB $\times$ HP, that is

$a^3 \times \overline{\frac{1}{\sqrt{2}} - \sqrt{1-n}} = a^2 n \times \text{OH} - a^3 \times \frac{\overline{1-n}^{\frac{3}{2}}}{3}$; and consequently HO $= \frac{a \times \overline{3 - \sqrt{18} \times \sqrt{1-n}} + a \times \sqrt{2} \times \overline{1-n}^{\frac{3}{2}}}{\sqrt{18} \times n}$; from which quantity taking away the line HG $= a \times \frac{1}{\sqrt{2}} - \sqrt{1-n} = \frac{3n - \sqrt{18n^2} \times \sqrt{1-n}}{\sqrt{18} \times n}$, there will remain the line GO $=$

$$\frac{a \times \overline{3 - 3n - \sqrt{18} \times \overline{1-n}^{\frac{3}{2}} + \sqrt{2} \times \overline{1-n}^{\frac{3}{2}}}}{\sqrt{18} \times n}.$$

Referring to the general expression, namely $\frac{\overline{AB}^3 s}{12D} - ds$, we obtain in the present case $\overline{AB}^3 = 8a^3 \times \overline{1-n}^{\frac{3}{2}}$, D $= a^2 n$, GO $= d = \frac{a \times \overline{3 - 3n - \sqrt{18} \times \overline{1-n}^{\frac{3}{2}} + \sqrt{2} \times \overline{1-n}^{\frac{3}{2}}}}{\sqrt{18} \times n}$; wherefore $\frac{\overline{AB}^3}{12D} - d = \frac{8a^3 \times \overline{1-n}^{\frac{3}{2}}}{12a^2 n} - \frac{a \times \overline{3 - 3n - \sqrt{18} \times \overline{1-n}^{\frac{3}{2}} + \sqrt{2} \times \overline{1-n}|^{\frac{3}{2}}}}{\sqrt{18} \times n}$; which quantity being put equal to o, in order to obtain the limit, and the whole being multiplied by $\frac{3n \times \sqrt{2}}{\overline{1-n} \times a}$, will give $2\sqrt{2} \times \sqrt{1-n} = 3 - 3\sqrt{2} \times \sqrt{1-n} + \sqrt{2} \times \sqrt{1-n}$, or $\sqrt{1-n} = \frac{3}{4\sqrt{2}}$; wherefore $1 - n = \frac{9}{32}$ or $n = \frac{23}{32}$, the limit required.

By the preceding determinations of the four limiting values of

the specific gravity, *i.e.* $\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{6}}$, $\frac{9}{32}$, $\frac{23}{32}$, & $\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{6}}$:
or .211, .281, .718, & .789, we find that if the specific gravity is less than .211, the square parallelopiped, when placed on the surface of the fluid with a flat surface upward and horizontal, floats permanently in that position, but oversets if the specific gravity is greater than .211, and less than .789. We observe also, that when the solid is placed on the fluid with an angle upward, if the specific gravity is less than .281 it oversets; if greater than .281 and less than .718, the solid floats permanently with the angle upward; but if the specific gravity exceeds .718, the solid oversets when placed on the fluid with an angle upward.

It is therefore evident at what depth of floating, depending on the specific gravity, the solid when placed on the fluid in the positions which have been described, begins or ceases to float with stability. But a material inquiry remains to be considered, which is, to ascertain in what position a square parallelopiped will dispose itself, in respect to the fluid's surface, when the specific gravity is of any intermediate values between the limits which have been determined. To resolve this question the preceding results are evidently inadequate, since from these we only know in what cases, depending on the values of the specific gravity, the solid when placed on the fluid either with a flat surface or an angle upward will float permanently; and in what cases it will overset. Suppose the latter event to take place, and that the solid, having been placed on the fluid in a position of unstable equilibrium, oversets or changes its position by revolving on its axis. To ascertain what position the solid so circumstanced will assume, in which

it will continue permanently to float, we must have recourse to the theorem for expressing the perpendicular distance between the two verticals, which pass through the centres of gravity of the solid and of the part immersed. For by putting this value = o, the resolution of an equation thence arising, will give the sine of the inclination from the position of equilibrium at which these two vertical lines coincide; that is, when the centres of gravity of the solid and of the part immersed are again in the same vertical line: in this case the solid will be situated in a position of equilibrium, which, according to the observations in page 63, must be an equilibrium of stability.

Let EFDC (fig. 6.) represent the vertical section passing through the centre of gravity G of an oblong solid or parallelopiped, the longer axis of which passes through the centre of gravity G in a direction perpendicular to the plane EFCD; LGS is drawn through G parallel to CE or DF; this solid is placed on the surface of a fluid IABK, with the line SGL vertical; and the specific gravity of the solid is such as causes it to sink to the depth under the fluid's surface SN.

The volume immersed under the fluid's surface is the space ACDB, of which the centre of gravity is O; and since the points G and O are situated in the same vertical line, the solid will be in a position of equilibrium, which, according to the present supposition, is assumed to be the equilibrium of instability; the solid will therefore spontaneously overset whenever external support is removed, and will change its position by revolving round an horizontal axis which passes through the centre of gravity in a direction perpendicular to the plane CDFE.

It is required to ascertain through what angle WGS, the solid will be inclined round its axis, when the centres of gravity of the solid and of the part immersed are again in the same vertical line. As in the former cases, this problem will be solved, by referring to the general expression for the distance between the two vertical lines which pass through the centres of gravity of the solid and of the part immersed.

Suppose then the solid to be inclined from its former position of equilibrium in an angle WGS, so as to become transferred from the position ECDF into the position YWHV; the part immersed will now be ZHVR; the line AB will also be transferred to PQ, and the space QXR, which was before above the fluid's surface, will now be immersed under it; and the space PXZ, which was before under the surface, will now be above it. Bisect the lines PZ, QR, in m and n, and join mX, nX; and take X$a = \frac{2}{3}$ of Xm and X$d = \frac{2}{3}$ of Xn; so shall a and d be the centres of gravity of the triangles PXZ, QXR, respectively; draw the lines ab, cd, perpendicular to the horizontal line AB. Referring to the quantity expressing the distance between the vertical lines which pass through the centres of gravity of the solid and of the part immersed, namely, $\frac{b\text{A}}{\text{V}} - ds$, there will be applicable to the present case, the space QXR = A; the space ZHVR or ACDB = V; $bc = b$; OG $= d$; the sine of the angle of inclination or WGO $= s$: let t be the tangent of the same angle to radius $= 1$; then, since the triangles ZXP, QXR, are similar, and the areas are equal by the supposition, the sides of the two triangles will be respectively equal; that is, QX will be equal to XP; ZP equal to QR; and ZX to XR. Let the height of the solid SL $= c$,

and the specific gravity $= n$ when that of the fluid is equal to 1, also make VW or XQ $= a$; then QR $= at$, and $Qn = \frac{at}{2}$; $Xn = \sqrt{a^2 + \frac{t^2 a^2}{4}}$, or $Xn = \frac{a}{2} \times \sqrt{4 + t^2}$.

To find the sine of the angle nXR, make the following proportion. As Rn or Qn $\left(\frac{ta}{2}\right)$: Xn $\left(\frac{a}{2} \times \sqrt{4+t^2}\right)$: : sine nXR : sine XRn : wherefore sine $n\text{XR} = \frac{\text{sine } n\text{RX} \times t}{\sqrt{4+t^2}}$; or because sine $n\text{RX} = \frac{1}{\sqrt{1+t^2}}$, sine $n\text{XR} = \frac{t}{\sqrt{4+t^2} \times \sqrt{1+t^2}}$; cos. $n\text{XR} = \frac{2+t^2}{\sqrt{4+t^2} \times \sqrt{1+t^2}}$: and since $\text{X}d = \frac{2}{3} \times \text{X}n = \frac{a \times \sqrt{4+t^2}}{3}$, it follows that $\text{X}c = \frac{a \times \sqrt{4+t^2} \times \overline{2+t^2}}{3n\sqrt{4+t^2} \times \sqrt{1+t^2}} = \frac{a}{3} \times \frac{2+t^2}{\sqrt{1+t^2}}$; and since the triangles XPZ, XQR, as also the triangles ZXm, RXn, are similar and equal, the line Xb = Xc; and consequently $bc = 2\text{X}c = \frac{2a \times \overline{2+t^2}}{3 \times \sqrt{1+t^2}}$; which quantity $=$ b in the general value $\frac{b\text{A}}{\text{V}} - ds$. And since the specific gravity of the solid is $= n$, the height SL $= c$, and the base CD $= 2a$, the immersed part or ACDB $= 2acn$, which in the general expression is denoted by V; and the volume QXR $= \frac{a^2 t}{2}$ is denoted by the letter A in such general value.

Substituting, therefore, in the expression $\frac{b\text{A}}{\text{V}} - ds$, $\frac{2a \times \overline{2+t^2}}{3 \times \sqrt{1+t^2}}$ for b; $\frac{a^2 t}{2}$ for A; and $2acn$ for V; the distance between the vertical lines passing through the centres of gravity of the solid and the centre of gravity of the part immersed, appears

to be $\frac{2a \times \overline{2 + t^2}}{3 \times \sqrt{1 + t^2}} \times \frac{a^2 t}{2 \times 2acn} - ds$, or $\frac{a^2 t \times \overline{2 + t^2}}{6cn \times \sqrt{1 + t^2}} - ds$; or since $d = \frac{c - cn}{2}$, the said distance $= \frac{a^2 t \times \overline{2 + t^2}}{6cn \times \sqrt{1 + t^2}} - \frac{\overline{c - cn} \times s}{2}$; or by substituting for t^2 its value $\frac{s^2}{1 - s^2}$, the distance $= \frac{a^2 s \times \overline{2 - s^2}}{6cn \times \overline{1 - s^2}} - \frac{\overline{c - cn} \times s}{2}$; in which expression a denotes half the breadth PQ; but as it may be more convenient to represent the whole breadth AB or PQ by the letter a, the expression will in this case be $= \frac{a^2 s \times \overline{2 - s^2}}{24cn \times \overline{1 - s^2}} - \frac{\overline{c - cn} \times s}{2}$; which quantity being put $= 0$, we obtain $s^2 = \frac{2a^2 - 12c^2 n + 12c^2 n^2}{12c^2 n^2 - 12c^2 n + a^2}$, or $s^2 = \frac{12c^2 n - 12c^2 n^2 - 2a^2}{12c^2 n - 12c^2 n^2 - a^2}$. From this equation the angle of inclination from the original position of equilibrium may be found, from having given the specific gravity; or conversely, the specific gravity may be found from having given the angle of inclination through which the solid must revolve, so as to be situated in a second position of equilibrium. As the instances given to illustrate the propositions already investigated have been adapted to the case of a square parallelopiped, the present result may be exemplified on the same supposition. Assuming then the height of the solid to be equal to the base, a will become $= c$ in the preceding expression, and consequently $s^2 = \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1}$.

We have seen in a foregoing proposition, that if the specific gravity of this square solid should be greater than .211 so as not to exceed .789, the solid placed on the fluid with a flat surface upward, would be situated in an equilibrium of instability, and consequently must change its position by revolving on

its axis till it settles in some other position wherein the equilibrium is permanent.

From the present proposition we shall be enabled to ascertain what that position is. Thus, let the specific gravity $n = .24$, which is between the limits .211 and .789; and will consequently place the solid with a flat surface upward and horizontal, in an equilibrium of instability. By referring to the equation $s^2 = \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1}$, and substituting .24 for n, we find that $s^2 = \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1} = \frac{.1888}{1.1888}$; and $s =$ the sine of 23° 29′. From this calculation it appears, that the solid after having overset from its position of unstable equilibrium, with the flat surface upward and horizontal, and having revolved through an angle of 23° 29′, will settle in a position of permanent equilibrium at that angular distance from its original situation; for by the solution, when the solid has revolved through that angle, the centres of gravity of the solid and of the part immersed are again situated in the same vertical line, and consequently the solid is then situated in a position of equilibrium, which must be the equilibrium of stability, because the original position from which the solid inclined, was that of instability; and it has been observed previously, that when a solid changes its position by revolving on an axis on the surface of a fluid, any position of equilibrium is always succeeded by a position of equilibrium which is of a contrary description.

If the angle of inclination from the upright position with a flat surface horizontal should be given, the specific gravity of the solid may be inferred from the preceding equation, which will cause the solid to float in a position of equilibrium at

that given angle of inclination; for by solving the equation $s^2 = \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1}$ we obtain $n = \frac{1}{2} \pm \sqrt{\frac{1 - 2s^2}{12 - 12s^2}}$. Thus, if it should be required to ascertain the specific gravity which will cause the solid to float *in equilibrio* at the angular distance of 23° 29′ from the upright, we have $\sqrt{\frac{1 - 2s^2}{12 - 12s^2}} =$ 0.26000, and the specific gravity required, that is, $n = .5 + .26 = .76$, or $n = .5 - .26 = .24$. Thus we find from this calculation that there are two specific gravities which will cause the solid to float in a position of equilibrium at the same angular distance 23° 29′ from the original situation with a flat surface horizontal; a conclusion which it is easy to verify by substituting .76 for n in the equation $\frac{12n - 12n^2 - 2}{12n - 12n^2 - 1} = s^2$: the result is that $s^2 = \frac{.1888}{1.1888}$, the same as in the former instance, when n was assumed $= .24$.

In the application of analytical investigation to the solution of problems, it is always necessary to keep distinctly in view the conditions on which the investigation has been founded; for however correct the solution may otherwise have been, any inadvertence in this respect will unavoidably lead to error and inconsistency. The investigation by which the floating position of the solid is determined after it has changed its position from an equilibrium of instability, when one of the flat surfaces was parallel to the horizon, has proceeded on a supposition that the surface of the fluid intersects the parallel surfaces YH, WV, (fig. 6.) in the points R and Z; but if the two surfaces intersected by the fluid should be the inclined sides HV, VW, or in other words, if the point of intersection Z should be situated between H and V, neither the geometrical construction

nor the analytical investigation depending on it, can be applied, so as to ascertain the required position of equilibrium, a solution altogether different being required to determine the position in which a solid under these conditions will float permanently. It is, however, certain, that as long as the point of intersection Z is not lower than the point of the base H, the preceding solution will be applicable: it will be therefore material to find both the angle of inclination from the original position of unstable equilibrium, and the specific gravity of the solid when it floats permanently, with this condition annexed, *i. e.* that the surface of the fluid shall pass through one of the extremities of the base: the result of this solution will form a limiting value both of the angle of inclination and of the specific gravity, beyond which the preceding investigation not being applicable, another solution is required.

Let AECD (Tab. IV. fig. 7.) represent a vertical section of the square parallelopiped which rests permanently on the surface of the fluid IKDH, passing through the extremity of the base D. It is required to find the angle of inclination KDC from a position of equilibrium with a flat surface horizontal, and the specific gravity of the solid, when it floats in a state of equilibrium. Let the tangent of the required angle KDC be to radius as t to 1, and put $CD = a$; let the specific gravity of the solid be to that of the fluid as n to 1. Then $KC = at$, and the area $KCD = \frac{a^2 t}{2}$: and because as the area KCD is to the area AECD, so is n to 1, it follows that $n = \frac{t}{2}$; and since by the preceding investigation* $s^2 = \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1}$, where s represents the sine of the angle of inclination from the

* Page 79.

upright position, which is the angle KDC in the present case; substituting for n its value $\frac{t}{2}$, the equation will now become $s^2 = \frac{6t - 3t^2 - 2}{6t - 3t^2 - 1}$, or because $s^2 = \frac{t^2}{1 + t^2}$, $\frac{t^2}{1 + t^2} = \frac{6t - 3t^2 - 2}{6t - 3t^2 - 1}$, or $6t^3 - 3t^4 - t^2 = 6t - 3t^2 - 2 + 6t^3 - 3t^4 - 2t^2$; or $4t^2 = 6t - 2$; which equation being resolved, gives $t = \frac{3}{4} \pm \frac{1}{4}$, that is, $t = \frac{1}{2}$ or $t = 1$. By this solution it appears, that there are two angles at which the solid may be inclined from its upright position of unstable equilibrium with the flat surface upward, so as to rest permanently on the surface of the fluid, when that surface passes through one extremity of the base: 1st, when the angle of inclination is KDC $= 26° \, 33', \, 51'',4$, or about $26° \, 34'$, of which the tangent is to radius as 1 to 2; and secondly, (fig. 8.) when the angle of inclination KDC $= 45°$, of which the tangent is equal to the radius. When the solid floats permanently on the fluid at the angle of inclination KDC $= 26° \, 34'$ from the upright position, the part immersed, or KCD, is to the whole volume ABCD as 1 to 4; and therefore the specific gravity of the solid is to that of the fluid as 1 to 4, or resuming the former notation applied to the present case, the specific gravity of the solid or $n = \frac{1}{4}$, when that of the fluid is $= 1$. That the position of equilibrium here determined is that of stability, appears from attending to the limiting value of the specific gravity, determined in page 69, where it is shewn that when the square parallelopiped is placed on the surface of a fluid with one of the flat surfaces horizontal, and the specific gravity of the solid is greater than .211, so as not to exceed .789, the equilibrium will be that of instability, and consequently the solid will overset. It has

been just shewn, that after the body has revolved through an angle of 26° 34′ it will be again in a position of equilibrium, which must therefore be the equilibrium of stability. Similar consequences follow from supposing the specific gravity $= \frac{1}{2}$; in this case if the solid is placed on the fluid with a flat surface upward, the equilibrium will be that of instability; and it appears from the preceding solution, that after revolving through an angle of 45°, (fig. 8.) it will again be in a position of equilibrium, which therefore will be stable and permanent. By a similar investigation, the angle of inclination ABK (fig. 9.) from the original position of equilibrium may be found when the solid floats permanently, and the fluid's surface intersects one of the extremities of the upper side of the square AB: for the notation remaining, by putting the tangent of the angle of inclination ABK $= t$, the area ABK $= \frac{a^2 t}{2}$, area KCDB $= \frac{2a^2 - a^2 t}{2}$, wherefore the specific gravity or $n = \frac{2 - t}{2}$; which quantity being substituted for n in the equation $\frac{t^2}{1 + t^2}$* $= \frac{12n - 12n^2 - 2}{12n - 12n^2 - 1}$, there will arise the equation $\frac{t^2}{1 + t^2} = \frac{6t - 3t^2 - 2}{6t - 3t^2 - 1}$, exactly the same as in the former case; and by solving this equation it appears that $t = \frac{3}{4} \pm \frac{1}{4}$, and consequently the specific gravity of the solid, or $n = \frac{2 - t}{2} = \frac{3}{4}$ or $n = \frac{1}{2}$.

The only inquiry remaining to complete the investigation respecting the floating positions of the square parallelopiped, is to ascertain in what position the solid will float permanently.

* Because s being the sine, and t being the tangent of the angle ABK, it follows that $s^2 = \frac{t^2}{1 + t^2}$.

with a plane angle obliquely upward, when the specific gravity is between the limits $\frac{8}{32}$ and $\frac{9}{32}$, or between the limits $\frac{23}{32}$ and $\frac{24}{32}$. It has been seen in a former investigation, that if the solid is placed on the fluid with an angle upward, and the specific gravity is $\frac{9}{32}$, it will just begin to float with stability, and ceases to float with stability when the specific gravity exceeds $\frac{23}{32}$. When the specific gravity is $\frac{1}{4} = \frac{8}{32}$ or $\frac{3}{4} = \frac{23}{32}$, it floats permanently with the surface of the fluid coincident with an extremity of one of the sides: if, therefore, the specific gravity is between the limits $\frac{8}{32}$ and $\frac{9}{32}$, or between $\frac{23}{32}$ and $\frac{24}{32}$, the solid will float permanently, with the diagonal line inclined to the vertical. This angle may be determined by finding an equation which expresses the relation between the given specific gravity and the sine or tangent of the required angle to radius $= 1$. Let a square parallelopiped IVCF (fig. 10.) float with an angle obliquely upward, so that the diagonal line shall make an angle with the vertical; suppose that angle to be OGT, the line GT being perpendicular to the horizon; let the surface of the fluid coincide with the line DE perpendicular to GT; take CB a mean proportional between EC and CD, and draw BA parallel to GV, intersecting the line GC in H; so shall CH be the depth to which the solid sinks in the fluid when the diagonal line CI is vertical, and consequently the area BXE is equal to the area XDA; take CO $= \frac{2}{3}$ CH; O will be the centre of gravity of the volume ABC; bisect EB in K, and AD in B; draw XR and XK; and take XM $= \frac{2}{3}$ of XR, and HL $= \frac{2}{3}$ of XK; M will be the centre of gravity of the

triangle XAD, and L will be the centre of gravity of the triangle BXE; through the points M, L, draw the lines MP, QL, perpendicular to the horizontal line DE; make PQ $= b$, the sine of BXE $= s$; the tangent of BEX $= t$ to radius $= 1$, and let EC $= a$.

Then CD $= ta$; and CB $= \sqrt{ta^2}$; CH $= \sqrt{\frac{ta^2}{2}}$; CO $= \frac{2}{3} \times$ CH $= \sqrt{\frac{2ta^2}{9}}$; the area ABC $= \overline{CH}^2 = \frac{ta^2}{2}$; put the area BXE $= u$; then to find the distance OT, the following proportion is to be made; as the area CDE or ABC is to the area BXE : : so is PQ* to OT; or as $\frac{ta^2}{2} : u : : b :$ OT $= \frac{2bu}{ta^2}$; and OG $= \frac{2bu}{ta^2 s}$; and since CO $= \sqrt{\frac{2ta^2}{9}}$, it follows that CG $= \frac{2bu}{ta^2 s} + \sqrt{\frac{2ta^2}{9}}$; and therefore CV $= \frac{\sqrt{8} \times bu}{ta^2 s} + \sqrt{\frac{4ta^2}{9}} = \frac{\sqrt{72} \times bu + \sqrt{4t^3 a^6 s^2}}{3ta^2 s}$; and the specific gravity being $= n$, $\sqrt{n} = \frac{\text{CH}}{\text{CV}} = \sqrt{\frac{ta^2}{2}} \times \frac{3ta^2 s}{\sqrt{72} \times bu + \sqrt{4t^3 a^6 s^2}} = \frac{3t^{\frac{3}{2}} a^3 s}{12bu + 2\sqrt{2}\, t^{\frac{3}{2}} a^3 s}$: thus, if the angle at which the diagonal line IC is inclined to the vertical line TN or OGT $=$ BXE should be 15°, the angle XEC $= 30°$; wherefore in the preceding expression, $t =$ tangent 30° to radius 1; $s =$ sine 15°; if CE or a is assumed $= 1$, on making the proper trigonometrical computations, the area BXE $= u = .039395$, and PQ $= b = 0.73089$; from substituting these quantities for their values in the equation $\sqrt{n} = \frac{3t^{\frac{3}{2}} a^3 s}{12bu + 2\sqrt{2t^{\frac{3}{2}} a^3 s}}$, it appears that $\sqrt{n} = \frac{.34063}{.34552 + .32114} = 0.51094$, and $n = 0.261$ the specific gravity which causes the solid to float on the fluid in a position of equilibrium with a

* Page 59.

diagonal line obliquely upward, being inclined to the vertical at an angle of 15°; the equilibrium is that of stability, because when the diagonal is vertical, the solid floats in a position of unstable equilibrium, the specific gravity 0.261 being less than $\frac{9}{32}$ or .281, the limiting value which separates the cases of permanent and unstable equilibrium when the solid is placed on the fluid with a diagonal line vertical.

It is curious to observe the conclusions which arise in the extreme case when the angle of inclination from the vertical is assumed $= 0$; and consequently the angle XEC $= 45°$; for in this case CB $=$ CE $= a$; $t = 1$; and BH $= \frac{a}{\sqrt{2}}$; therefore u or the area BXE * $= \frac{\overline{BH}^2 \times s}{2} = \frac{sa^2}{4}$; and since $b = PQ = \frac{4a}{3\sqrt{2}}$, it follows that $bu = \frac{a^3 s}{3\sqrt{2}}$; and $12bu = \frac{4a^3 s}{\sqrt{2}} = \sqrt{2}a^3 s$; which quantities being substituted for their values, the equation $\sqrt{n} = \frac{3t^{\frac{3}{2}} a^3 s}{12bu + 2\sqrt{2}t^{\frac{3}{2}} a^3 s}$ will become $\sqrt{n} = \frac{3a^3 s}{2\sqrt{2}a^3 s + 2\sqrt{2}a^3 s} = \frac{3}{4\sqrt{2}}$; and therefore $n = \frac{9}{32}$, agreeing † precisely with the specific gravity inferred by a different method from the same data.

The equation $\sqrt{n} = \frac{3t^{\frac{3}{2}} s}{2\sqrt{2}t^{\frac{3}{2}} s + 12bu}$ (the line CE $= a$ being assumed $= 1$) expresses the relation between the specific gravity of the solid and the fraction representing the sine of the angle of inclination from the upright position: if, therefore, that

* Because the point of intersection X coincides with H when the angle BXE vanishes.

† Page 72.

angle is given, the specific gravity will be known. If it should be required to find the sine of the angle of inclination from having given the specific gravity, it is evident from the nature of the equation, that such determination would require analytical operations extremely complex and troublesome, which may be avoided by having recourse to well known methods of approximation. By assuming the quantities s and t by estimation, let the value of $\sqrt{n}$ be calculated from the equation, which being compared with the given value of $\sqrt{n}$, the difference will be the error arising from the error in the assumed values of s and t, which are therefore to be corrected, and the operation repeated until the value of $\sqrt{n}$, deduced from calculation, coincides with its true value; from which method of proceeding, the angle of inclination from the original position of equilibrium will be known.

This solution is evidently applicable to all cases in which the specific gravity of the solid is between the limits $\frac{8}{32}$ and $\frac{9}{32}$, and by an investigation entirely similar, an equation is deduced expressing the relation of the specific gravity of the solid and the sine or tangent of the angle of inclination from the perpendicular, when the specific gravity of the solid is between $\frac{23}{32}$ and $\frac{24}{32}$; in which case the solid will float permanently with the diagonal line IC obliquely upward, being inclined to the vertical at some angle between the limits 0 and $18° 26' 8'',6$.

These determinations comprehend all the positions in which a square parallelopiped can be placed on the surface of a fluid in a position of equilibrium, provided the solid is moveable only round one axis, namely, that which passes through the centre

of gravity perpendicular to the planes of the square sections; and this condition is insured by making the axis of sufficient length; for instance, if it is two or three times longer than one of the sides, the solid will not spontaneously revolve on any other axis. When the axis is diminished considerably, it is certain the body will be spontaneously moveable round some other axis; but it is unnecessary to enter into a detail of multiplied instances, since the exposition of principles is the material object in disquisitions of this kind.

The various positions which the square parallelopiped assumes when floating freely on a fluid's surface depending on its specific gravity, are brought under one point of view in the following abstract, the line IK denoting the surface of the fluid in the figures from fig. 11 to fig. 24.

If the specific gravity of the solid should be between the limits 0 and $\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{1}{6}}$, (fig. 11, 12, and 13.) that is, between 0 and 0.211, the solid floats permanently on the fluid with a flat surface upward, and parallel to the horizon.

If the specific gravity is between the limits .211 and .25 (fig. 13, 14, and 15.), the solid floats permanently with a flat surface upward, but inclined to the horizon at sundry angles of which the limits are 0°, corresponding to the specific gravity .211 and 26° 34′, corresponding to the specific gravity .25.

If the specific gravity is between the limits $.25 = \frac{8}{32}$ and $\frac{9}{32}$, (Tab. IV. and Tab. V. fig. 15, 16, 17.) the solid floats with one angle only immersed under the fluid's surface, the diagonal line being inclined to the vertical at various angles depending on the specific gravity, the limits of which angles are 18° 26′, cor-

responding to the specific gravity $.25 = \frac{8}{32}$, and o, corresponding to the specific gravity $\frac{9}{32}$.

When the specific gravity is increased beyond $\frac{9}{32}$, (fig. 17, 18.) the solid floats permanently with a diagonal line vertical, till the specific gravity becomes $= \frac{23}{32}$.

If the specific gravity is of any magnitude between $\frac{23}{32}$ and $\frac{24}{32}$ the solid floats with the diagonal line inclined to the vertical at sundry angles depending on the specific gravity, (fig. 18, 19, 20.) the limits of which angles are o, corresponding to the specific gravity $\frac{23}{32}$, and 18° 26′ corresponding to the specific gravity $\frac{24}{32}$, three angles of the solid being immersed under the fluid's surface.

If the specific gravity is between the limits $\frac{24}{32}$ and .789, (fig. 20, 21, 22.) the solid floats with a flat surface upward, and inclined to the horizon at sundry angles depending on the specific gravity, the limits of which angles are 26° 34′ corresponding to the specific gravity $\frac{24}{32}$ or .75, and o corresponding to the specific gravity .789.

When the specific gravity is of any magnitude between .789 and 1, the solid floats permanently with a flat surface parallel to the horizon.

From these determinations we also collect that while the solid in question, floating on the fluid's surface, revolves round its longer axis through 360°, it passes through either 16 or 8 positions of equilibrium. If the specific gravity should be between the limits .211 and .281, or between the limits .719 and

.789, the number of those positions will be sixteen; of which eight will be positions of permanent, and the remaining eight positions of unstable equilibrium; these different species of equilibrium succeeding each other alternately while the solid revolves round its axis. If the specific gravity should be of any value not included within these limits, the solid in revolving through 360° will pass through 8 positions of equilibrium only; of which four are positions of permanent, and four of unstable equilibrium.

In the investigations which have preceded, the solid is supposed to be of uniform figure in respect to the axis of motion, so as to make all the vertical sections drawn perpendicular to the axis equal. But when the floating body is of such a form that the sections drawn through it perpendicular to the axis at various points thereof are unequal, a different process, depending however on the same principles, will be necessary; both for determining whether the solid will float permanently or overset, and for ascertaining the several positions in which it will float on the surface of a fluid.

Let EFCD (fig. 23.) represent a cylinder* placed on the surface of a fluid with the axis NP vertical. Suppose the specific gravity to be such as causes the solid to sink to the depth QP; let it be required to determine in what cases, depending on the dimensions and specific gravity of the cylinder, it will float permanently in that position, and in what cases it will overset. Put the radius QA $= r$; the specific gravity of the solid $= n$, that of

* In this and the following propositions, the plane surfaces which terminate the solid are always understood to be perpendicular to the axis.

the fluid being $= 1$; let the centre of gravity be G; the centre of gravity of the immersed part $=$ O; GO $= d$; let AIBHSA represent a circular section of the cylinder coincident with the fluid's surface; draw any diameter IS; and a diameter AB perpendicular to IS; let the axis passing through the centre of gravity round which the cylinder is moveable be parallel to IS; through any point W of the diameter IS draw the ordinate KW perpendicular to IS, and produce KW till it intersects the circle in the point H; make QW $= z$; NP $= l$; $\pi = 3.14159$. It appears from page 66 that the solid will float permanently in the given position of equilibrium with the axis vertical, when the fluent of $\frac{\overline{KH}^3 \times \dot{z}}{12V}$ is greater than d, the letter V signifying the volume immersed under the fluid's surface; it is also shewn in page 66, that if d is greater than $\frac{\text{fluent of } \overline{KH}^3 \times \dot{z}}{12V}$, the equilibrium will be unstable; when the fluent of $\frac{\overline{KH}^3 \times \dot{z}}{12V} = d$, the equilibrium will be the limit separating the cases in which the solid floats with stability from those in which it is momentary and unstable. To ascertain the limit in the present case it is necessary to find the fluent of $\frac{\overline{KH}^3 \times \dot{z}}{12V}$. Since QS $= r$, and QW $= z$, WH $= \sqrt{r^2 - z^2}$, KH $= 2 \times \sqrt{r^2 - z^2}$, and $\overline{KH}^3 \dot{z} = 8 \times \overline{r^2 - z^2}|^{\frac{3}{2}} \times \dot{z}$; the fluent of which quantity, while z increases from o to r is $\frac{8 \times 3\pi r^4}{16}$,* and for both semicircles, the

* Fluent of $\overline{r^2 - z^2}|^{\frac{3}{2}} \dot{z} =$ fluent of $r^2 \times \overline{r^2 - z^2}^{\frac{1}{2}} \dot{z} -$ fluent of $\overline{r^2 - z^2}^{\frac{1}{2}} z^2 \dot{z}$.

Fluent of $r^2 \times \overline{r^2 - z^2}^{\frac{1}{2}} \times \dot{z} = r^2 \times$ the area QBHW. (fig. 23.)

fluent of $\overline{KH}^3 \times \dot{z} = 3\pi r^4$; and because $PQ = ln$, and the area of the circle AIBHSA is πr^2, the volume of the part immersed V is $= \pi r^2 ln$; moreover $GP = \frac{l}{2}$; and $OP = \frac{ln}{2}$; wherefore $GO = \frac{l - ln}{2} = d$: and since the $\frac{\text{fluent of } \overline{KH}^3 \times \dot{z}}{12V} = \frac{3\pi r^4}{12\pi r^2 ln}$; making $\frac{\text{fluent of } \overline{KH}^3 \times \dot{z}}{12V} = d$, in order to obtain the limit or limits which separate the cases of permanent and unstable equilibrium, we obtain the equation $\frac{3\pi r^4}{12\pi r^2 ln} = \frac{\overline{l - ln \times}}{2}$ or $\frac{r^2}{2l^2} = n - n^2$; $n^2 - n = -\frac{r^2}{2l^2}$; or if $2r$ is put $= b =$ the diameter of the base, $n^2 - n = \frac{-b^2}{8l^2}$ and $n = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{b^2}{8l^2}}$.

If therefore the diameter of the base bears to the axis a greater proportion than that of $\sqrt{2}$ to 1, no value can be given to the solid's specific gravity, which will cause it to float in a state of insensible equilibrium; or in other words, there is no specific gravity separating the cases in which the cylinder will float permanently, from those in which it will overset when the

— Fluent of $\overline{r^2 - z^2}^{\frac{3}{2}} z^2 \dot{z} = \frac{r^2 z^2}{8} \times \sqrt{\frac{r^2 - z^2}{z^2}} - \frac{2z^4}{8} \times \sqrt{\frac{r^2 - z^2}{z^2}} + \frac{r^4}{8} \times \frac{\text{arc HS}}{r}$.

This quantity ought to be $= 0$, when $z = 0$; wherefore the entire fluent of $\overline{r^2 - z^2}|^{\frac{3}{2}} \dot{z} = r^2 \times \text{area QBWH} + \frac{r^2 z^2}{8} \times \sqrt{\frac{r^2 z^2}{z^2}} - \frac{2z^4}{8} \times \sqrt{\frac{r^2 - z^2}{z^2}} + \frac{r^4}{8} \times \frac{\text{arc HS}}{r} - \frac{\pi r^4}{16}$, because the arc $\frac{HS}{r} = \frac{\pi}{2}$ when $z = 0$, or $SH = SB$; when $z = r$, this fluent, that is, the fluent of $\overline{r^2 - z^2}|^{\frac{3}{2}} \dot{z}$ while z increases from 0 to r is $= r^2 \times \text{area SBQ} - \frac{\pi r^4}{16} = \frac{\pi r^4}{4} - \frac{\pi r^4}{16} = \frac{3\pi r^4}{16}$.

axis is placed vertically; the cylinder, under these circumstances, must always float permanently with its axis vertical.

When the diameter of the base bears to the length a less proportion than that of $\sqrt{2}$ to 1, two values of the specific gravity may always be assigned, which will be the limits of the cases in which the solid floats with stability or oversets; *i.e.* $n = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{b^2}{8l^2}}$. If the specific gravity should be given, the proportion of the cylinder's length to the diameter of the base may be defined which limits the cases of stability or instability of floating with the axis vertical; for since $n - n^2 = \frac{b^2}{8l^2}$, it follows that $\frac{b}{l} = \sqrt{8n - 8n^2}$; consequently n being given, if the diameter of the base should be to the length of the axis in a greater proportion than that of $\sqrt{8n - 8n^2}$ to 1, the solid will float permanently with the axis upward; but if the base should be to the length of the axis in a less proportion than that of $\sqrt{8n - 8n^2}$ to 1, the solid will overset. Thus if n* $= \frac{3}{4}$, $\sqrt{8n - 8n^2} = \sqrt{\frac{3}{2}} = 1.2247$; if therefore the diameter of the base should be in a greater proportion to the length of the axis than 1.2247 to 1, it will float permanently with the axis vertical, if in a less proportion, it will overset from that position.

Suppose a parabolic conoid CEDK (fig. 24.) of given dimensions and specific gravity, should be placed on the surface of a fluid with the vertex downward, and the axis vertical; to ascertain the limits (depending on the length of the axis, the parameter of the parabola from which the conoid is formed, and the specific gravity,) which separate the cases in which the solid

* Page 48.

will float permanently with the axis vertical, or will overset, the plane of the base being supposed perpendicular to the axis: Let CED represent a plane section of this solid passing through the axis, which section will therefore be a parabola. Suppose the specific gravity to be such as causes the solid to sink to the depth FE. AIBHA represents a circular section of the solid which coincides with the fluid's surface; draw any diameter HI, and the diameter AB perpendicular to HI. Through any point W, in the radius FH, draw the ordinate KM perpendicular to FH; and suppose the solid to be moveable round an axis of motion parallel to the diameter HI; put the parameter of the parabola $= p$; the length of the axis KE $= a$, FW $= z$; the specific gravity $= n$; $\pi = 3.14159$; also let G be the centre of gravity of the solid, and O the centre of gravity of the part immersed. Then, since the volume immersed AEB is to the volume CED as $\overline{AB}^2 \times EF$ is to $\overline{CD}^2 \times EK$, or as $\overline{EF}^2$ to $\overline{EK}^2$; and since the volume immersed AEB is to the volume CED as n to 1, it follows that as $\overline{EF}^2 : \overline{EK}^2 = a^2 :: n$ to 1, and therefore EF $= a\sqrt{n}$, and $\overline{FB}^2 = pa\sqrt{n}$; referring to the expression for determining the stability of floating bodies when the inclinations from a position of equilibrium are very small, or $\frac{\text{fluent of } \overline{KM}^3 \dot{z} \times s}{12V} - ds$, we have, applicable to the present case, the entire fluent of $\overline{KM}^3 \dot{z} = 3\pi \times \overline{FB}^4$; or, because $\overline{FB}^4 = p^2 a^2 n$, the fluent of $\overline{KM}^3 \dot{z} = 3\pi p^2 a^2 n$: V or the volume immersed $= \frac{\pi a^2 pn}{2}$; and since by the properties of the figure, GE $= \frac{2a}{3}$ and OE $= \frac{2a\sqrt{n}}{3}$, we have GO $= \frac{2a - 2a\sqrt{n}}{3} = d$, these substitutions being made in the general value, $\frac{\text{fluent of } \overline{KM}^3 \dot{z} \times s}{12V} - ds$; this quantity becomes $= \frac{3\pi p^2 a^2 n \times 2 \times s}{12 \times \pi a^2 pn}$

$- \frac{\overline{2a - 2a\sqrt{n} \times s}}{3}$, which being put $= 0$, in order to obtain the limiting value required, we obtain $\frac{p}{2} - \frac{2a - 2a\sqrt{n}}{3} = \frac{3p - 4a - 4a\sqrt{n}}{6} = 0$, and $\sqrt{n} = \frac{4a - 3p}{4a}$; consequently $\sqrt{n} : 1 :: a - \frac{3p}{4} : a$.

From this determination it appears, that if the axis should be to the parameter in a proportion less than that of 3 to 4, no specific gravity can be given to the solid which will make it float in the equilibrium, which is the limit between the stability and instability of floating; secondly, if the specific gravity of the solid bears a greater proportion to that of the fluid than the proportion which the square of the difference between the axis and $\frac{3}{4}$ of the parameter bears to the square of the axis; when the axis is placed vertical, the solid will float with stability in that position; and thirdly, if the specific gravity of the solid bears a less proportion to the specific gravity of the fluid than that which the square of the aforesaid difference bears to the square of the axis, the solid will overset when placed on the fluid with the axis vertical, and will settle permanently with the axis inclined to the vertical line. These limits agree precisely with those which are demonstrated by ARCHIMEDES, in the second book of his tract, intituled *De iis quæ in humido vehuntur*,* prop. iii. and prop. iv.

* The demonstrations of ARCHIMEDES, which relate to the parabolic conoid, are founded on a supposition that this solid is generated by the revolution of a rectangular parabola on its axis; that is, of a parabola which is the section of a rectangular cone; in which case the line, called by the author (or rather by his translator, the original of this treatise being lost) " ea quæ usque ad axem," is half the principal parameter, being equal to the perpendicular distance between the plane which touches the cone, and the plane parallel to it, which is coincident with the parabola. This solid is termed by ARCHIMEDES, " conois rectangula," but the limitation appears to be unnecessary, because the demonstrations of the author are equally applicable to a solid generated by

If the specific gravity of the parabolic conoid should be less than the limit which has just been investigated, and if the axis should be to the parameter in a proportion greater than that of 3 to 4, and less than that of 15 to 8, it will float permanently on the fluid with the axis inclined to the horizon, and with the base wholly extant above the surface at some angle less than 90°; which angle may be determined by the following geometrical construction, subject to the limitation which will appear from the construction itself, or rather from the computation founded upon it.

Let ASBTD (Tab. VI. fig. 25.) represent a section of the parabolic conoid which passes through the axis; which section will be a parabola. Let the axis BE be divided into three equal parts, one of which is EF. By the properties of this figure, F will be the centre of gravity of the solid. In the line FB take FH equal to half of the parameter, and through H draw the indefinite line ΓGZ perpendicular to BE, and in the line GZ take HK = FB; in the line HΓ take HI, which shall be to HK in the proportion of the specific gravity of the solid to that of the fluid; and bisect IK in the point L; with the centre L and radius LI describe the semicircle KOI, intersecting the axis BE in the point O; through O draw OC parallel to KI, intersecting the parabola in the point C, and let PCN be drawn touching the parabola in the point C. Through C draw the indefinite line CR parallel to BE, intersecting the line KI in the point G; in

the revolution of a parabola, which is the section of any cone, whatever may be the angle at the vertex, half the parameter being substituted instead of the line, called by ARCHIMEDES "ea quæ usque ad axem;" and it is a property of conics easily demonstrable, that any parabola being given, a similar and equal parabola may be formed from the section of any cone, whatever may be the angle at the vertex, the axis being of sufficient length.

the line CR take GQ equal to half GC; and through Q draw SQT parallel to PCN. When the conoid floats permanently and at rest, the surface of the fluid will coincide with the line SQT, and the axis will be inclined to the horizon at the angle ONC: through the points F and G draw the indefinite line FGM.

The order of the demonstration will be as follows. First, to shew that, according to the construction, the volume of the immersed part SCBT is to the whole magnitude of the solid in the proportion which the specific gravity of the solid bears to that of the fluid: secondly, to shew that the centre of gravity of the solid and the centre of gravity of the part immersed are in the same vertical line; and consequently the construction will place the solid in a position of equilibrium: thirdly, to demonstrate that the equilibrium so constituted is that of stability.

Since by the properties of the circle, HI is to HK as the square of HO is to the square of HK; and the square of HO is to the square of HK as the square of CQ ($= \frac{3}{2} \times$ HO) is to the square of BE ($= \frac{3}{2}$ BF): therefore, since by the construction the specific gravity of the solid is to that of the fluid as HI to HK, it follows, that as the specific gravity of the solid is to the specific gravity of the fluid, so is the square of CQ to the square of BE: but by the properties of the parabolic conoid the magnitude of the segment SCBT is to the magnitude of the whole solid ACBTD as the square of CQ to the square of BE; and consequently it is proved that when the solid floats according to the position described in the construction, the volume immersed SCPT will be to the whole magnitude as the specific gravity of the solid is to that of the fluid, which was in the first place to be de-

monstrated. Secondly, because CQ is the abscissa of the segment SCT corresponding to the vertex C and ordinate SQ, and by the construction $CG = 2\,GQ$, it follows from the properties of the solid that G is the centre of gravity of the segment or part immersed SCBT. By the properties of the parabola, as ON is to CO so is CO to half the parameter, that is, as $ON : CO :: CO = GH : FH$; therefore since the triangles GHF, CON, have one right angle each, and the sides round the equal angles are proportional, the triangles will be similar; consequently the angle OCN = the angle NFG: the sum of the angles FNC, NFC, is therefore a right angle, and the line FGM is perpendicular to the horizontal line PCN; and since F by construction is the centre of gravity of the parabolic conoid, and G has been proved to be the centre of gravity of the part immersed, and the line FGM is vertical, it follows, that the centres of gravity of the entire solid and of the part immersed are in the same vertical line, and consequently the solid is in a position of equilibrium, according to the construction. Thirdly, this equilibrium is that of stability; for let the solid be conceived to be turned round an axis passing through the centre of gravity, through a small angle, in such a direction as to depress the parts towards D, and to elevate those near to A; in that case the lowest point of the curve will be situated between C and B; suppose it to be at W, draw $WX = CQ$, parallel to BE, and take $Wg = \frac{2}{3}$ of WX. Then since* $\overline{CQ}^2$ is to $\overline{BE}^2$ as the specific gravity of the solid to that of the fluid, it is evident that however the axis BE is inclined to the horizon, $\overline{CQ}^2$ and consequently CQ must

* Page 98.

always continue of the same value, and therefore $\frac{2}{3}$ of CQ $= \frac{2}{3}$ of WX or CG $=$ Wg; consequently g is the centre of gravity of the part immersed after the inclination. And since the abscissa or portion of the diameter intercepted between the lowest point and surface of the fluid must always be of the same magnitude while the specific gravity remains the same; and by the construction Wx is made equal to the abscissa CQ; it follows, that when the solid has been so inclined, that the lowest point shall coincide with W, CG $= wg$, and consequently wg is always less than wV; if therefore a line gz is drawn through the centre of gravity g perpendicular to the horizon, the point of intersection z with the horizontal line RU will be between the points F and U; and the pressure of the fluid acting in the direction of the line gz will cause an angular motion in the solid,* which elevates the point D and depresses the point A, or, in other words, will counteract the inclination of the solid, by which it is deflected from its position of equilibrium. By the same method of argument it is shewn, that if the solid is inclined on the contrary direction, a force is created by the position of the centre of gravity of the part immersed, which restores the solid to its former situation, as found by the construction; which therefore places the solid in a position of equilibrium which is permanent.

The several conditions by which this construction is limited will be more easily deduced from analytical investigation, than from having recourse to geometrical constructions.

To represent in general terms the angle CNO, at which the axis of the solid is inclined to the horizon, let BE $= a$; 2HF or the parameter $= p$; also let the specific gravity of the solid be to that of the fluid as n to 1; consequently

* Page 63.

$FH = \frac{p}{2}$; $FB = \frac{2a}{3}$; $BH = \frac{2a}{3} - \frac{p}{2} = \frac{4a-3p}{6}$; and since by the construction $KH : HI : 1 : n$, and $KH = BF = \frac{2a}{3}$, it follows that $HI = \frac{2an}{3}$, and $HO = \frac{2a\sqrt{n}}{3}$; consequently $OB = HB - HO = \frac{4a-3p}{6} - \frac{2a\sqrt{n}}{3} = \frac{4a-3p-4a\sqrt{n}}{6}$; $CO = \sqrt{\frac{4a-3p-4a\sqrt{n}}{6}} \times \sqrt{p}$: and because $ON = \frac{4a-3p-4a\sqrt{n}}{3}$, it appears that CO will be to ON, that is, the tangent of the angle of inclination CNO, will be to radius as $\sqrt{\frac{4a-3p-4a\sqrt{n}}{6}} \times \sqrt{p}$ to $\frac{4a-3p-4a\sqrt{n}}{3}$, or as $\sqrt{\frac{\frac{3}{2} \times p}{4a-3p-4a\sqrt{n}}}$ to 1. When therefore the angle CNO becomes equal to 90°, that is, when the solid floats with the axis in a vertical position, the tangent of inclination $= \sqrt{\frac{\frac{3}{2}p}{4a-3p-4a\sqrt{n}}}$ becomes infinite, or, which is the same thing, $4a - 3p - 4a\sqrt{n} = 0$, and consequently $\sqrt{n} = \frac{4a-3p}{4a}$, precisely coinciding with the limit deduced by a different method* of investigation.

But another inquiry is here suggested. It is evident that this construction is applicable only while the solid floats in such a manner that the whole of the base AD shall be extant above the fluid's surface. To know in what cases this condition takes place, it will be necessary to investigate what must be the value of the solid's specific gravity, and the proportion of the axis to the parameter when the solid floats permanently, so that the surface of the fluid shall pass through one of the extremities of the base A. The result will shew the limit, or limits, if there are more than one, which sepa-

* Page 96.

rate the cases in which the solid floats permanently with the base entirely extant above the fluid's surface, from those in which a part of the base is immersed under it.

The notation remaining as before, since OB or BN $= \frac{4a - 3p - 4a\sqrt{n}}{6}$ and EB $= a$, (fig. 26.) by addition EN $= \frac{10a - 3p - 4a\sqrt{n}}{6}$; and because NW $=$ CQ* $= a\sqrt{n}$, it follows that EW $=$ EN $-$ NW $= \frac{10a - 3p - 10a\sqrt{n}}{6}$: and since AE $= \sqrt{ap}$, the tangent of the angle CNO or AWE, is to radius, as EA to EW, or as $\sqrt{ap} : \frac{10a - 3p - 10a\sqrt{n}}{6}$, that is, making the radius $= 1$, the tangent of the angle AWE or CNO $= \frac{\sqrt{ap} \times 6}{10a - 3p - 10a\sqrt{n}}$; but the tangent† of CNO $= \sqrt{\frac{\frac{3}{2} \times p}{4a - 3p - 4a\sqrt{n}}}$, which two quantities are therefore equal, or $\frac{\sqrt{ap} \times 6}{10a - 3p - 10a\sqrt{n}} = \sqrt{\frac{\frac{3}{2}p}{4a - 3p - 4a\sqrt{n}}}$; or if $1 - \sqrt{n}$ is put $= m$, $\frac{\sqrt{ap} \times 6}{10ma - 3p} = \sqrt{\frac{\frac{3}{2} \times p}{4ma - 3p}}$; and by squaring both sides, $\frac{36ap}{100m^2a^2 - 60map + 9p^2} = \frac{3p}{2 \times 4ma - 3p}$, or $\frac{24a}{100m^2a^2 - 60map + 9p^2} = \frac{1}{4ma - 3p}$, which is reduced to the equation, $100m^2a^2 - 60m\,ap + 9p^2 = 96m\,a^2 - 72ap$; or $m^2 - \frac{60pa + 96a^2}{100a^2} \times m = \frac{-9p^2 - 72ap}{100a^2}$. Wherefore $m = \frac{30pa + 48a^2}{100a^2} \pm \sqrt{\left|\frac{30ap + 48a^2}{100a^2}\right|^2 - \frac{9p^2 + 72ap}{100a^2}} = \frac{30p + 48a}{100a}$

* By the preceding investigation in appears, that HO $= \frac{2a\sqrt{n}}{3}$, and since HO $=$ GC and GQ $= \frac{1}{2}$ GC, it follows that CQ or NW $= a\sqrt{n}$.

† Page 101.

$\pm \sqrt{\frac{576a^2 - 1080ap}{2500a^2}} = \frac{15p + 24a \pm \sqrt{576a^2 - 1080pa}}{50a}$; consequently, restoring the value of $m = 1 - \sqrt{n}$, $1 - \sqrt{n} = \frac{15p + 24a \pm \sqrt{576a^2 - 1080pa}}{50a}$; and therefore $\sqrt{n} = \frac{26a - 15p \pm 6\sqrt{2a} \times \sqrt{8a - 15p}}{50a}$.

Various inferences follow from this determination. In the first place, although the object of the preceding investigation was, to find a single value only of the specific gravity, which would cause the solid to float permanently with the extremity of the base coincident with the fluid's surface, yet by the result it appears, that there are two values of the specific gravity which will answer this condition under a certain limitation, which is also discovered by the solution; this is, that the axis (a) shall be to the parameter (p) in a proportion greater than that of 15 to 8; for if that proportion should be less, $8a$ will be less than $15p$; in which case the quantity $\sqrt{8a - 15p}$ becomes impossible. From which circumstance it may be inferred, that whenever the axis is to the parameter in a less proportion than of 15 to 8, the solid will float permanently on the fluid with the whole of the base extant above the fluid's surface, whatever may be the specific gravity of the solid. This limit is precisely the same with that which is demonstrated by ARCHIMEDES, in the second book of his tract, intituled *de iis quæ in humido vehuntur*, prop. vi. When the axis bears a greater proportion to the parameter than that of 15 : 8, the solid will float either with the base entirely out of the fluid, or partly immersed under it, according to the specific gravity. Having given the axis a in a greater proportion to the parameter p than 15 to 8, by making the

specific gravity $n = \left(\frac{26a - 15p + 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50a}\right)^2$ or $n = \left(\frac{26 - 15 - 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50a}\right)^2$, the specific gravity of the fluid being $= 1$, the solid will float with the extremity of the base in contact with the fluid's surface. If the specific gravity is greater than $\left(\frac{26a - 15p + 6 \sqrt{2a} \times \sqrt{8a - 15p}}{50a}\right)^2$, or less than $\left(\frac{26a - 15p - 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50a}\right)^2$, the solid will float with the base wholly above the surface. If the specific gravity of the solid is to that of the fluid in any proportion between the limits $\left(\frac{26a - 15p + 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50}\right)^2$ to a^2, and $\left(\frac{26a - 15p - 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50}\right)^2$ to a^2, the solid will float with the base partly immersed beneath the fluid's surface.

These limits are determined by geometrical construction in the treatise before quoted (lib. 11. prop. x. *et seq.*) to which construction the preceding investigation may serve as a comment and analysis; and some elucidation of this kind may perhaps be deemed the more requisite, since no traces are to be found in the work referred to of the method of investigation or train of reasoning, by which a problem of so much difficulty was solved, without assistance from analytical operations, at least from any that would seem competent to such an inquiry.*

* Before any proposition can be demonstrated synthetically, it must have been investigated or discovered by some previous train of reasoning: it has been supposed that the ancient geometricians purposely concealed the analysis of their propositions; but as no satisfactory evidence is produced to support this conjecture, it is probable that the supposed concealment arose from the want of a proper notation, by which analytical investigations might be conveniently expressed.

This construction of Archimedes* may be justly regarded as one of the most curious remains of the ancient geometrical synthesis, and is here inserted, in order that the agreement between the solutions by analytical investigation and geometrical construction, may appear in the most satisfactory point of view.

Having given the parabola APBL, (fig. 27.) which is a section of a conoid passing through the axis BD, and having given the axis BD, which is to the parameter in a greater proportion than 15 to 8, it is required to express, by geometrical construction, the two proportions which the specific gravity of the conoid must bear to that of the fluid, so that the solid may float permanently on the fluid when the surface passes through an extremity of the base.

BD represents the axis of the conoid, DA is the greatest ordinate to the axis; join the points B and A, and bisect BA in T; draw TH perpendicular to AD; and with the axis TH, and ordinate AH, describe the parabola ATD; in the axis BD set off DK $= \frac{1}{3}$ of DB, and make KR $= \frac{1}{2}$ the parameter; also set off KC to DB in the proportion of 4 to 15: consequently DB bears a greater proportion to KR than 15 to 4; and since KR is half the parameter, it follows that the axis is to the parameter in a greater proportion than that of 15 to 8. Through C draw CE parallel to DA intersecting BA in E, and draw EZ perpendicular to AD. With the ordinate AZ and axis ZE describe the parabola AEI, and through R draw the line RGY, intersecting the parabola AEI in the points G and Y; through the points G and Y draw the lines ON, PQ, perpendicular to AD, intersecting the parabola ATD in the points X and F.

* *De iis quæ in humido vehuntur,* Lib. ii. prop. x.

Then the proposition affirms that the solid will float permanently on the fluid with the surface thereof in contact with one extremity of the base, when the specific gravity of the solid is to that of the fluid as the square of the line OX is to the square of the axis BD, or as the square of the line PF is to the square of the axis BD.

Instead of inserting the geometrical demonstration of this construction, it will be more expedient, in the present instance, to proceed by a contrary method of argument, *i. e.* by assuming the construction as true, and inferring from it the proportions of the specific gravities in question, and comparing the proportions so inferred with those which have been already found by analytical investigation. Proceeding, according to this method, through the points X and O draw the lines SX, OY, parallel to AD; and since the axis $DB = a$, and $DK = \frac{a}{3}$ by construction, and $KC = \frac{4a}{15}$, it follows that $DC = \frac{9a}{15}$, and $BC = \frac{6a}{15}$; moreover, by the properties of the parabola $DA = \sqrt{ap}$, and the triangles ABD, ECB, being similar, $EC = \frac{DA \times BC}{BD} = \frac{\sqrt{ap} \times 6}{15} = ZD$; moreover, as $DB : ZE$ or $DC :: DA : ZA$, that is, as $a : \frac{9a}{15} :: \sqrt{ap} : ZA = \frac{9\sqrt{ap}}{15}$; and consequently the parameter of the parabola $AEI = \frac{\overline{AZ}^2}{ZE} = \frac{\overline{AZ}^2}{DC} = \frac{9 \times 9 \times ap}{15 \times 15} \times \frac{15}{9a} = \frac{9p}{15}$. And because $RC = EM = KC - KR = \frac{8a - 15p}{30}$, and $\overline{ZN}^2 =$ parameter of the parabola $AEI \times ME$, it follows that $ZN = \sqrt{\frac{ME \times 9p}{15}} = \sqrt{\frac{8pa - 15p^2}{50}}$, and $ND = ZD - ZN = EC - ZN = \frac{\sqrt{ap} \times 6}{15} - \sqrt{\frac{8ap - 15p^2}{50}} = \frac{\sqrt{8ap} - \sqrt{8ap - 15p^2}}{\sqrt{50}}$,

BV

and $BY = \frac{ND^2}{p} = \frac{16a - 15p - 4\sqrt{2a} \times \sqrt{8a - 15p}}{50}$, and $YD = ON$

$= a - \frac{16a - 15p - 4\sqrt{2a} \times \sqrt{8a - 15p}}{50} = \frac{34a + 15p + 4\sqrt{2a} \times \sqrt{8a - 15p}}{50}$;

and since $HN = HD^* - ND$, we shall obtain $HN = HD$

$= HA = \frac{\sqrt{ap}}{2} - \frac{\sqrt{8ap} - \sqrt{8ap - 15p^2}}{\sqrt{50}} = \frac{\sqrt{ap} + \sqrt{16ap - 30p^2}}{10}$,

and $TS = \frac{\overline{HN}^2}{\text{parameter of ATD}} = \frac{17a - 30p + 2\sqrt{2a} \times \sqrt{8a - 15p}}{50}$;

wherefore since $TH = \frac{a}{2}$, and $NX = TH - TS$, it follows

that $NX = \frac{a}{2} - \frac{17a - 30p + 2\sqrt{2a} \times \sqrt{8a - 15p}}{50} =$

$\frac{8a + 30p - 2\sqrt{2a} \times \sqrt{8a - 15p}}{50}$; or finally, $OX = ON - NX =$

$\frac{34a + 15p + 4 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50} - \frac{8a + 30p - 2 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50}$

$= \frac{26a - 15p + 6\sqrt{2a} \times \sqrt{8a - 15p}}{50}$.

By a computation similar to the preceding it is found, that the line $PF = \frac{26a - 15p - 6\sqrt{2a} \times \sqrt{8a - 15p}}{50}$.

It is therefore a consequence, from the geometrical construction assumed as true, that the parabolic conoid will float permanently with the extremity of the base in contact with the fluid's surface; if the specific gravity of the solid is to that of the fluid, either in the proportion of

$\left|\frac{26a - 15p + 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50}\right|^2$ to a^2, or in that of

$\left|\frac{26a - 15p - 6 \times \sqrt{2a} \times \sqrt{8a - 15p}}{50}\right|^2$ to a^2; precisely agreeing with the proportions which were deduced from analytical investigation:†

* $HD = HA = \frac{\sqrt{ap}}{2}$.

† Page 104.

by which agreement both the construction and investigation receive the most satisfactory confirmation.

It has been observed in the course of the preceding pages, that the theorems* investigated to discover the floating positions of bodies, are no less applicable to ascertain the stability of floating, or the resistance which the fluid's pressure opposes to any force applied to incline a floating body from its position of equilibrium. This latter branch of statics is a subject deserving of every attention which science and practical experience can bestow upon it, from the immediate relation it bears to the motion and equilibrium of ships at sea. By this principle, the wind's impulses become effectual in propelling vessels, which, in default of stability, are rather inclined from the perpendicular than moved forward by the force of the wind: and when a ship has been nearly overset by the violence of the elements, it is the power of stability which still sustains, and (if sufficient) at length restores it to the upright position.

The stability of a floating body when inclined through any angle from the perpendicular, has been obtained by investigating a general value of the perpendicular distance GZ† $= \frac{bA}{V} - ds$; (fig. 2.) for the distance between the two vertical lines, one of which passes through the centre of gravity of the solid, and the other through the centre of gravity of the volume immersed. This principle is now to be applied to ascertain the stability of ships: this will be effected by finding either by construction or by calculation, the length of the line GZ: and if the vessel's weight should be W, the measure of stability will be GZ × W, by which it is plainly seen, that if any force M should be applied at a distance from the centre of gravity SG, (fig. 2.) and in a direction perpendicular to SG, to balance

* Page 61. † Page 60.

or counterpoise the force of stability, there will arise the equation $M \times SG = W \times GZ$.

In the particular case, when the angles at which a floating solid is inclined from the position of equilibrium are very small, the line GZ (fig. 2.) has been found* $= \frac{\text{fluent of } \overline{AB}^3 \times \dot{z} \times s}{12V} - ds$: in which expression $\dot{z}$ is a small portion of a line drawn coincident with the fluid's surface, and parallel to the axis of motion; AB is the breadth of the solid at the water's surface, corresponding to the line z parallel to the axis; V is the total displacement or volume immersed; d is the distance GO; and s the sine of the small angle of inclination from the position of equilibrium. Respecting this expression it is observable, that since $\frac{\text{fluent of } \overline{AB}^3 \times s \times \dot{z}}{12V} = ET$, (fig. 2.) and $d = OG = EG$, it follows that $\frac{\text{fluent of } \overline{AB}^3 \dot{z}}{12V} = ES$, and $\frac{\text{fluent of } \overline{AB}^3 \dot{z}}{12V} - d = GS$; which quantity is invariably the same whatever may be the inclination of the floating body from the position of equilibrium, provided that inclination is very small; that is, the point S is immoveable in respect of the point G, while the floating body revolves through any different small angles round the axis, passing through the centre of gravity G in a direction perpendicular to the plane ADHB. Since, therefore, the measure of stability $GZ \times W$ is $\overline{\frac{\text{fluent of } \overline{AB}^3 \times \dot{z}\, s}{12V} - ds} \times W$ and $\frac{\text{fluent of } \overline{AB}^3 \times \dot{z}}{12V} - d = GS$, (fig. 2.) it follows that the measure of stability $= W \times SG \times s$, agreeing with the value which Euler has deduced by other methods for expressing the stability of vessels when the angles of inclination are evanescent.†

* Page 66.

† *Théorie complette de la Construction des Vaisseaux*, chap. viii.

If SG = o, that is, if the centre of gravity of the solid coincides with the point of equipoise S, otherwise called the metacentre,* or centre of equilibrium, the stability will be = o, or in other words, the solid will float in all positions alike, without effort to restore the upright position when inclined, or to incline itself further; it being remembered that the angles of inclination are very small. When the centre of gravity is situated beneath the metacentre, the solid must always float with stability, the measure of which is $W \times SG \times s$, in which case this force acts on the solid to turn it in a direction contrary to that in which it is inclined from the upright position; but when the centre of gravity is placed above the metacentre, (fig. 2.) the quantity $W \times SG \times s$ having passed through o, becomes a force which acts to turn the solid in the same direction in which it is inclined, and will therefore constitute the equilibrium of instability. The determination of the point S becomes, for these reasons, of consequence in estimating the stability of vessels and other bodies when the angles of inclination are very small, and is particularly of use in ascertaining whether a solid, when placed on a fluid in a given position of equilibrium, will float permanently in that position, or will overset. Because it depends on the stability or instability† of floating when the angles of inclination are of evanescent magnitude, whether the solid will continue to float in a position of equilibrium or will revolve on an axis until it settles in some other. These theorems, however, for the measure of stability being applicable only in those cases when the angles of inclination from the position of equilibrium are extremely small, when a ship or other body is inclined 10°, 15°, or 20°, the stability of floating is to be ob-

* Bouguer. Liv. i. sect. iii. chap. iv.

† Page 66.

tained by having recourse to the theorem demonstrated in page 59, where it is shewn, that the stability of a vessel is truly measured by its weight, and the distance between the two vertical lines which pass through the centres of gravity of the vessel and the centre of gravity of the immersed volume; or if s be put to represent the sine of the angle of inclination from the perpendicular, V = the total displacement or volume immersed; A = the volume immersed in consequence of the inclination; b = the horizontal line bc; d = the line GO, (fig. 2.) and W = the weight of the vessel, the measure of the vessel's stability appears by this theorem to be $W \times GZ = \overline{\frac{bA}{V} - ds} \times W$. In applying this expression to any case in practice, it is supposed that the position of the centre of gravity of the ship, and the position of the centre of gravity of the immersed volume, when the ship floats in an upright position, are both known, and consequently the distance of those points, represented by the line GO = d, is a given or ascertained quantity. The total displacement is supposed to have been determined by previous measurements, which quantity is denoted by the letter V; and consequently the weight of a quantity of water, the volume of which is V, will be = W, or the vessel's weight. s, the sine of the angle of inclination from the upright position, is necessarily given from the nature of the case, and may be of any magnitude. The only quantity which remains to be determined, for ascertaining the measure of the vessel's stability, is bA. To facilitate this determination the following observations are premised. If a line be conceived to pass through the centre of gravity parallel to the horizon from the head to the stern, when the ship floats in an upright position, that line is termed the longer axis, to distinguish it from another line, also horizontal, which passes

through the centre of gravity in a direction perpendicular to the former, and is called the shorter or transverse axe. A vertical plane drawn through the longer axe when the vessel floats upright divides it into two parts perfectly similar and equal; in which particular the figures of ships may be termed regular; although in other respects they are of forms not restrained to any uniform proportions. From the equality of these two divisions of a vessel, it must necessarily happen that when it floats in a quiescent position the similar parts on the opposite sides will be equally elevated above the water's surface. A ship thus floating in a position of equilibrium may be conceived to be divided into two parts, by the horizontal plane which is coincident with the water's surface; and the section formed by this plane passing through the body of the vessel is termed the principal section of the water, and is represented in fig. 2. as coincident with the line AB: when the ship is caused to heel, by being inclined round the longer axe through any angle SGK or NXB, (fig. 2.) the plane in the ship represented by the line AB will be transferred to the position IN, and the section of the water will now pass through the vessel in the direction of a plane coincident with AP, inclined to the former plane in the angle NXP, and may be termed, merely for the sake of distinction, the secondary section of the water. These two planes intersect each other in the line denoted by the point X, or rather in the line which is projected into the point X on the plane ABDH. Since the vessel is supposed to be inclined round the longer axe, it follows, that the line of intersection denoted by X will be parallel to that axis. And since from the laws of hydrostatics the volume PXN, which has been immersed in consequence of the inclination, is equal to the volume IXW, which has been elevated

above the water's surface by the same cause are precisely equal, the position of the line represented by the point X (always parallel to the axis) will depend on the figure which is given to the sides of the vessel PN, WI. It has been seen that when the figure is a parallelopiped floating with two plane angles thereof immersed, the point X (fig. 6.) bisects the lines corresponding to AB or IN in fig. 2: when the same solid floats with one plane angle only immersed, (fig. 10.) the point X is removed nearer to those parts of the solid which are more immersed by the inclination. In a ship, the breadth of which continually alters from the head to the stern, and in no regular proportion expressible by geometrical laws, it is evident that the position of the point X, representing the line in which the water's surface intersects the vessel in its two positions, must be determined practically by methods of approximation, from which, at the same time, the other requisites for this solution will be obtained. Since to find the value of the quantity bA in the expression $W \times \overline{\frac{bA}{V} - ds}$, it is necessary that the position of the point X should previously be known: to determine this particular it will be expedient to conceive the volume (fig. 2. and 28.) NXP, which has been immersed in consequence of the inclination, and that which has been elevated above the fluid's surface, or IXW, to be divided into segments, by vertical planes passing perpendicular to the longer axis, and at a distance of a few feet from each other, for instance, 2 or 3 feet; each of these segments will be of a wedge-like form, (fig. 28.) contained between two planes, $XxPp$ and $XxNn$, inclined to each other at the given angle of inclination NXP; two vertical parallel planes NXP, nxp, which are nearly equal, and the portion $NPnp$, of the ship's side.

The distance between the planes NXP, *nxp*, is the line Xx = Nn = Pp; Xx produced, is the line in which the two sections of the water intersect each other, and is therefore coincident with the water's surface, and is parallel to the longer axis. The dimensions of the vessel being supposed known, the lines AB, NI, will be known in fig. 2: from these data the lines NX, PX, (fig. 2. and 28.) are to be assumed by estimation, and the angle NXP being given by the supposition, the area NXP is known from the rules of trigonometry, and the area PTNP may be inferred by the known methods of approximation.*

In like manner the area *xptn* is to be determined, and a mean of the two areas being multiplied into the thickness or

* Stirling. *De Interpolatione Serierum*, prop. xxxi. Chapman. *Traité de la Construction des Vaisseaux*, ch. i.

Methods of approximating to the areas of curves, founded on the differential serieses, are given by several authors, particularly by Stirling and Simpson. Admiral Chapman proposes a very ingenious method of approximation, depending on the properties of the parabola; either is sufficiently exact for the purposes of practical geometry, as appears by the instance inserted underneath: but of the two methods that of Mr. Stirling is the most correct. The two methods of approximation are severally applied and compared in the following example of finding the curvilinear area, which is comprehended between an arc of 30° and the radius, sine, and cosine of the said arc: to obtain this area by approximation, 5 equidistant ordinates are given; *i. e.* 1st. ordinate = radius = 8, 2d. = $\sqrt{63}$: 3d. = $\sqrt{60}$, 4th. = $\sqrt{55}$, 5th. = $\sqrt{48}$.

The approximate area is,

According to the method of Stirling, -	30.61153	According to the method of Chapman, -	30.61131
Correct area - -	30.61156		30.61156
Error of approximation —	.00003	—	.00025

The same method by which the areas of curves are found by approximation, may be applied with equal exactness to determine the solid contents of space, and the position of the centre of gravity.

distance Xx will be the solid contents of this segment, to a degree of exactness fully sufficient for the purposes of this approximation. In the same manner the solid contents of all the segments which are elevated above the surface are to be obtained by making $XI = AB - NX$, $XW = AB - PX$, and proceeding as in the former case. If the aggregate of the segments NXP representing the part immersed, in consequence of the vessel's inclination, should not be equal to the aggregate of the segments IAW, (fig. 2.) which are elevated above the surface, the position of the point X, or rather of the line which that point denotes, must be altered, and the same operations repeated till the sums of the segments on each side of the said line of inclination are precisely equal.

This having been effected, the magnitude of the volume immersed, denoted by A in the expression $W \times \overline{\frac{bA}{V} - ds}$, will be known; and the magnitude of each of the individual segments NXPnxp and IXWixw, &c. will also be known; the quantity bA will be found in the following manner. The area PXNTP and its centre of gravity d are to be determined by methods of approximation. Through d draw dc perpendicular to the horizontal line PX; Xc* will be the

* The solution of problems by geometrical construction has been little practised since methods of calculation have been so much improved by the invention of logarithms and other facilities: the solutions of difficult cases are, however, sometimes obtained with sufficient exactness by construction, which would be more troublesome by any other method: in the present instance, after the area PTNP and its centre of gravity have been determined, the position of the centre of gravity d, of the entire area XNTP, and the length of the line Xc, may be most easily ascertained by the method of construction. If the line PN is bisected in the point C, the centre of gravity of the triangle PXN will be situated at the distance of $\frac{1}{3}$ CX from the point C: the centre of gravity of the triangle PXN being thus constructed with geometrical exactness, it follows, that the centre of gravity of the entire area PXNTP, which is the

distance of the centre of gravity d from the point X, estimated in the direction of the horizontal line PX.

The same operations being applied to the area $xptn$, will give the distance ex of the centre of gravity of the area $xptn$, from the point x, estimated in the direction of the horizontal line px; the mean of the two distances so found will be the distance of the centre of gravity of the solid segment XPNxpn, from the line Xx, estimated in the direction of the horizontal line XP or xp, to a degree of exactness entirely sufficient for this approximation.

Similar distances of the centres of gravity of all the segments (fig. 2. and 28.) PXNpxn, corresponding to the line Xx produced, having been found, also of all the segments IXWixw, if each of these segments is multiplied into the distance of its centre of gravity from the line Xx, estimated in a horizontal direction, the sum of the products so formed will be the value of the quantity bA in the expression $W \times \overline{\frac{bA}{V} - ds}$, which is the measure of the vessel's stability, when inclined from its upright position through an angle PXN of which the sine is to radius as s to 1: and the quantities* W, V, and d, having been previously determined, it is evident that from the methods which have been described, the vessel's stability when inclined to the given angle will be obtained.

It would be improper, in a disquisition not written on the practice of naval architecture, to enter into further detail on this subject. By what has preceded, it is evidently seen that the stability of vessels may be determined for any angles at which they are inclined from the position of equilibrium, as well as for those which are very small. In both cases

common centre of gravity of the areas PXN and PNTP, is capable of being determined with very great precision.

* Page 111.

it is necessary that the position of the centre of gravity of the ship, and that of the part immersed, when the ship floats upright, should be known; practical methods of mensuration are required, in both cases, to ascertain these points. When the angles of inclination are very small, to find the ship's stability, it is necessary to measure* the successive ordinates or breadths of the ship on a level with the water's surface, and when the angles of heeling are not limited, but are considered as being of any magnitude, the requisite mensurations are indeed more troublesome, but are not liable to more errors in execution than in the former case, when the angles are limited to those which are evanescent.

The theorems for measuring the stability of ships, which are founded on assuming the angles of inclination from the position of equilibrium evanescent, explain, in the most satisfactory manner, the principles on which the stability of ships, when heeled to small angles of inclination, is founded; they also ascertain when ships or other bodies float on the water permanently in a given position of equilibrium, or overset. But this can scarcely ever be an object of inquiry in respect of ships, which are always constructed so as to float upright, even before any ballast or lading has been added to them.

Mons. Romme, in his valuable work on naval architecture, intituled *L'Art de la Marine*, published at Paris in the year 1787, informs his readers (p. 106), that the French ship of the line of 74 guns, called Le Scipion, was first fitted for sea at Rochfort in the year 1779. As soon as the ship was floated in deep water, a suspicion arose that she wanted stability; to ascertain this point the guns were run out on one side, and drawn in at the other; in consequence, the ship heeled 13

* Chapman, chap. i. Clairbois *Architecture Navale*, part. ii. sect. i.

inches (probably meaning at the greatest measure on the side of the vessel): by adding the weight of the men brought to the same side, the depth of heeling increased to 24 inches. This being a degree of instability, which was deemed too great to be admitted in a ship of war, the ship was ordered into port, that some remedy might be applied to the defect which had been discovered. M. Romme proceeds to relate, that a difference of opinion prevailed amongst the engineers respecting the cause of this imperfection in the ship, and the remedies by which it might be corrected. The chief engineer, who was sent from Paris to Rochfort to direct what measures ought to be adopted on this occasion, and for rectifying the like fault in two other ships of war, L'Hercule and Le Pluton, was of opinion, that the stability of the ship Le Scipion would be sufficiently increased by altering the quality and disposition of the ballast. The original ballast of the Scipio had been 84 tons of iron and 100 tons of stone; according to the new arrangement of the chief engineer, the ballast was composed of 198 tons of iron and 122 tons of stone. But as a ship of war does not admit of any alteration in the total displacement or immersed volume, to compensate for the additional weight of ballast, amounting to 136 tons, the quantity of water with which the ship had been supplied was diminished by the weight of 136 tons. This alteration must necessarily have the effect of lowering the centre of gravity of the vessel, and thereby of increasing its stability: but, on trial, this increase was by no means sufficient; the diminution of heeling measured on the vessel's side being only 4 inches. After this and other ineffectual attempts, the defect of stability was at length remedied by applying a bandage or sheathing of light wood to the exterior sides of the vessel, from 1 foot to 4 inches in

thickness, extending throughout the whole length of the water line, and 10 feet beneath it.

This account shews that the theory of stability, restrained to cases in which the angles of inclination, or heeling, are very small, cannot be relied on for ascertaining the requisite stability of ships in the practice of navigation. It must be supposed that the weight and dimensions of every part of this ship were exactly known to the engineers, yet we observe that the instability was not certainly ascertained, but suspected only to exist when the ship was first set afloat in deep water; and after this defect had been discovered by the experiment which has been related, the cause was sought for in vain, and the remedy at length was stumbled upon by accident, rather than adopted from any knowledge of the principles by which the application of it might have been directed.

It seems allowable to suppose, that if rules for ascertaining stability correspondent to any different angles of heeling, similar to those which are demonstrated in page 60, and exemplified in page 115 of this tract, had been applied to the case in question, they would have discovered that an error in the form* given to the sides of the vessel was the principal cause of the defective stability, and would have suggested the remedy accordingly; or rather would have prevented the necessity of having recourse to it, by previously shewing the original defects in the plan of the ship.

The force of stability by which ships, when inclined round

* Mr. ROMME observes, page 108, that the defect of stability in the Scipio was not occasioned by any want of breadth in the principal section of the vessel; for other ships of the same force, *i. e.* Le Magnifique, Le Sceptre, Le Minotaur, L'Intrepide, the breadths of which were the same, or rather less, than that of the Scipio, carried their sail perfectly well.

the longer axis from their position of equilibrium through different angles, endeavour to regain that position, is to be considered in two points of view respecting the motion of a vessel at sea; first, in relation to the resistance by which it opposes any force that may be applied to incline the ship, for instance, that of the wind; in which case the ship's stability, and the impulse of the wind, constitute a species of equilibrium as long as the wind continues of the same intensity. Secondly, the force of stability is to be considered as operating on the ship, after the force by which it has been inclined ceases, to restore the vessel to its upright position; the ship being continually impelled by the force of stability, revolves round an horizontal axis, passing through the centre of gravity with an increasing velocity, till it arrives at its upright position; and afterwards with a velocity continually retarded, till it arrives at the greatest inclination on the other side. This rolling of the ship, with alternate acceleration and retardation of the angular velocity, will evidently depend on the force by which the angular motion is generated; that is, on the force of stability, and its variation corresponding to the several angular distances of the vessel from its upright position; from this cause arises one of the principal difficulties in the practice of naval architecture; *i. e.* to give a vessel a sufficient degree of stability, and at the same time to avoid the inconveniences which proceed from an angular velocity of rolling, increasing and decreasing too rapidly. It is certain that the variation of the force of stability depends principally on the shape given to the sides of the vessel, which admit of being so constructed (all other circumstances permitting) that the force shall increase either slowly or rapidly to its limit.

From the preceding investigations we observe that some float-

ing bodies, during their inclination from 0° to 90°, pass through a position of equilibrium, in which the force of stability becomes evanescent: in other bodies, no limit of this kind takes place; a difference which depends partly on their forms, and partly on the disposition of the centres of gravity of the solids and of the immersed volumes. It may be satisfactory to consider, in a general view, the effects produced on the motion of ships by the different proportions of their stability while they are inclined round the longer axes. If a vessel* should be of a cylindrical form, floating with its axis horizontal, the vertical sections must necessarily be equal circles: supposing the centre of gravity of such a cylinder to be situated out of the axis, the vessel will float permanently with its centre of gravity, and the centre of the section passing through it, in the same vertical line: if such a vessel should be inclined from the upright by external force, it will be impelled in a contrary direction by the force of stability, which increases exactly in the proportion of the sine of the angle of inclination: it is plain, therefore, that a vessel of this description, during its inclination by heeling, cannot arrive at any limit where the force of stability is evanescent; on the contrary, it must continually increase until the inclination is augmented to 90°, where it will have become greater than at any other angle.

Let another case be assumed: suppose the form of the vessel to be a square parallelopiped, floating permanently with one of the flat surfaces upward; when this solid has been inclined round the longer axis through 45 degrees, the stability will be evanescent, and the least inclination greater than that angle

* This is evidently an hypothetical case, stated with a view of illustrating the subject.

will cause the vessel to overset: in this case, as the vessel is gradually inclined from the upright, the stability will first increase to a maximum, and afterwards decrease; differing altogether from the variation of the stability in the preceding case, when the vessel was supposed to be of a cylindrical form. Although vessels are usually so constructed that during any inclination from 0° to 90° they do not pass through a position of equilibrium; yet there seems reason to suppose that in some vessels the stability increases to a maximum, and afterwards decreases when the angle of inclination is farther augmented: whenever a vessel of this description should be inclined beyond the angle where the stability is greatest, the following consequence must necessarily ensue; if the angular velocity should be considerable, the rolling of the ship will be extended to large angles of inclination, because when the stability is more and more diminished as the angle of inclination is augmented, more time will be required for the diminished force to react against the ponderous mass of the vessel, in order to restore it to the upright. It is certain that the angle, as well as the celerity or slowness of rolling, depend on other elements, as well as on the stability, particularly on the weight and extent of the masts and sails, and the position of the ballast and lading: but in comparing the vibrations of the same vessel through different arcs, those elements are the same, while the force of stability alters continually as the angles of inclination are increased or diminished.

These alternate vibrations of a ship in rolling have been deemed analogous to the oscillations of a pendulum; and in order to reduce to some kind of measure so essential a quality of vessels, M. BOUGUER and other writers propose to find a pendulum isochronal to the oscillations of a ship. This pro-

blem seems to imply both that the pendulum sought, and the vessel itself, shall vibrate in arcs that are extremely small; for otherwise the analogy altogether fails: no oscillating body can describe arcs of unequal lengths in equal times, unless it is impelled by forces which are in the direct ratio of the distances from the quiescent point; and therefore the oscillations of a vessel vibrating in different finite angles are evidently not isochronal with each other, since the force of stability varies in a proportion so different from that of the distances from quiescence; nor can they be isochronal with any pendulum, unless the arcs of vibration are of evanescent magnitude; in which case the force of stability being in the direct proportion of the angles of inclination from the upright, has the effect of producing an equality in the times of oscillation: to ascertain a pendulum vibrating in small arcs which is isochronal to the oscillations of a vessel, under these restrictions, is a problem which may be solved with sufficient exactness; but unless the limitation that has been mentioned should be specified, it is a question without the necessary conditions. Mons. BOUGUER* in his chapter intituled, *que les Oscillations sont Isochrones*, does not expressly mention this limitation, but we must allow it probable that he conceived it to be implied.

From the reasons that have been stated it seems to follow, that in order to form a satisfactory opinion of the qualities and performance of a vessel at sea as depending on the plan of its construction, the forces of stability at the several angles of inclination from 0 to the greatest limit ought to be ascertained, particularly the measure of the greatest stability, and the angle of heeling at which it takes place.

* Liv. i. sect. iii. chap. vii.

In these general remarks the water's resistance has not been considered, which must necessarily have some effect in retarding the oscillations of the vessel, and more in the larger arcs than in the smaller: it is however observable, that the resistance to the rolling of vessels is of a very different kind to that which is opposed to their progress through the water, in which case a volume of the fluid proportional to the vessel's bulk and velocity is entirely displaced during its motion; whereas in the rolling of ships a far less quantity of water suffers an alteration of place by the ship's oscillations, which is therefore the less retarded on this account.

Another observation occurs on this subject. The entire stability of a ship has been shewn to consist of the aggregate stabilities of the several vertical sections into which it can be divided. Let it be supposed that the ship has been inclined round the longer axis through a given angle, and that the vessel returns through the same angle of inclination by the force of its stability; if the forces arising from the several sections do not act in their due proportion on each side of the centre of gravity, in respect to the longer axis, the ship will not return to its position of equilibrium by revolving round the longer axis; but will be inclined round various successive horizontal lines between the longer and shorter axes; a circumstance that must create irregular motions and impulses, to which a vessel in all respects well constructed is not liable.

The theory of statics and mechanics was, I believe, first applied to explain the construction and management of vessels toward the latter end of the last century, in a work intituled *Théorie de la Construction des Vaisseaux, par* P. Paul Hoste, printed at Lyons in the year 1696. Several eminent mathematicians have since prosecuted this difficult subject,

particularly JOHN BERNOUILLI, BOUGUER, and the excellent M. EULER, whose treatise, intituled *Théorie complette de la Construction & Manœuvre des Vaisseaux*, is a work correspondent to the title, entirely theoretical. In this elaborate performance the author has not only endeavoured to explain the complicated laws which influence the motion of ships at sea, but proceeds to investigate, on the ground of such data as the subject affords, the dimensions and position of the most essential parts of vessels which combine to give them every possible advantage in the practice of navigation.

Several inquiries are suggested by the perusal of these theoretical works; first, whether the proportions and dispositions of parts in ships resulting from theory have been found to differ from, or to agree with, those which had been previously established in the practice of naval architecture; secondly, if disagreement should have been discovered, whether any adequate and satisfactory trials have been made to ascertain the advantages which result from adhering to the constructions prescribed by practice, compared with those which are consequences of following the deductions from theory; and lastly, if any new forms of vessels, disposition of parts, or other varieties of construction, have been discovered by considering this subject in a theoretical view, and in what degree these inventions have been found advantageous when applied in practice.

Exclusive of the application of geometrical principles,* by

* Practical treatises on ship-building have been published by various authors, particularly by M. CLAIRBOIS, ROMME, and FRED. CHAPMAN. In these useful works theory is occasionally applied to explain and illustrate the principles of naval architecture: but no accounts are to be found in either of these volumes, as far as my researches extend, by which the construction of vessels, founded on theoretic investi-

which the forms of vessels and the disposition of their most essential parts are ascertained, theory may be considered as bearing to naval architecture a two-fold relation: first, as depending on the pure laws of mechanics, a subject on which the preceding cursory observations have been offered: secondly, the practice of naval architecture is guided, in most parts of the world, by a species of theory or systematic rule which individuals form to themselves from experience and observation alone: it is founded on the experimental knowledge in naval constructions, which has been transmitted from preceding times, combined with the more recent improvements, and includes whatever inventions of skill and ingenuity are applicable to the various machinery that is employed in the construction and management of vessels: by repeated observation on the forms, proportions, and equipment of ships, and by attention to their excellencies and defects when afloat at sea, faults are remedied, good qualities are improved, and rules of practice are by degrees established according to principles, well perceived and understood, without much assistance from the theories of mechanics, statics, and geometry, on which such principles are founded: for in this, as well as other instances, it is well known that skilful practice, aided by long experience, arrives at determinations which it is very difficult (sometimes impossible) for theory to infer: on the other hand it must be allowed, that pure theory, depending on the laws of motion, the subject of disquisition in

gation, have been subjected to practical examination during voyages. M. CHAPMAN, in page 79 of his work (Paris edit.), expresses the proportions and disposition of parts in vessels by algebraic quantities, which, however, are not to be mistaken for deductions from theory; since the author has not pointed out any mode of investigation, or train of reasoning, by which those expressions can be deduced from the principles of mechanics.

the works of M. EULER and BOUGUER, is of great importance to the advancement of this science: for by such investigation, so far as the data are sufficient, the qualities of vessels are traced to their true causes, and are explained by general laws; whereas the principles derived from mere observation are scarcely ever applicable beyond the cases in which they have been experienced in practice.

Whatever may have been the means by which naval architecture receives progressive improvement, it seems to be generally allowed, that the art of constructing vessels has, at the present period, attained to a degree of perfection far surpassing any that has been known to former times, either ancient or modern; yet it is equally certain, that some principles, by which the construction of vessels is materially influenced, still remain to be developed and explained. It is frequently remarked by navigators, as well as by naval architects, that alterations apparently the most trivial, in the form of a vessel, in the distribution of the ballast, or in the position and extent of the masts and sails, will wholly change the qualities of a ship from bad to good, or the reverse. As these changes cannot be attributed to fortuitous causes, it is necessary to allow that they are consequences of principles certain and definite, though in many cases unknown, or imperfectly estimated by conjecture. The proportions and disposition of parts, which operate to produce good or bad effects on the sailing of ships, are probably in these instances so intricately combined as to make it scarcely possible from mere observation, however extended and diversified, to account satisfactorily for changes so remarkable: it must also be acknowledged, that some of the data on which the theory of naval architecture is founded, being imperfectly known, parti-

cularly the laws of the different resistances to the ship's motion,* it would be unsafe to rely entirely on deductions *a priori* for explaining this subject.

* The laws of resistances, opposed to bodies which move in fluids, and varying in a duplicate ratio of the body's velocities, are demonstrated by Sir Isaac Newton, in the second book of the *Principia,* on conditions restrained to the particular case in which the motion of the resisted body is extremely slow, and the fluid perfectly compressed. On these conditions, the pressure which resists the motion of the body is exactly balanced by the pressure on the posterior part, and consequently the only force opposed to the body's motion, is the *inertia* of the fluid, which is displaced while the body moves through it: for the resistance of friction depending on the body's velocity must be, in a physical sense, evanescent, when the motion is very slow. It is evident, that the theory of resistances founded on these principles ought not to be applied to the solution of cases in which the velocity is much increased, without great care and circumspection; for by the increase of velocity, three different forces begin to have operation, of which the Newtonian theory takes no account; *i. e.* the pressure on the anterior part of the body, the pressure on the posterior part, and the resistance of friction. The pressure on the anterior part will evidently be a constant or invariable quantity as long as the moving body continues at the same depth. The pressure on the posterior part will depend on the velocity of the body's motion, and when that velocity is $= 0$, the pressure will be precisely equal, and contrary to that which acts on the anterior part. Moreover, when the body's velocity is equal to that with which the fluid rushes into empty space, the pressure on the posterior part will be $= 0$, and of consequence all the pressures on the posterior surface, corresponding to the intermediate velocities, must be found between these limits. When the surfaces of the moving body are smooth, it has been supposed that the effects of friction are not very considerable. This opinion is however disproved, to the satisfaction of any one who consults the account of the very accurate and well devised experiments on the motion of bodies through the water, made under the direction of the committee of the Society for the Improvement of Naval Architecture, and published by their order. I have examined these experiments with a good deal of attention, particularly those which were made on oblong beams or parallelopipeds, denoted in the account of the experiments by the letters A, B, &c.; and find, that although the surfaces of the moving body were planed very smooth, the resistance of friction was equal to a weight of no less than ninety pounds, on a surface of 258 square feet, when the body moved with a velocity of 8 feet in a second. It appears also, by methods of calculation, founded on Sir Isaac Newton's rule for drawing a parabolic line through any number of given points situate in the same plane, and applied to the above-named experiments, that the resistance of friction varies in no power of the velocity expressible by less than three di-

These difficulties will appear still greater, if it be considered that the causes which influence the motion of ships at sea are not separate and independent, but operate on each other, as well as immediately on the motion of the vessel: thus, if the position of the centre of gravity is altered by moving the ballast or lading nearer to the head or stern, this alteration will have the effect of changing the section of the water, and the form of the immersed part of the vessel; on which account, the resistance opposed by the water to the ship's motion must necessarily be changed; the centre of gravity of the part immersed will also be differently situated, which must combine

mensions thereof, that is, if z is put to denote the resistance of friction, and v to denote the velocity, the resistance requires an equation of the form $z = au + bu^2 + cu^3$; in which a, b, and c, are invariable quantities: the force also of pressure on the posterior surface is expressed by an equation equally complex: to these difficulties another is to be added, which is, that the resistance varies with the depth of the moving body, as appears by the experiments referred to. On these considerations it seems manifest, that investigations on the subject of naval architecture, founded on the theory of motion, which takes into account the resistances of the water, considering the velocity to be such as ships usually sail with, must involve algebraic expressions so complicated, as to make it very difficult, perhaps impossible, to infer any useful practical conclusions from this mode of considering the subject. EULER and BOUGUER, the principal authors who have attempted to apply the theory of resistances to naval architecture, suppose the resistance to be in a duplicate ratio of the velocities; a law evidently different from that according to which vessels at sea are opposed by the medium in which they move: and one of these most eminent authors,* doubts whether this theory is not too imperfect to be relied on, when it is applied to ascertain the motion of ships at sea. Notwithstanding the impediments which arise from the complicated laws of resistance and friction, the general principles investigated in the works of these authors are no doubt capable of being applied to the solution of many difficulties which occur in considering the subject of naval architecture, due allowance being made for those irregular forces which cannot be included in the theoretic solutions.

* EULER. *Théorie complette de la Construction des Vaisseaux,* English edition, p. 93, 94.

with the alteration of the centre of gravity of the vessel, and the section of the water, to increase or diminish the stability of the ship; and it must be added, that the inclination of the masts and sails to the horizon, and the direction in which the wind impinges on them, will suffer alteration from the same cause.

Although theory alone may not be adequate to the solution of these difficulties, yet, when combined with experiments and obervations, it may be probably employed with great advantage in these researches. If the proportions and dimensions adopted in the construction of individual vessels are obtained by exact geometrical mensurations, and calculations founded on them, and observations are made on the performance of these vessels at sea; experiments of this kind, sufficiently diversified and extended, seem to be the proper grounds on which theory may be effectually applied in developing and reducing to system those intricate, subtile, and hitherto unperceived causes, which contribute to impart the greatest degree of excellence to vessels of every species and description. Since naval architecture is reckoned amongst the practical branches of science, every voyage may be considered as an experiment, or rather as a series of experiments, from which useful truths are to be inferred towards perfecting the art of constructing vessels: but inferences of this kind, consistently with the preceding remark, cannot well be obtained, except by acquiring a perfect knowledge of all the proportions and dimensions of each part of the ship; and secondly, by making and recording sufficiently numerous observations on the qualities of the vessel, in all the varieties of situation to which a ship is usually liable in the practice of navigation.

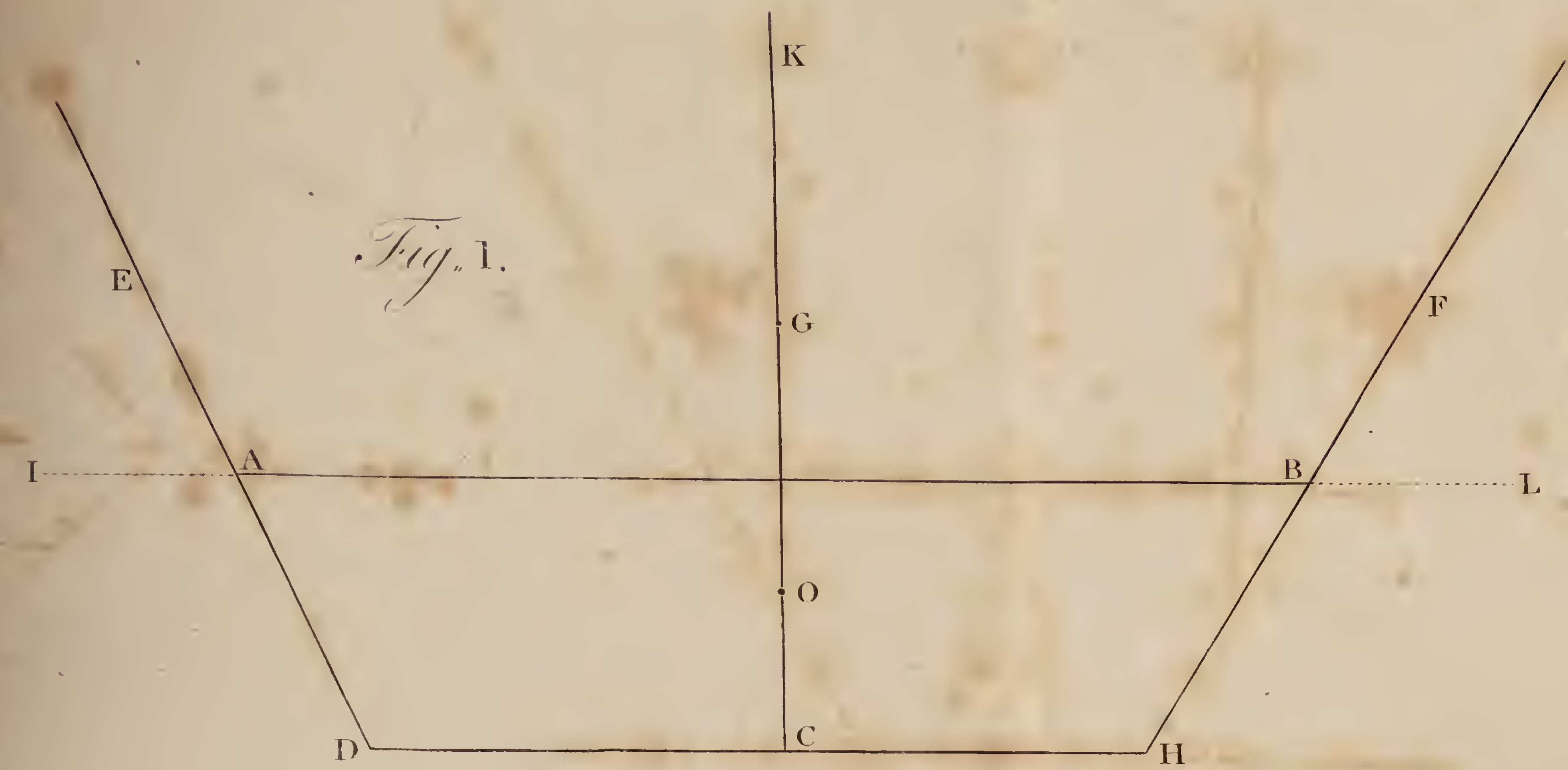
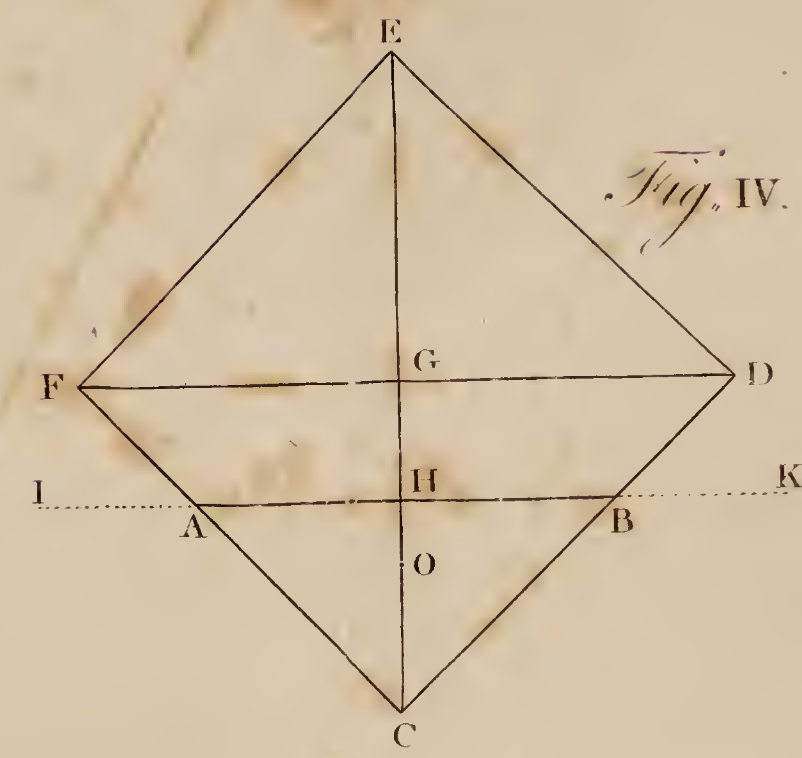


Fig. I.



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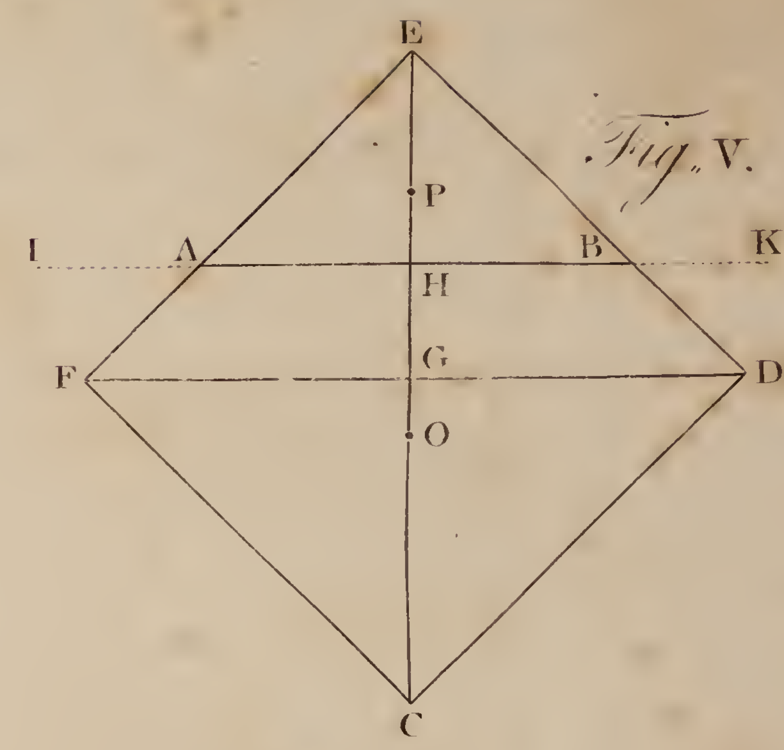

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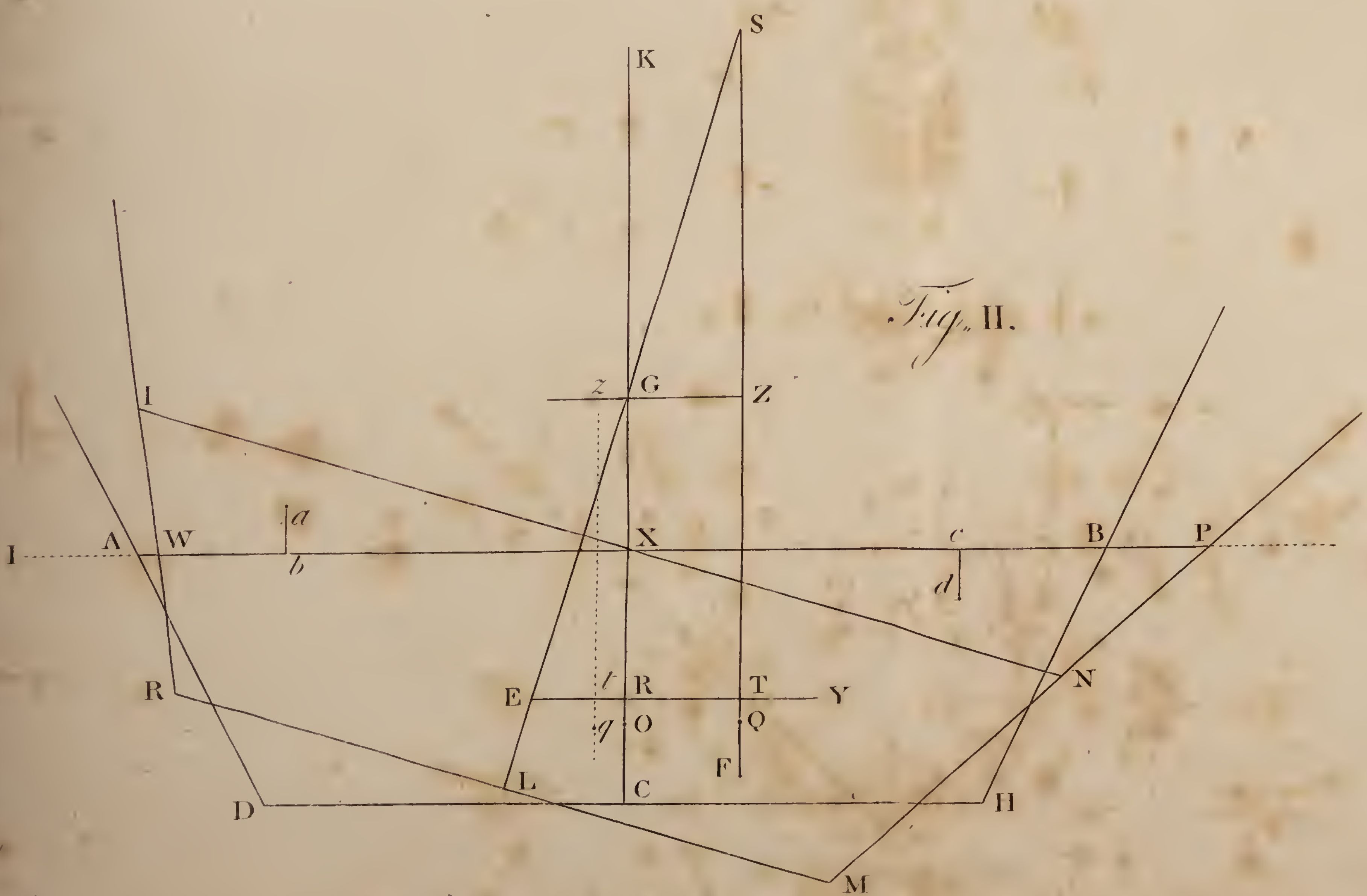

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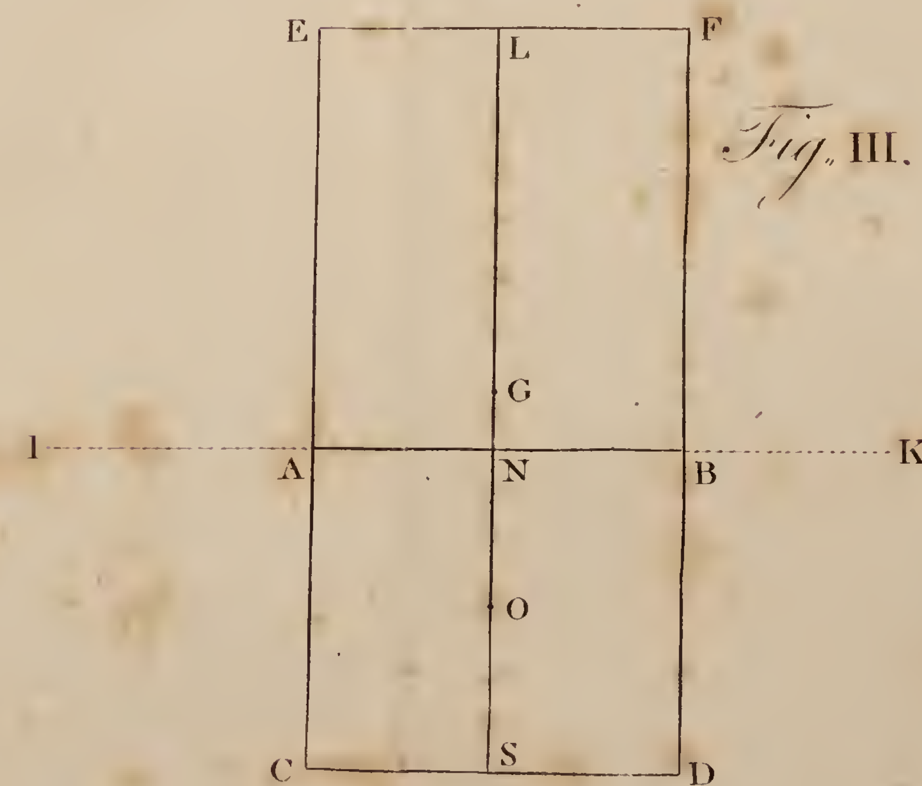

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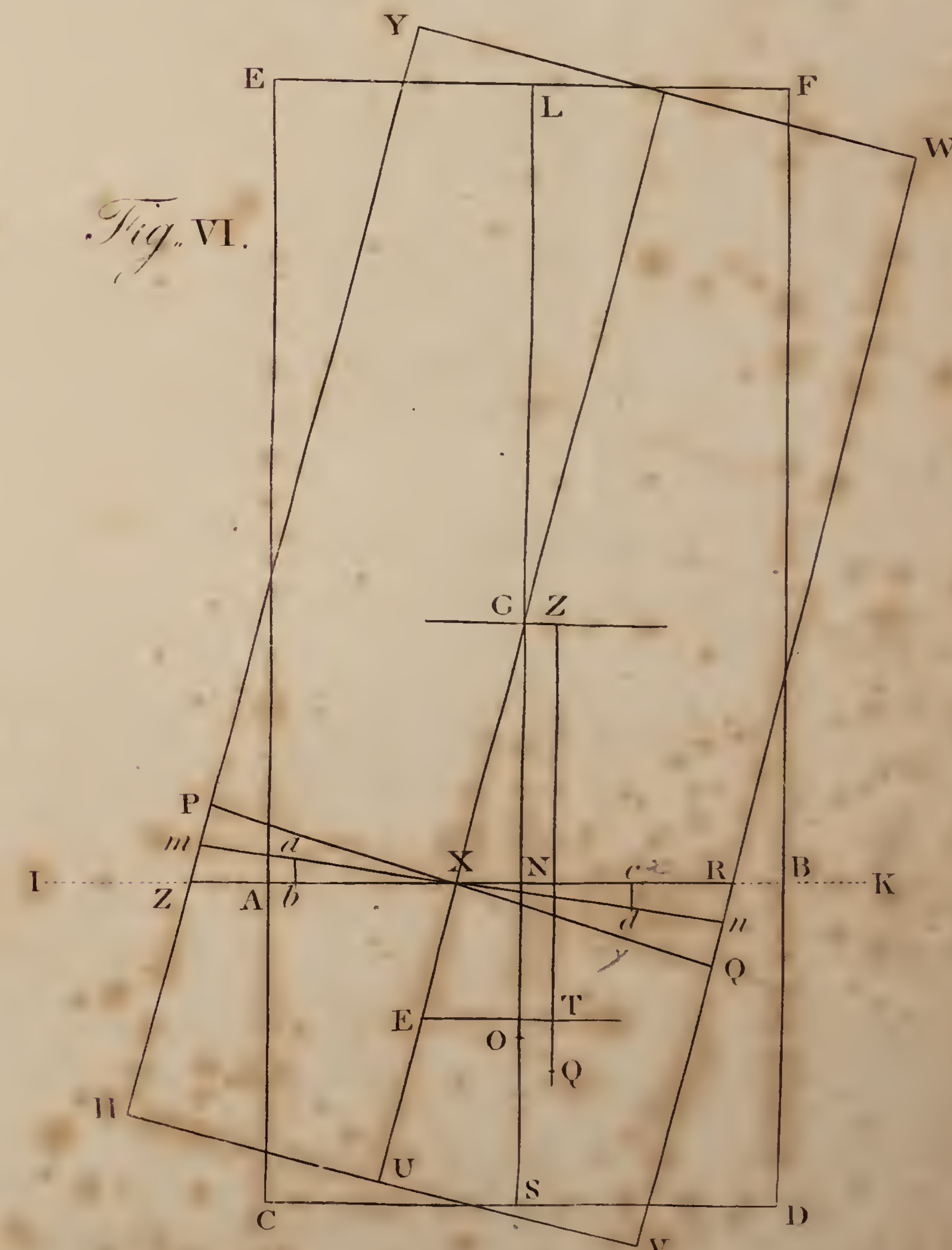

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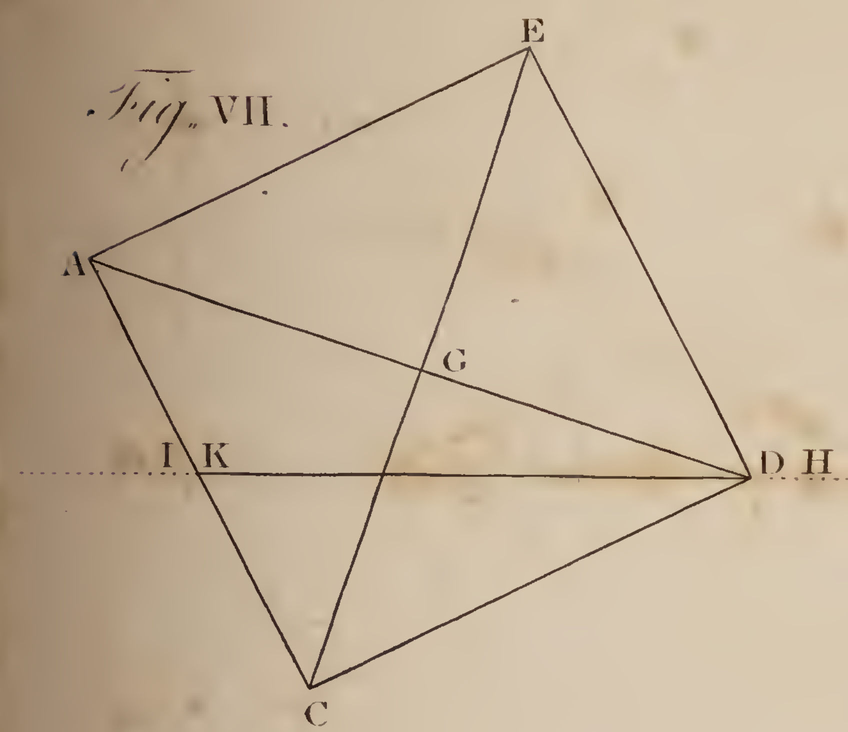

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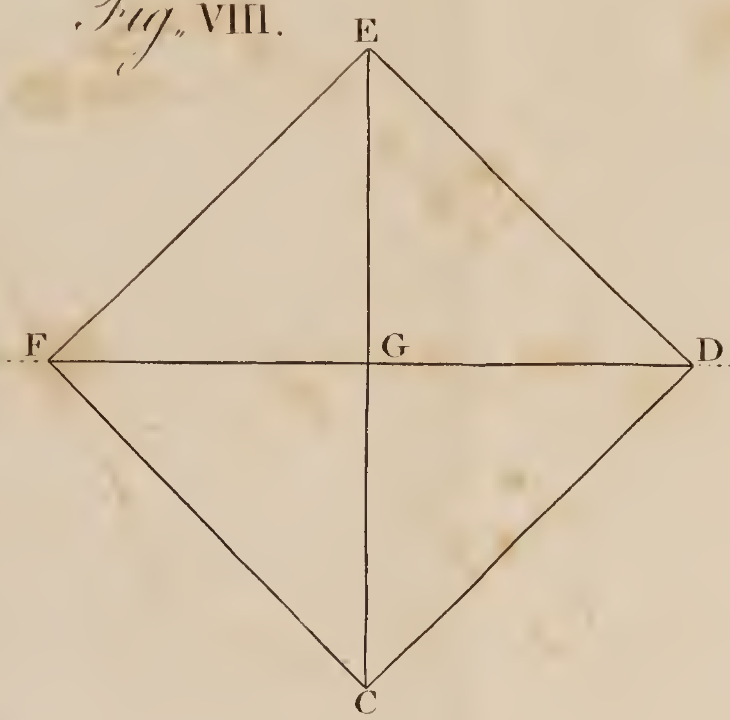

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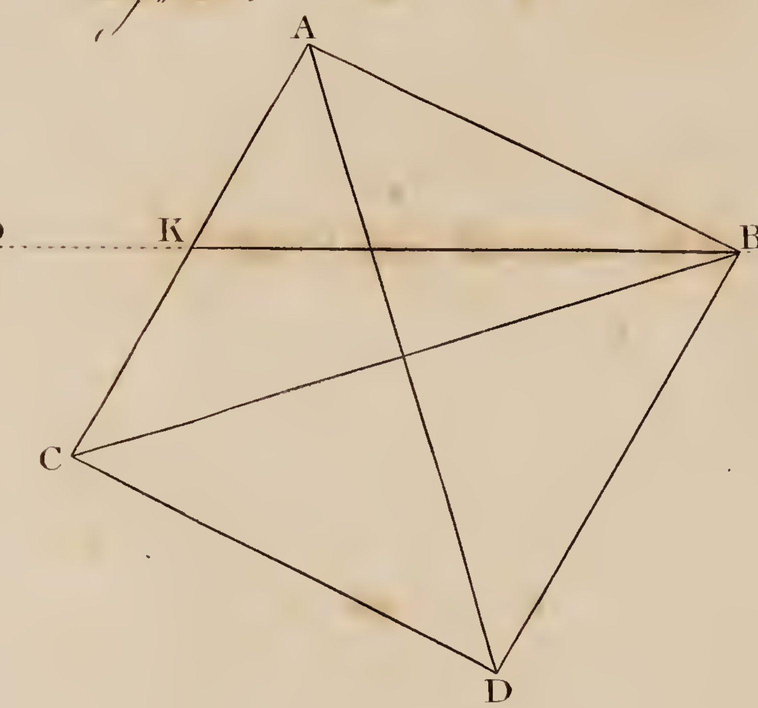

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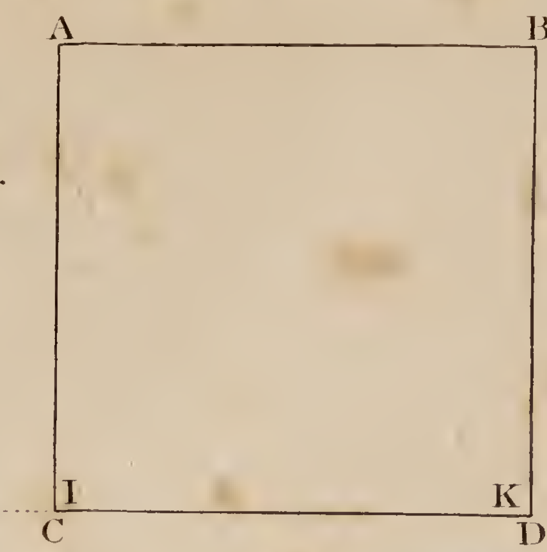
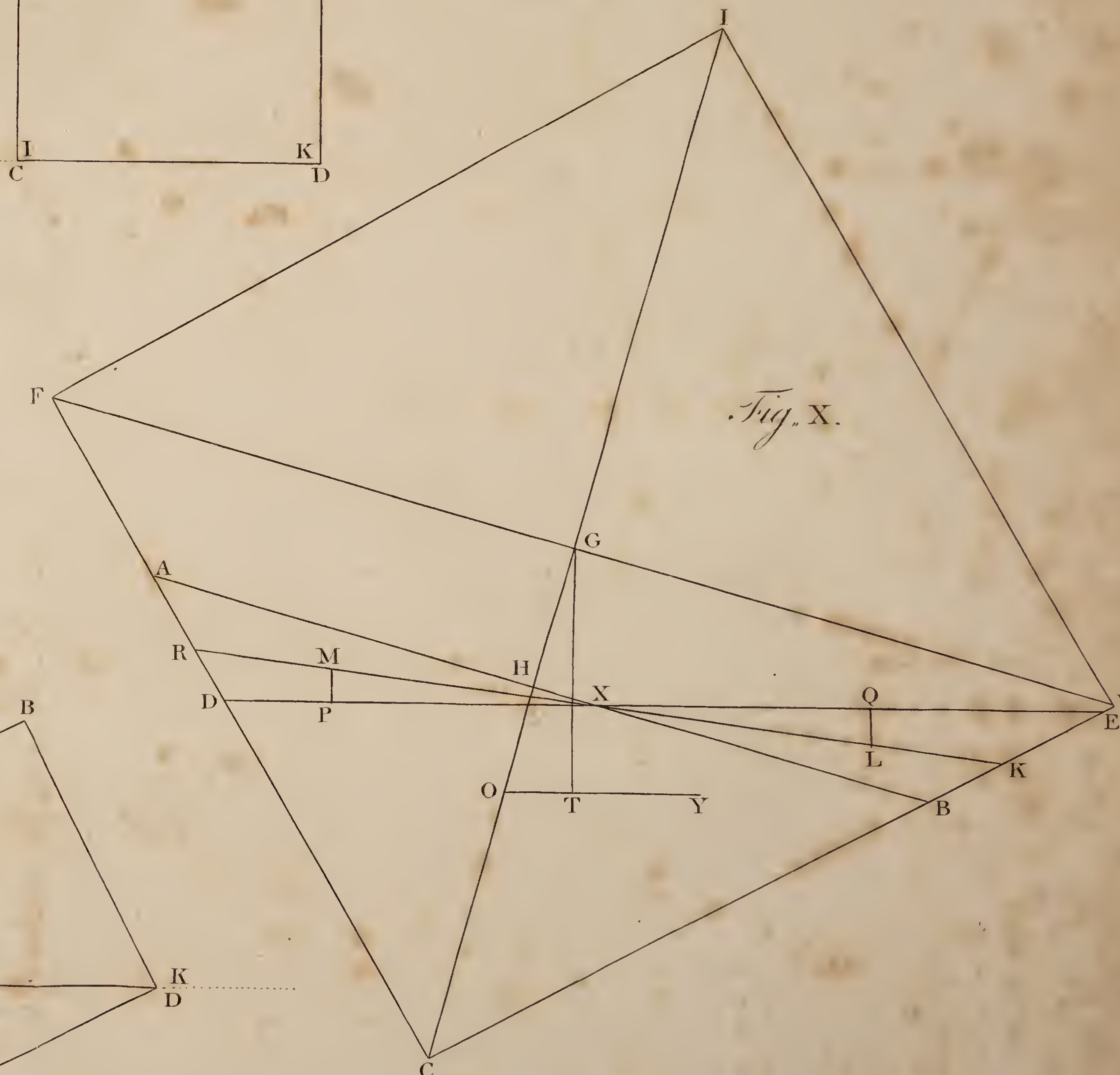


Fig. X.



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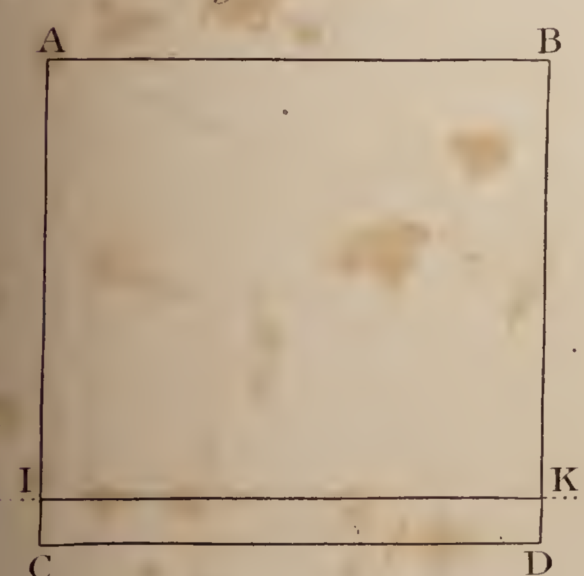

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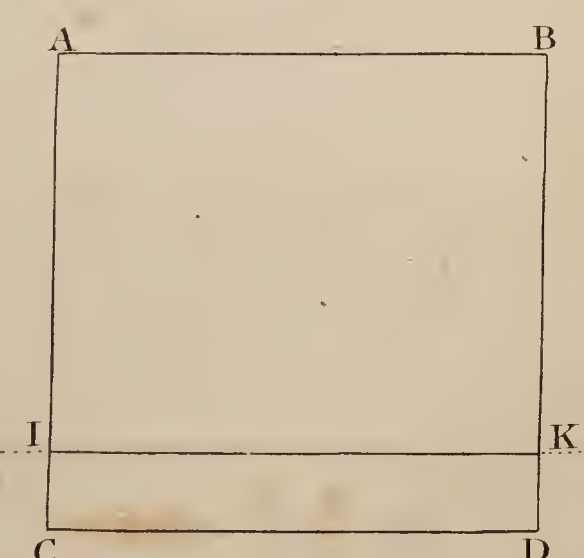

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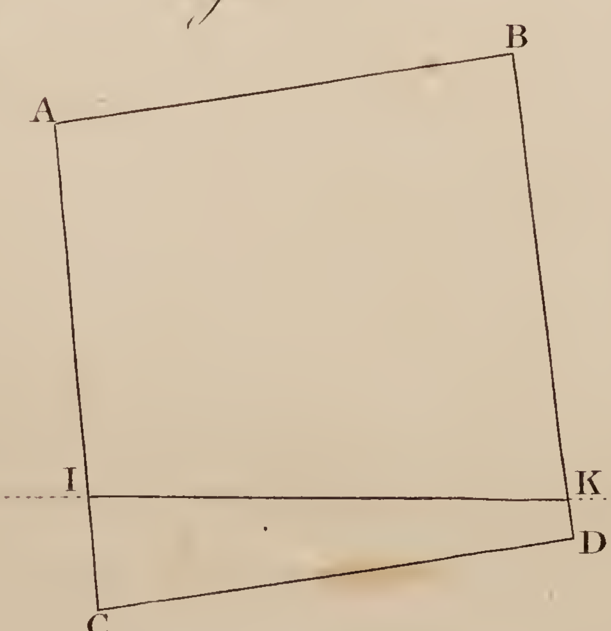

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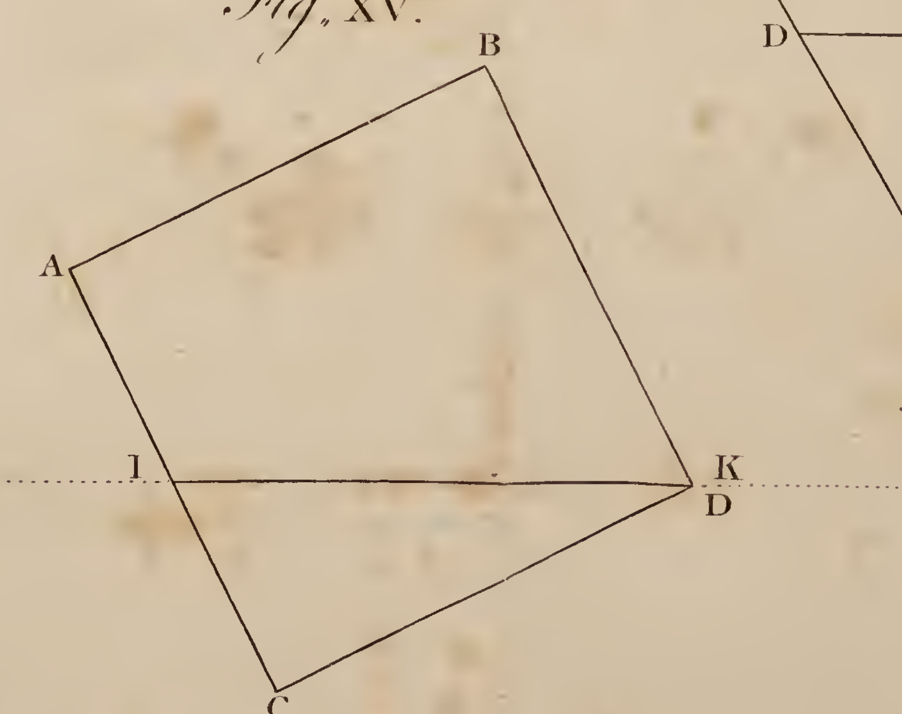

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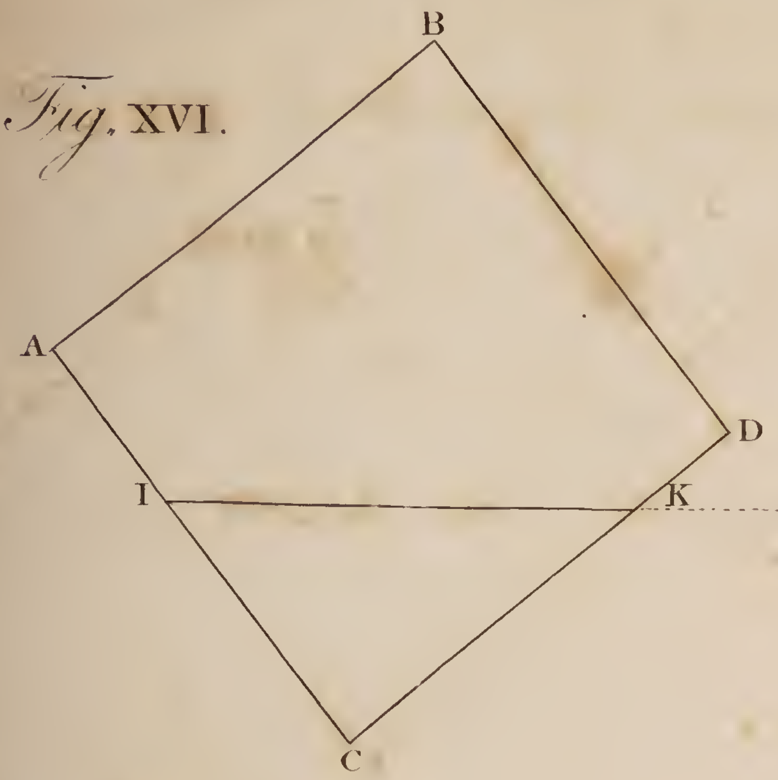

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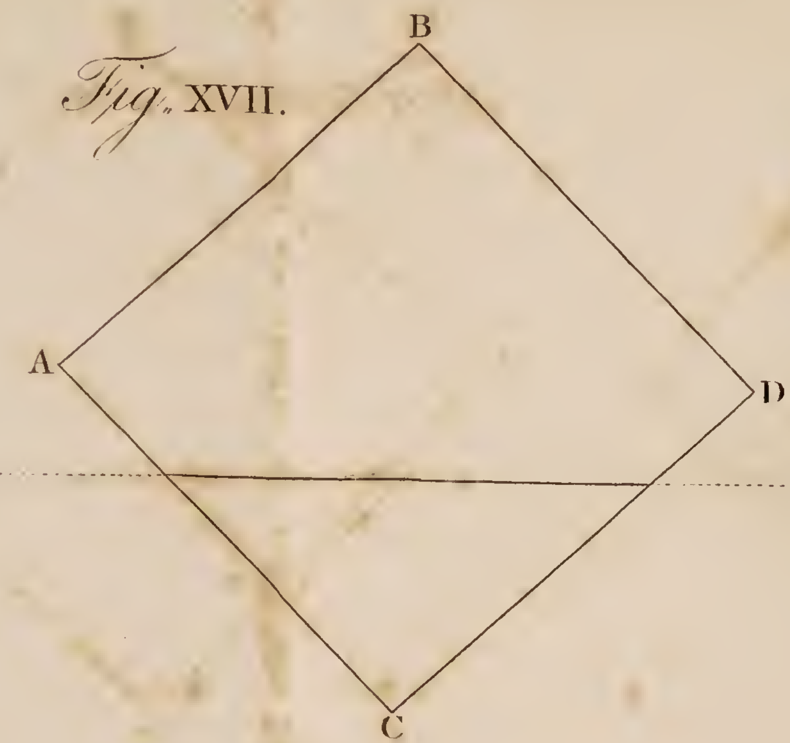

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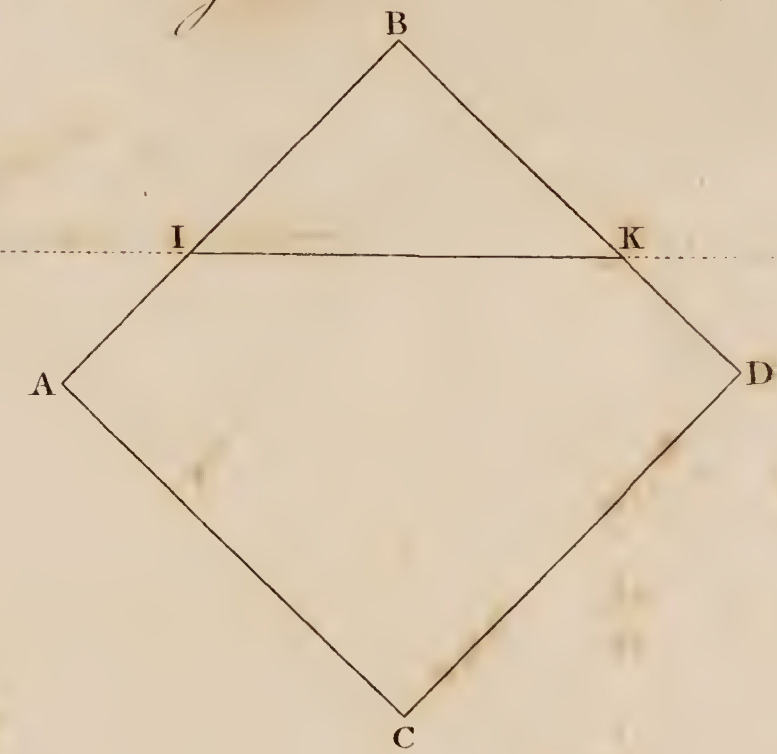

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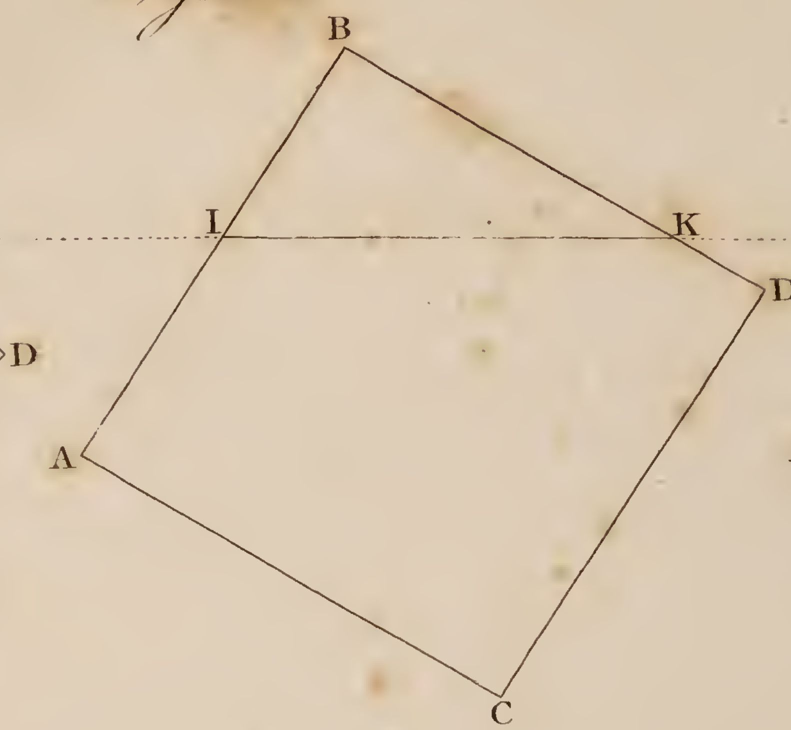

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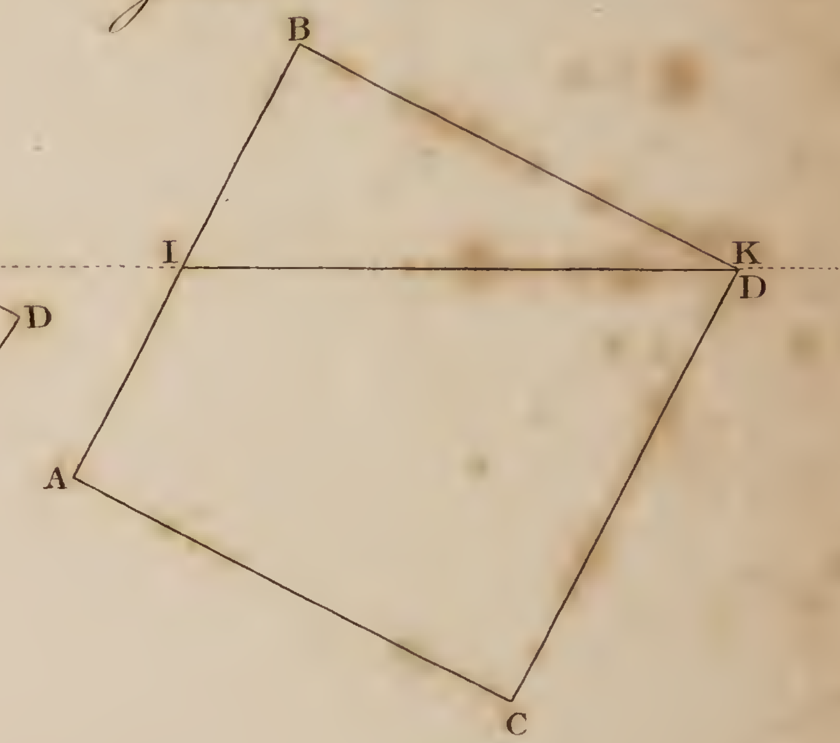
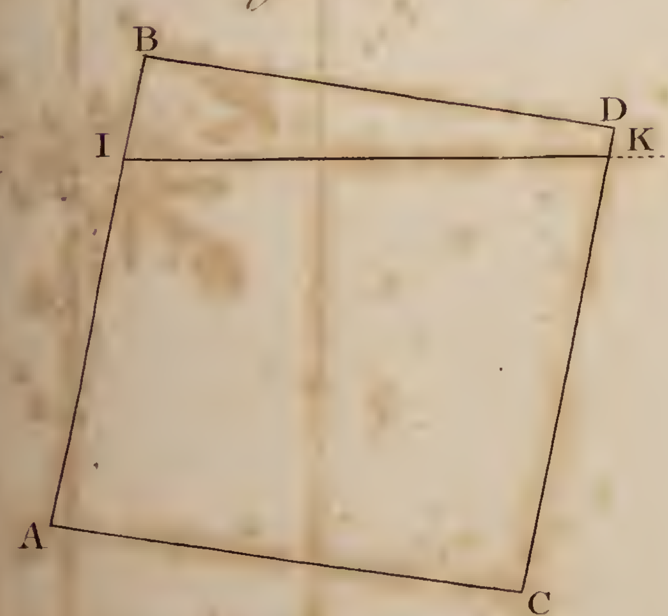


Fig. XXI.



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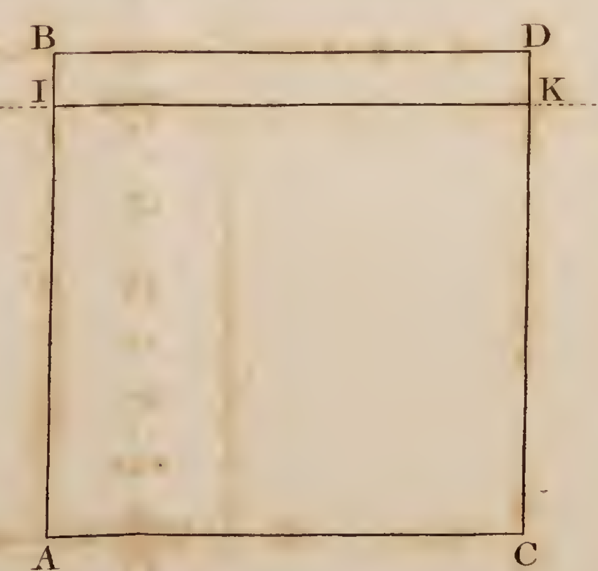

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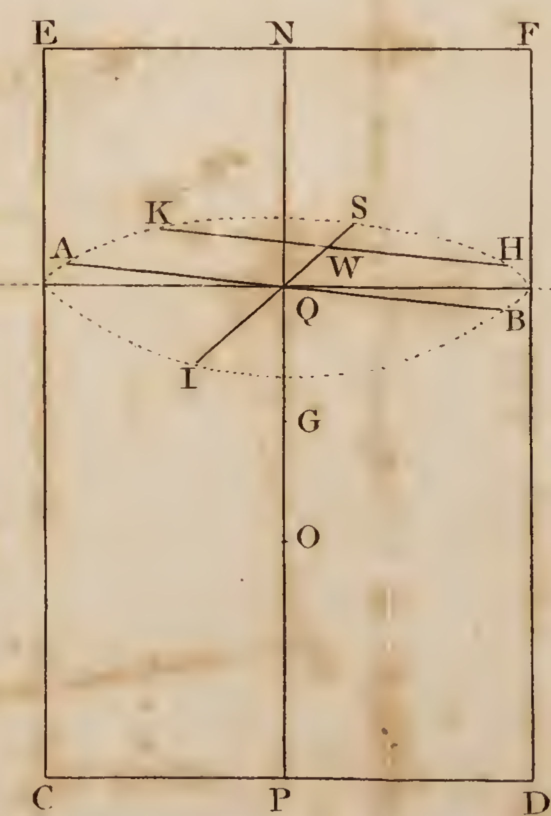

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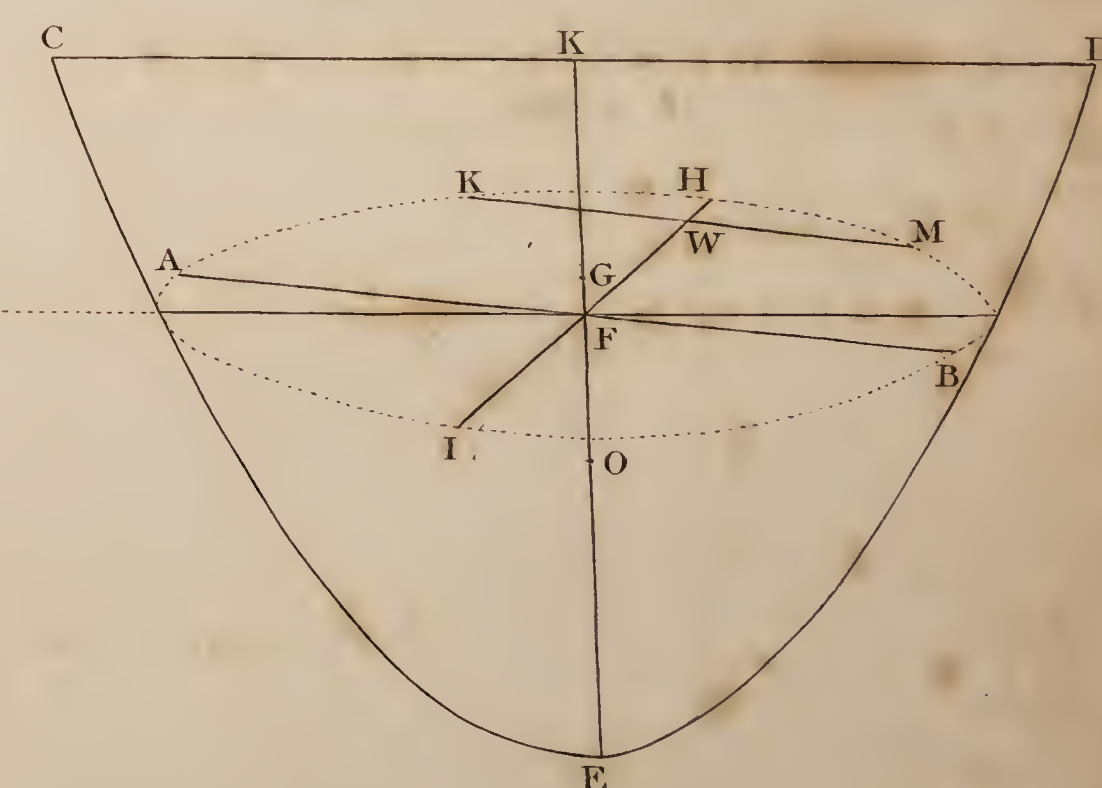

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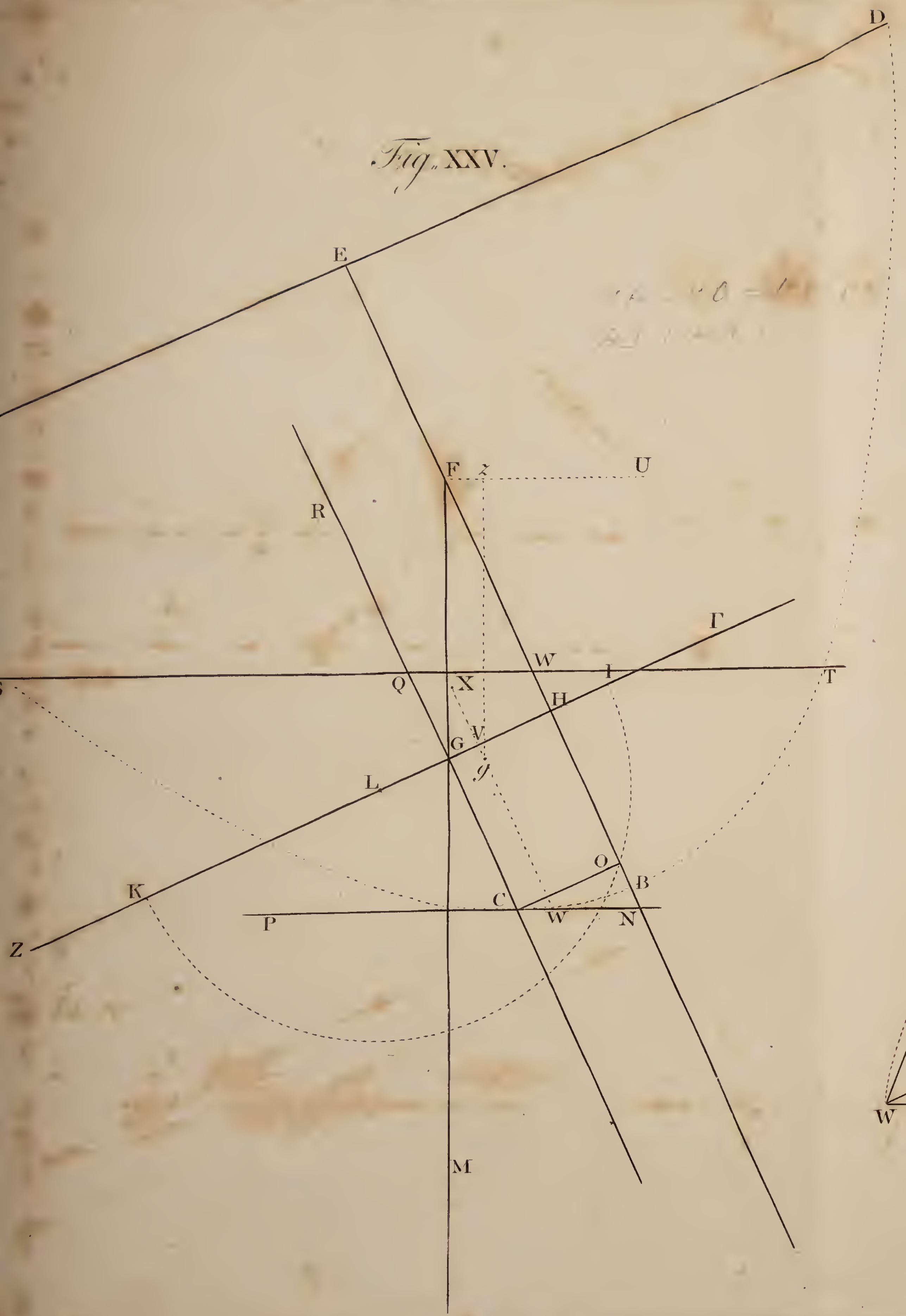

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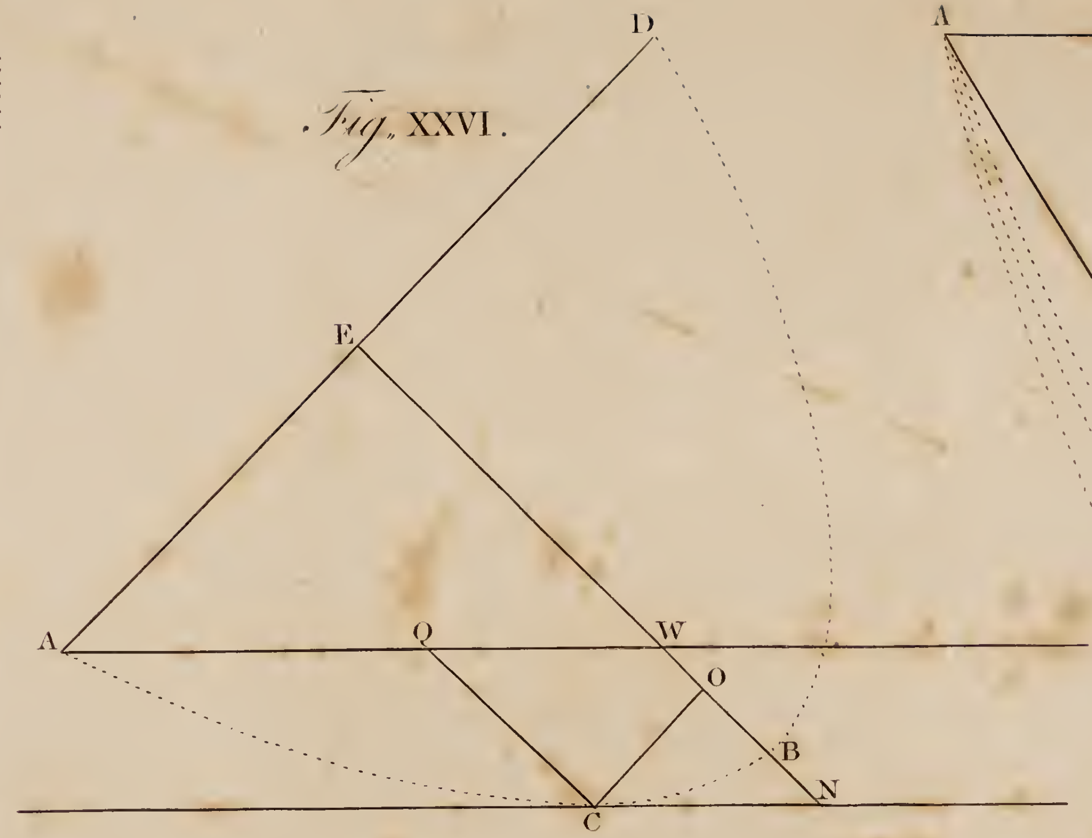

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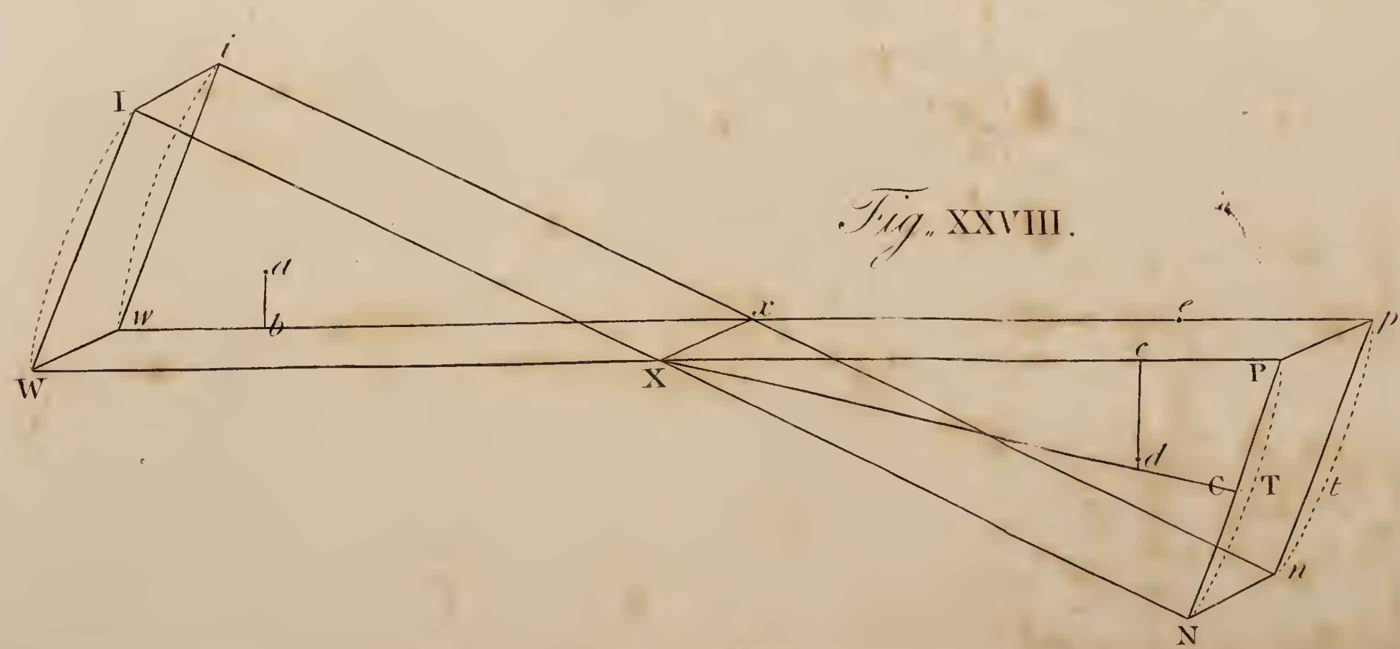

VI. *Account of the Discovery of a new Comet. By Miss* Caroline Herschel. *In a Letter to Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read November 12, 1795.

SIR, Slough, November 8, 1795.

LAST night, in sweeping over a part of the heavens with my 5-feet reflector, I met with a telescopic comet. To point out its situation I transcribe my Brother's observations upon it from his Journal.

November 7, 1795.

0^h 33′ Sidereal time. Place of the comet 2° 20′ *np.* 37 (γ) Cygni, in a line continued from 66 (υ) through γ nearly. It is just visible to the naked eye.

0^h 44′. It is in a line between two small stars at a considerable distance from each other, to which are perpendicular the two extreme stars of three other stars, that form a small arch approaching to a straight line.

0^h 49′. The comet precedes the point in the line where the perpendicular of the arch crosses the line of the two stars one-fifth of the distance of the bisecting point from the preceding star.

1^h 25′. The comet is visibly moved from the place where it was 0^h 49′.

The direction of its motion seems to be towards the south preceding side, and it is about 5 or 6′ removed from its former place.

1^h 51′. The diameter of the comet is about 5′. It has no kind of nucleus, and has the appearance of an ill-defined haziness, which is rather strongest about the middle.

2^h 16′. The comet is about 2° 38′ *np* γ, in a line continued from 69 through 37 Cygni.

3^h 37′. The comet is about 2° 50′ *np* γ, in a line continued from 65 through 37 Cygni, or, perhaps more accurate, in a line from 70 continued through 37 Cygni.

It will probably pass between the head of the Swan and the constellation of the Lyre, in its descent towards the sun.

The direction of its motion is retrograde.

Place of the comet deduced from the above.

Nov. 7.	$0^h\ 33'$	RA $20^h\ 3'\ 48''$	PD $49°\ 17'\ 18''$
	3 37	20 0 58	49 37 18

As the appearance of one of these objects is almost become a novelty, I flatter myself that this intelligence will not be uninteresting to astronomers.

I have the honour to be, &c.

CAROLINA HERSCHEL.

Additional Observations on the Comet. By William Herschel, *LL.D. F. R. S.*

November 8, 1795.

0^h 10′. The comet is about 42′ north of 22 Cygni, in a line continued from 21 (η) through 22 nearly; it is not quite come to the line.

It is exactly in a line with 22 and a north following star 1° 34′ from 22 towards 21.

0^h 31′. Distance of the comet from 19 Cygni, 1° 10′. From 22 Cygni, about 42′. From 25 Cygni, 2° 10′. From 15 Cygni, exactly 3°.

2^h 27′. The comet is 36′ from 22 Cygni; its motion has been very nearly in the line pointed out before. It will however not pass over 22, but go by it towards 19 Cygni, having left the line pointed out, a little on the following side.

November 9, 1795.

20^h 45′. The comet is about 17 or 18′ from 15 Cygni.

21^h 59′. The comet is now centrally upon a small star north following 15 Cygni. It is a small telescopic star of about the 11th or 12th magnitude, and is double, very unequal, the smallest of the two being much smaller than the largest.

With a power of 287 I can see the smallest of the two stars perfectly well; this shews how little density there is in the comet, which is evidently nothing but what may be called a collection of vapours.

Angle of the small north following star with respect to 15 Cygni, 71° 55′ north following.

Position of the small star belonging to the double star, a few degrees south following.

The north following star must be of the 11th or 12th magnitude at least, for it is not visible in my achromatic finder, and its smaller companion therefore is an extremely small star indeed.

The double star is about 5 or 6′ from 15 Cygni.

November 10, 1795.

21^h 55′. The comet is about 1° 5′ north of, and a little following, 8 Cygni, and exactly 1° 30′ south following 4 Cygni.

2^h 16′. The comet is about 40′ from 8 Cygni, in the line between 8 and 4, but rather past the line.

It is about 1° 26′ from 4 Cygni.

VII. *Mr.* Jones's *Computation of the Hyperbolic Logarithm of* 10 *improved: being a Transformation of the Series which he used in that Computation to others which converge by the Powers of* 80. *To which is added a Postscript, containing an Improvement of Mr.* Emerson's *Computation of the same Logarithm. By the Rev.* John Hellins, *Vicar of* Potter's Pury, *in* Northamptonshire. *Communicated by* Nevil Maskelyne, *D.D. F. R. S. and Astronomer Royal.*

Read February 18, 1796.

1. THE method of computing by series is so extensive and useful a part of the mathematics, that any device which facilitates the operation by them will undoubtedly be acceptable to those who are proper judges of these matters. In this persuasion I have employed an hour of that little leisure which my present situation affords me, in improving a calculation of NAPIER's, for finding the hyperbolic logarithm of 10, which was given by the justly celebrated WILLIAM JONES, Esq. F.R.S. in p. 180 of his *Synopsis Palmariorum Matheseos.* The same computation, described in a manner better suited to the capacities of beginners, was also published many years afterward by the learned Dr. SAUNDERSON, in the second volume of his Elements of Algebra, p. 633 and 634. Since Dr. SAUNDERSON's time the doctrine of series has been much improved. My present intention is, to exhibit a transformation of the series by which Mr. JONES computed the hyperbolic logarithm of 10

to others, the terms of which decrease by the powers of 80; so that their convergency is swift, and the divisions by 80 are easily made.

2. Mr. JONES considered the number 10 as composed of $2 \times 2 \times 2 \times \frac{5}{4}$; and consequently obtained the logarithm of 10 by adding three times the logarithm of 2 to the logarithm of $\frac{5}{4}$. The algebraic series which he used on this occasion was $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}$, &c. and the numerical value of $\frac{d}{s}$ was $\frac{1}{3}$ for the logarithm of 2, and $\frac{1}{9}$ for the logarithm of $\frac{5}{4}$; so that he has

$$\text{Sum of} \left\{ \begin{array}{l} 3\,\text{L.}\,2 = \frac{6}{3} + \frac{6}{3 \cdot 3^3} + \frac{6}{5 \cdot 3^5} + \frac{6}{7 \cdot 3^7}, \text{ \&c.} \\ \text{L.}\,\frac{5}{4} = \frac{2}{9} + \frac{2}{3 \cdot 9^3} + \frac{2}{5 \cdot 9^5} + \frac{2}{7 \cdot 9^7}, \text{ \&c.} \end{array} \right\} = \text{L.}\,10.$$

3. Now the series $\frac{6}{3} + \frac{6}{3 \cdot 3^3} + \frac{6}{5 \cdot 3^5} + \frac{6}{7 \cdot 3^7}$, &c. ($= 3$L. 2) is evidently $= \frac{6}{3} \times : 1 + \frac{1}{3 \cdot 3^2} + \frac{1}{5 \cdot 3^4} + \frac{1}{7 \cdot 3^6}$, &c. $= 2 \times : 1 + \frac{1}{3 \cdot 9} + \frac{1}{5 \cdot 9^2} + \frac{1}{7 \cdot 9^3}$, &c. And if the first, third, fifth, &c. term of this series be written in one line, and the second, fourth, sixth, &c. in another, we shall have

$$3\text{L.}\,2 = \left\{ \begin{array}{l} 2 \times : 1 + \frac{1}{5 \cdot 9^2} + \frac{1}{9 \cdot 9^4} + \frac{1}{13 \cdot 9^6}, \text{ \&c.} \\ + 2 \times : \frac{1}{3 \cdot 9} + \frac{1}{7 \cdot 9^3} + \frac{1}{11 \cdot 9^5} + \frac{1}{15 \cdot 9^7}, \text{ \&c.} \end{array} \right.$$

which two series are evidently

$$= \left\{ \begin{array}{l} 2 \times : 1 + \frac{1}{5 \cdot 81} + \frac{1}{9 \cdot 81^2} + \frac{1}{13 \cdot 81^3}, \text{ \&c.} \\ + \frac{2}{9} \times : \frac{1}{3} + \frac{1}{7 \cdot 81} + \frac{1}{11 \cdot 81^2} + \frac{1}{15 \cdot 81^3}, \text{ \&c.} \end{array} \right.$$

And Mr. JONES's other series, $\frac{2}{9} + \frac{2}{3 \cdot 9^3} + \frac{2}{5 \cdot 9^5} + \frac{2}{7 \cdot 9^7}$, &c. $\left(= \text{L.}\,\frac{5}{4}\right)$ is evidently $= \frac{2}{9} \times : 1 + \frac{1}{3 \cdot 9^2} + \frac{1}{5 \cdot 9^4} + \frac{1}{7 \cdot 9^6}$, &c. $=$

$\frac{2}{9} \times : 1 + \frac{1}{3.81} + \frac{1}{5.81^2} + \frac{1}{7.81^3}$, &c. We therefore now have $3\text{L}.\ 2 + \text{L}.\ \frac{5}{4}$ equal to the sum of these three series,

$$2 \times : 1 + \frac{1}{5.81} + \frac{1}{9.81^2} + \frac{1}{13.81^3}, \&c.$$

$$\frac{2}{9} \times : \frac{1}{3} + \frac{1}{7.81} + \frac{1}{11.81^2} + \frac{1}{15.81^3}, \&c.$$

$$\frac{2}{9} \times : 1 + \frac{1}{3.81} + \frac{1}{5.81^2} + \frac{1}{7.81^3}, \&c.$$

which sum is also equal to the hyperbolic logarithm of 10.

4. The form to which Mr. JONES's series are now brought is evidently the same with the general form $a \times : \frac{1}{m} + \frac{x^n}{m+n} + \frac{x^{2n}}{m+2n} + \frac{x^{3n}}{m+3n}$, &c. the value of which, while m and n are affirmative numbers, and x sufficiently small, will be given by the series $a \times : \frac{1}{m.\ \overline{1-x^n}} - \frac{nx^n}{m.\ \overline{m+n}.\ \overline{1-x^n}|^2} + \frac{n.\ 2n.\ x^{2n}}{m.\ \overline{m+n}.\ \overline{m+2n}.\ \overline{1-x^n}|^3} - \frac{n.\ 2n.\ 3n.\ x^{3n}}{m.\ \overline{m+n}.\ \overline{m+2n}.\ \overline{m+3n}.\ \overline{1-x^n}|^4}$,* &c. And this series, if we call the first, second, third, &c. terms of it A, B, C, &c. respectively, and put $\frac{x^n}{1-x^n} = z$, will be more concisely expressed thus; $a \times : \frac{1}{m.\ \overline{1-x^n}} - \frac{nzA}{m+n} + \frac{2nzB}{m+2n} - \frac{3nzC}{m+3n} + \frac{4nzD}{m+4n}$, &c. which form is well adapted to arithmetical calculation.

Now, by comparing the three series at the end of the last article with the general series here given, we shall find that, in the first and last of these series, the value of m is 1, and in the second of them it is 3. The value of n in the first and

* See Phil. Trans. for 1794, Part 2d. p. 218, where this matter is more fully explained.

second series is 4, and in the third it is 2. The values of a are obviously 2 in the first series, and $\frac{2}{9}$ in the second and third. But in each of them $z, = \frac{x^n}{1 - x^n}$, is $= \frac{\frac{1}{81}}{1 - \frac{1}{81}} = \frac{1}{80}$. These values of the letters being written for them in the second general form, we have three new series, viz.

$$\frac{2.81}{80} - \frac{4A}{5.80} + \frac{8B}{9.80} - \frac{12C}{13.80} + \frac{16D}{17.80}, \text{ \&c.}$$

and $$\frac{2.81}{9.3.80} - \frac{4A}{7.80} + \frac{8B}{11.80} - \frac{12C}{15.80} + \frac{16D}{19.80}, \text{ \&c.}$$

and $$\frac{2.81}{9.80} - \frac{2A}{3.80} + \frac{4B}{5.80} - \frac{6C}{7.80} + \frac{8D}{9.80}, \text{ \&c.}$$

which three series are equal in value to those in art. 3, and to the hyperbolic logarithm of 10.

5. With respect to the convergency of these new series, it is evidently somewhat swifter than by the powers of 80. For even in the first series, which has the slowest convergency of the three, the coefficients $\frac{4}{5}$, $\frac{8}{9}$, $\frac{12}{13}$, &c. are each of them less than 1.

6. But another advantage of these new series is, that their numerators and denominators may be reduced to simpler terms, in consequence of which the arithmetical operation by them is further facilitated. In the first and second series, every term after the first is divisible by 4; and every term in the third series admits of a similar reduction by the number 2. The three series then, when these reductions are made, and their first terms are also abbreviated, will stand as below, (each still converging somewhat faster than by the powers of 80); and we shall have the hyperbolic logarithm of 10

$$=\begin{cases} \dfrac{81}{40}-\dfrac{A}{5.20}+\dfrac{2B}{9.20}-\dfrac{3C}{13.20}+\dfrac{4D}{17.20}, \text{ \&c.} \\ \dfrac{3}{40}-\dfrac{A}{7.20}+\dfrac{2B}{11.20}-\dfrac{3C}{15.20}+\dfrac{4D}{19.20}, \text{ \&c.} \\ \dfrac{9}{40}-\dfrac{A}{3.40}+\dfrac{2B}{5.40}-\dfrac{3C}{7.40}+\dfrac{4D}{9.40}, \text{ \&c.} \end{cases}$$

The arithmetical operation by the new series is undoubtedly easier than by the original series; yet it is evident, by inspection, that half the number of divisions by 20, (although easy operations), in the first and second series, may be exchanged for divisions by 10, which are no more than so many removals of the decimal point; and that, in the third series, half the number of divisions by 40, (the first excepted) may be exchanged for easier ones, one-fourth of them for divisions by 20, and the other fourth for divisions by 10. The new series then, still converging somewhat quicker than by the powers of 80, may stand thus:

$$\frac{81}{40}-\frac{A}{5.20}+\frac{B}{9.10}-\frac{3C}{13.20}+\frac{2D}{17.10}, \text{ \&c.}$$

and $\dfrac{3}{40}-\dfrac{A}{7.20}+\dfrac{B}{11.10}-\dfrac{3C}{15.20}+\dfrac{2D}{19.10}$, &c.

and $\dfrac{9}{40}-\dfrac{A}{3.40}+\dfrac{B}{5.20}-\dfrac{3C}{7.40}+\dfrac{D}{9.10}$, &c.

And even yet one might still facilitate the computation of the value of some of the terms. Thus, $\frac{3}{20}$ is $=\frac{1+\frac{1}{2}}{10}$; $\frac{3}{40}$ is $=\frac{1-\frac{1}{4}}{10}$; $\frac{5}{40}$ is $=\frac{1}{8}$; and $\frac{3}{15.20}$ is $=\frac{1}{100}$, &c.

By these expedients the sum of the three new series, which is equal to the hyperbolic logarithm of 10, may quickly be found.

P. S. *Containing an Improvement of Mr.* EMERSON's *Computation of the Hyperbolic Logarithm of* 10.

7. Since the above paper was written, on looking into EMERSON's Fluxions, I have found, at p. 137 of the first edition,* another computation of the hyperbolic logarithm of 10, which is preferable to Mr. JONES's, on account of the swifter convergency of one of the series used in it, as will appear presently. These series also admit of a transformation to others, by which the constant divisors 81 and 64009, used by Mr. EMERSON, are exchanged for 40 and 32000, while nearly the same rate of convergency is retained; which is another remarkable instance of the utility of transformations of this kind.

Mr. EMERSON, considering the number 10 as composed of $\frac{5^{10}\times 2^{30}}{4^{10}\times 10^{9}} = \frac{5^{10}}{4^{10}} \times \overline{\frac{1024}{1000}}\Big|^{3}$, and using the same algebraic series as Mr. JONES used on this occasion, finds the hyperbolic logarithm of 10 to be $= 10$ L. of $\frac{5}{4} + 3$ L. of $\frac{1024}{1000}$,

$$=\left\{\begin{array}{l} \frac{20}{9} + \frac{20}{3.9.81} + \frac{20}{5.9.81^{2}} + \frac{20}{7.9.81^{3}}, \text{ \&c.} \\ + \frac{18}{253} + \frac{18.9}{3.253.64009} + \frac{18.9^{2}}{5.253.64009^{2}} + \frac{18.9^{3}}{7.253.64009^{3}}, \text{ \&c.} \end{array}\right.$$

where, instead of a series converging by the powers of $\frac{1}{9}$,† as in Mr. JONES's calculation, we have that which converges by the powers of $\frac{9}{64009}$, or above four times as swiftly. But what renders this very swiftly converging series still more useful is, that it admits of a transformation, by the theorem in article 4,

* See also page 197 of 3d edition.

† See article 3.

to another series which converges by the powers of $\frac{9}{64000}$, by which the numerical calculation is greatly facilitated.

8. For the two series in the preceding article (the sum of which is = H. L. of 10), are evidently =

$$\frac{20}{9} \times : 1 + \frac{1}{3.81} + \frac{1}{5.81^2} + \frac{1}{7.81^3}, \text{ \&c.}$$

and $\frac{18}{253} \times : 1 + \frac{9}{3.64009} + \frac{9^2}{5.64009^2} + \frac{9^3}{7.64009^3}$, &c.

And these two series, when transformed by the theorem abovementioned, and the terms abbreviated, become

$$\frac{9}{4} - \frac{A}{3.40} + \frac{2B}{5.40} - \frac{3C}{7.40} + \frac{4D}{9.40}, \text{ \&c.}$$

and $\frac{9.253}{32000} - \frac{9A}{3.32000} + \frac{2.9B}{5.32000} - \frac{3.9C}{7.32000} + \frac{4.9D}{9.32000}$, &c.

Which series admit of some other abbreviations similar to those pointed out in article 6; and by them may the hyperbolic logarithm of 10 be very easily and expeditiously computed.

June 24, 1795.

VIII. *Manière élémentaire d'obtenir les Suites par lesquelles s'expriment les Quantités exponentielles et les Fonctions trigonométriques des Arcs circulaires. Par M.* Simon L'Huilier, *F. R. S.*

Read February 18, 1796.

L'USAGE des logarithmes, et celui des fonctions trigonométriques des arcs de cercle, tels que sont les sinus, cosinus, tangentes, &c. sont si frequens dans les parties les plus élémentaires des mathématiques, soit pures soit mixtes, qu'on doit regarder ces quantités comme apartenant aux élémens; et que leur calcul doit entrer dans un traité élémentaire.

En s'en tenant à la manière ordinaire de présenter les logarithmes dans les élémens, on fait comprendre la possibilité de leur calcul plutôt qu'on ne peut le développer. Outre celui, la dépendance mutuelle des opérations successives par lesquelles on parvient à un resultat approché, exige dans chacune d'elles une exactitude, qui complique et prolonge le calcul, au point qu'il est bien peu de mathématiciens modernes, qui eussent pour les progrès des sciences le dévouement qu'ont eu l'auteur de cette belle découverte, et ceux qui ont poursuivi et complété ce calcul penible, avant qu'on eût trouvé les voïes directes et indépendantes les unes des autres de parvenir aux mêmes résultats.

Les mêmes difficultés et les mêmes longueurs se présentent dans les calculs des fonctions des arcs circulaires, quand on s'en tient aux voïes élémentaires développées jusqu'à présent. Le nombre des extràctions de racines, et leur dépendance mutuelle, sont telles, qu'elles exigent un travail affraïant, qui devient superflu, quand le calcul de chaque fonction est rendu direct.

Aussi dans le dévelopement de l'une et de l'autre de ces matières, est-on obligé d'abandonner ces voïes longues et penibles ; et en profitant des tables heureusement déja calculées, ou a coutume de renvoïer aux calculs supérieurs l'exposition des procédés abbregés et directs par lesquels on parvient aux mêmes résultats. Quelques mathématiciens il est vrai, et en particulier Euler dans son *Introductio*, ont exposé ces derniers procédés d'une manière qui paroît les raprocher des élémens. Mais, si on examine avec quelque soin la marche de ce mathématicien, on trouvera qu'elle est entièrement fondée sur le principe de l'infini ; tout au moins trop obscur, pour qu'on puisse regarder comme élémentaires des méthodes auxquelles il sert de base.

J'espère avoir evité ces inconveniens ; et avoir rendu l'exposition de la théorie generale des quantités exponentielles, et des fonctions des arcs circulaires, entièrement élémentaire et indépendante de toute idée de l'infini. La liaison intime qui règne entre les procédés par lesquels je calcule l'une et l'autre de ces espèces de fonctions, est remarquable, et propre à éclaircir l'analogie qui règne entr'elles. Cette analogie, il est vrai, est connue depuis longtems des mathématiciens ; mais, comme elle a été réduite aux expressions imaginaires d'une

de ces espèces de fonctions dans l'autre, elle meritoit d'être présentée d'une manière plus lumineuse.

La méthode que je vais exposer est conforme à celle qu'a emploïée un mathématicien trop modeste et trop peu connu, mon ami M. Pfleiderer, Professeur à Tubingue, en démontrant le théorème de Taylor dans sa dissertation intitulée *Theorematis Tayloriani Demonstratio:* Tubingæ, 1789.

§ 1. *Lemme.* Les différences des puissances des nombres naturels d'un ordre exprimé par l'exposant de ces puissances, est une quantité constante; savoir, le produit continuel des nombres naturels depuis l'unité jusqu'à cet exposant; et partant, les différences des mêmes puissances d'un ordre supérieur évanouissent.

Je pourrois regarder cette proposition comme connue. Mais, comme elle est une des principales de ce mémoire, et que sa démonstration est facile, je crois devoir l'exposer en abrégé.

1. Les différences premières des nombres naturels sont égales à l'unité, et les différences suivantes évanouissent.

2. Les différences premières des nombres quarrés, sont $n^2 - (n-1)^2 = 2n - 1$. Donc, les différences secondes des nombres quarrés sont les doubles des différences premières des nombres naturels; et partant 1.2. Et les différences suivantes évanouissent.

3. Les différences premières des cubes, sont $n^3 - (n-1)^3 = 3nn - 3n + 1$. Donc, les différences troisièmes, qui sont les différences secondes des différences premières, sont les triples des différences secondes des nombres quarrés; savoir 1.2.3; et les différences suivantes évanouissent.

En general. Les différences premières des puissances dont

l'exposant est m, sont $n^m - (n-1)^m = \frac{m}{1} n^{m-1} - \frac{m}{1}, \frac{m-1}{2} n^{m-2} + \frac{m}{1} \ldots \frac{m-2}{3} n^{m-3} \ldots$. Les différences m^{mes} de ces puissances, qui sont les différences $m-1$ des différences premières, sont les différences $m-1^{mes}$ des puissances des nombres naturels dont les exposans sont $m-1$ ou plus petits que $m-1$; affectées de coefficiens constans. Partant, s'il a été prouvé que les différences $m-1^{mes}$ des $m-1^{mes}$ puissances des nombres naturels, sont la quantité constante $1.2.3 \ldots m-1$; et que les différences du même ordre des puissances inférieures évanouissent; on obtient aussi que les différences m^{mes} des m^{mes} puissances, valent m fois le produit $1.2.3 \ldots m-1$; ou sont le produit $1.2.3 \ldots m$; et que les différences des ordres supérieurs évanouissent.

Avis. Pour abréger, je désignerai par $\Delta^p\ (a^m \ldots - . n^m)$ les différences de l'ordre p des puissances m^{mes} des nombres naturels, depuis a jusqu'à n.

Première Partie. Sur les Logarithmes.

§ 2. *Lemme*. Soit une progression géométrique, commençante par l'unité, *p. ex.* Les différences de tous les ordres des termes de cette progression forment aussi une progression géométrique, dont l'exposant est le même que celui de la première, et dont les termes sont les produits des termes de la première progression par la différence des deux premiers termes elevée à une puissance dont l'exposant est égal à l'ordre de

cette différence. Soit 1, a, a^2, a^3, a^4, a^5 . . . . . a^{n-1} une progression géométrique :

La suite des différences premières, est

$$a-1, a^2-a, a^3-a^2, a^4-a^3, a^5-a^4 \ldots a^{n-1}-a^{n-2}; \text{ ou,}$$

$(a-1)$ $(1, \quad a, \quad a^2, \quad a^3, \quad a^4, \ldots\ldots a^{n-2}.)$ De-là, la suite des différences secondes, est

$(a-1)^2$ $(1, \quad a, \quad a^2, \quad a^3, \quad a^4, \ldots\ldots)$ La suite des differences troisièmes, est

$(a-1)^3$ $(1, \quad a, \quad a^2, \quad a^3, \quad a^4, \ldots\ldots)$ La suite des différences quatrièmes, est

$(a-1)^4$ $(1, \quad a, \quad a^2, \quad a^3 \quad a^4, \ldots\ldots)$

La suite des differences m^{mes}, est

$(a-1)^m$ $(1, \quad a, \quad a^2, \quad a^3, \quad a^4 \ldots\ldots)$

§ 3. *Lemme.* Soit a^z une quantité exponentielle dans laquelle a est plus grande que l'unité. Cette quantité est plus grande que l'unité ou plus petite que l'unité, suivant que z est positif ou négatif; et dans l'un et l'autre cas cette quantité approche de l'unité d'autant plus que z est plus petit: de manière que l'unité est la limite en grandeur ou en petitesse de a^z suivant que z est positif ou négatif.

Corollaire. a^z est une fonction de z de la forme $1+Az+Bz^2+Cz^3+Dz^4+\ldots$

§ 4. Soit donc proposée la quantité exponentielle a^z à exprimer dans son exposant z;

Soit $a^z = 1 + Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 + \ldots$

On aura aussi . . .

$$a^{2z} = 1 + 2Az + 2^2Bz^2 + 2^3Cz^3 + 2^4Dz^4 + 2^5Ez^5 + \ldots$$
$$a^{3z} = 1 + 3Az + 3^2Bz^2 + 3^3Cz^3 + 3^4Dz^4 + 3^5Ez^5 + \ldots$$
$$a^{4z} = 1 + 4Az + 4^2Bz^2 + 4^3Cz^3 + 4^4Dz^4 + 4^5Ez^5 + \ldots$$

Soient prises les différences premières ; on aura

$$(a^z - 1)\, a^z = Az + (2^2 - 1)\, Bz^2 + (2^3 - 1)\, Cz^3 + (2^4 - 1)\, Dz^4 + (2^5 - 1)\, Ez^5 + \ldots .$$

$$(a^z - 1)\, a^{2z} = Az + (3^2 - 2^2)\, Bz^2 + (3^3 - 2^3)\, Cz^3 + (3^4 - 2^4)\, Dz^4 + (3^5 - 2^5)\, Ez^5 + \ldots$$

$$(a^z - 1)\, a^{3z} = Az + (4^2 - 3^2)\, Bz^2 + (4^3 - 3^3)\, Cz^3 + (4^4 - 3^4)\, Dz^4 + (4^5 - 3^5)\, Ez^5 + \ldots$$

Or ; puisque $a^z = 1 + Az + Bz^2 + Cz^3 + Dz^4 + \ldots .$

$$a^z - 1 = \quad Az + Bz^2 + Cz^3 + Dz^4 + \ldots .$$

Et les premiers termes de tous les premiers membres des équations précédentes sont Az. Mais, les premiers termes de tous les seconds membres des mêmes équations, sont aussi Az. Donc, on a pour ces premiers termes des expressions identiques, desquelles on ne peut rien déterminer pour la valeur de A. Partant, le coefficient A demeure indéterminé.

Soient prises les différences secondes ; on aura

$$(a^z - 1)^2.\, a^z = \Delta^{ii}\, (3^2 \ldots 1^2)\, Bz^2 + \Delta^{ii}\, (3^3 \ldots 1^3)\, Cz^3 + \Delta^{ii}\, (3^4 \ldots 1^4)\, Dz^4 + \Delta^{ii}\, (3^5 .. 1^5)\, Ez^5 + \ldots$$

$$(a^z - 1)^2\, a^{2z} = \Delta^{ii}\, (4^2 \ldots 2^2)\, Bz^2 + \Delta^{ii}\, (4^3 \ldots 2^3)\, Cz^3 + \Delta^{ii}\, (4^4 .. 2^4)\, Dz^4 + \Delta^{ii}\, (4^5 .. 2^5)\, Ez^5 + \ldots$$

$$(a^z - 1)^2.\, a^{3z} = \Delta^{ii}\, (5^2 \ldots 3^2)\, Bz^2 + \Delta^{ii}\, (5^3 \ldots 3^3)\, Cz^3 + \Delta^{ii}\, (5^4 \ldots 3^4)\, Dz^4 + \Delta^{ii}\, (5^5 .. 3^5)\, Ez^5 + \ldots$$

Puisque $a^z - 1 = Az + Bz^2 + Cz^3 + Dz^4 + \ldots\ldots\ldots$; le premier terme de tous les premiers membres des équations précédentes est AAz^2. Mais, le premier terme de tous les seconds membres, est (§ 1) $1.2\ Bz^2$. Donc, égalant les coefficiens des puissances secondes de z, on a $AA = 1.2\ B$; ou, $B = \frac{1}{1.....2} AA$.

Soient prises les différences troisièmes ; on obtient

$(a^z - 1)^3 a^z = \Delta^{iii}(4^3 \ldots 1^3) Cz^3 + \Delta^{iii}(4^4 \ldots 1^4) Dz^4 + \Delta^{iii}(4^5 \ldots 1^5)$
$Ez^5 + \Delta^{iii}(4^6 \ldots 1^6) Fz^6 + \ldots$

$(a^z - 1)^3 a^{2z} = \Delta^{iii}(5^3 \ldots 2^3) Cz^3 + \Delta^{iii}(5^4 \ldots 2^4) Dz^4 + \Delta^{iii}(5^5 \ldots 2^5)$
$Ez^5 + \Delta^{iii}(5^6 \ldots 2^6) Fz^6 + \ldots$

$(a^z - 1)^3 a^{3z} = \Delta^{iii}(6^3 \ldots 3^3) Cz^3 + \Delta^{iii}(6^4 \ldots 3^4) Dz^4 + \Delta^{iii}(6^5 \ldots 3^5)$
$Ez^5 + \Delta^{iii}(6^6 \ldots 3^6) Fz^6 + \ldots$

On montre de même ; que, le premier terme des premiers membres de toutes ces équations est $A^3 z^3$; tandis que le premier terme des seconds membres est $1.2.3\ Cz^3$. Donc ; $1.2.3\ C = A^3$; et $C = \frac{1}{1.2.3} A^3$.

Soient prises les différences quatrièmes ; on obtient

$(a^z - 1)^4 a^z = \Delta^{iv}(5^4 \ldots 1^4) Dz^4 + \Delta^{iv}(5^5 \ldots 1^5) Ez^5 + \Delta^{iv}(5^6 \ldots 1^6)$
$Fz^6 \ldots$

$(a^z - 1)^4 a^{2z} = \Delta^{iv}(6^4 \ldots 2^4) Dz^4 + \Delta^{iv}(6^5 \ldots 2^5) Ez^5 + \Delta^{iv}(6^6 \ldots 2^6)$
$Fz^6 \ldots$

$(a^z - 1)^4 a^{3z} = \Delta^{iv}(7^4 \ldots 3^4) Dz^4 + \Delta^{iv}(7^5 \ldots 3^5) Ez^5 + \Delta^{iv}(7^6 \ldots 3^6)$
$Fz^6 \ldots$

Les premiers termes des premiers membres de ces équations sont $A^4 z^4$. Mais, les premiers termes des seconds membres sont $1.2 \ldots 4Dz^4$. Donc ; $1.2 \ldots 4D = A^4$; et $D = \frac{1}{1.2 \ldots 4} A^4$.

On montre de la même manière ; en prenant les différences cinquièmes, sixièmes, septièmes ......
qu'on a les équations .... $A^5 = 1.2 \ldots 5E$; $A^6 = 1.2 \ldots 6F$; $A^7 = 1.2 \ldots 7G$ ......
et partant ; .... $E = \frac{1}{1.2 \ldots 5} A^5$; $F = \frac{1}{1.2 \ldots 6} A^6$; $G = \frac{1}{1.2 \ldots 7} A^7$ ......

On a donc ; $a^z = 1 + Az + \frac{A^2}{1.2} z^2 + \frac{A^3}{1.2.3} z^3 + \frac{A^4}{1.2...4} z^4 + \frac{A^5}{1.2...5} z^5 + \ldots$

De même ... $a^{-z} = 1 - Az + \frac{A^2}{1.2} z^2 - \frac{A^3}{1.2.3} z^3 + \frac{A^4}{1.2..4} z^4 - \frac{A^5}{1.2...5} z^5 + \ldots$

De-là ; .... $\frac{a^z + a^{-z}}{2} = 1 + \frac{A^2}{1.2} z^2 + \frac{A^4}{1.2..4} z^4 + \frac{A^6}{1.2..6} z^6 + \ldots$

$$\frac{a^z - a^{-z}}{2} = Az + \frac{A^3}{1.2\ 3} z^3 + \frac{A^5}{1.2..5} z^5 + \ldots$$

Remarque. On parvient donc par un procédé purement élémentaire, fondé sur une proprieté essentielle et première des progressions géométriques (§ 2.), aux suites qu'on a déduites jusqu'à présent des calculs supérieurs, ou du moins, de l'introduction de l'infini.

Il est connu ; que, a est la *base* du système logarîthmique ; que A en est le *module*, et faisant $z = 1$, la rélation de a à A, est exprimée par l'équation, $a = 1 + A + \frac{A^2}{1.2} + \frac{A^3}{1.2.3} + \frac{A^4}{1.2..4} + \ldots$ Faisant $A = 1$, le système est celui des logarithmes naturels, dont la base est désignée par e. De la dernière suite, on peut exprimer A en a, soit par la méthode du retour des suites ; ou plutôt, par la voïe que je développerai bientôt, après avoir fait sur ce qui précède quelques observations.

§ 5. Soit développé le binome $\left(1 + A\frac{z}{n}\right)^n$. On obtient

$$1 + Az + \frac{n}{1}, \frac{n-1}{2} A^2 \frac{z^2}{n^2} + \frac{n}{1}, \frac{n-1}{2}, \frac{n-2}{3}. A^3 \frac{z^3}{n^3} + \frac{n}{1} \ldots \frac{n-3}{4} A^4 \frac{z^4}{n^4} + \frac{n}{1} \ldots \frac{n-4}{5} A^5 \frac{z^5}{n^5} + \ldots$$

$= 1 + Az + \frac{1-\frac{1}{n}}{2} A^2 z^2 + \frac{1-\frac{1}{n}}{2}, \frac{1-\frac{2}{n}}{3} A^3 z^3 + \frac{1-\frac{1}{n}}{2}, \frac{1-\frac{2}{n}}{3},$

$\frac{1-\frac{3}{n}}{4} A^4 z^4 + \frac{1-\frac{1}{n}}{2} \ldots \frac{1-\frac{4}{n}}{5} A^5 z^5 + \ldots\ldots$

Plus n augmente, plus les facteurs $1 - \frac{1}{n}$, $1 - \frac{2}{n}$, $1 - \frac{3}{n}$, $1 - \frac{4}{n} \ldots$. approchent d'être égaux à l'unité; et partant, plus n est grand, plus la suite précédente approche d'être

$1 + Az + \frac{A^2}{1.2} z^2 + \frac{A^3}{1.2.3} z^3 + \frac{A^4}{1.2..4} z^4 + \frac{A^5}{1.2...5} + \ldots$; de manière que cette suite est la limite du binome $\left(1 + \frac{Az}{n}\right)^n$. Donc aussi, la quantité a^z est la limite du binome $\left(1 + \frac{Az}{n}\right)^n$; la quantité a^{-z} est la limite du binome $\left(1 - \frac{Az}{n}\right)^n$; et les quantités $\frac{a^z \pm a^{-z}}{2}$, sont les limites des quantités $\left(1 + A\frac{z}{n}\right)^n \pm \left(1 - A\frac{z}{n}\right)^n$.

§ 6. Je passe à l'expression de z en a et A.

Puisque $a^z = 1 + Az + \frac{A^2}{1.2} z^2 + \frac{A^3}{1.2.3} z^3 + \frac{A^4}{1.2..4} z^4 + \frac{A^5}{1.2...5} z^5 + \ldots.$

Soit $z = n\Delta z$; on aura aussi,

$a^{\Delta z} = 1 + A\Delta z + \frac{A^2}{1.2} \Delta z^2 + \frac{A^3}{1.2.3} \Delta z^3 + \frac{A^4}{1.2..4} \Delta z^4 + \frac{A^5}{1.2..5} \Delta z^5 + \ldots.$

et $a^z = a^{n\Delta z} = (a^{\Delta z})^n = \left(1 + A\Delta z + \frac{A^2}{1.2} \Delta z^2 + \frac{A^3}{1.2.3} \Delta z^3 + \frac{A^4}{1.2..4} \Delta z^4 + \frac{A^5}{1.2..5} \Delta z^5 + \ldots\right)^n. = 1 + v.$

De-là; $\left(A\Delta z + \frac{A^2}{1.2} \Delta z^2 + \frac{A^3}{1.2.3} \Delta z^3 + \frac{A^4}{1.2..4} \Delta z^4 + \frac{A^5}{1.2..5} \Delta z^5 + \ldots\right) = (1 + v)^{\frac{1}{n}} - 1,$

et $An\Delta z\ (1+\frac{A}{1.2}\ \Delta z+\frac{A^2}{1.2.3}\ \Delta z^2+\frac{A^3}{1.2..4}\ \Delta z^3+\frac{A^4}{1.2..5}$
$\Delta z^4+\ldots)=n\left((1+v)^{\frac{1}{n}}-1\right)$

$=\left(v-\frac{1-\frac{1}{n}}{1.2}v^2+\frac{1-\frac{1}{n}}{1.2}.\frac{2-\frac{1}{n}}{3}v^3-\frac{1-\frac{1}{n}}{1.2}.\frac{2-\frac{1}{n}}{3}.\frac{3-\frac{1}{n}}{4}v^4+\right.$
$\left.\frac{1-\frac{1}{n}}{1.2}\ldots\frac{4-\frac{1}{n}}{5}v^5-\ldots.\right)$

Or, (par supp.) $n\log.\ (1+A\Delta z+\frac{A^2}{1.2}\ \Delta z^2+\frac{A^3}{1.2.3}.\ \Delta z^3+$
$\frac{A^4}{1.2..4}\ \Delta z^4+\frac{A^5}{1.2...5}\ \Delta z^5+\ldots)=\log.\ (1+v)$

ou, $n\ \log.\ a^{\Delta z}=\log.\ (1+v)$; et $n\Delta z\ \log.\ a=\log.\ 1+v$;
ou, faisant a la base du systême, et partant $\log.\ a=1$,

$A\log.\ (1+v)\ (1+\frac{A}{1.2}\Delta z+\frac{A^2}{1.2.3}\Delta z^2+\frac{A^3}{1.2.3}\Delta z^3+\frac{A^4}{1.2..4}$
$\Delta z^4+\frac{A^5}{1.2..5}\Delta z^5+\ldots.)$

$=v-\frac{1-\frac{1}{n}}{1.2}v^2+\frac{1-\frac{1}{n}}{1.2}.\frac{2-\frac{1}{n}}{3}v^3-\frac{1-\frac{1}{n}}{1.2}.\frac{2-\frac{1}{n}}{3}.\frac{3-\frac{1}{n}}{4}v^4+$
$\frac{1-\frac{1}{n}}{1.2}.\frac{2-\frac{1}{n}}{3}\ldots.\frac{4-\frac{1}{n}}{5}v^5-\ldots..$

$=v-\frac{1-\frac{\Delta z}{z}}{1.2}v^2+\frac{1-\frac{\Delta z}{z}}{1.2}.\frac{2-\frac{\Delta z}{z}}{3}v^3-\frac{1-\frac{\Delta z}{z}}{1.2}\ldots\frac{3-\frac{\Delta z}{z}}{4}v^4+$
$\frac{1-\frac{\Delta z}{z}}{1.2}\ldots\frac{4-\frac{\Delta z}{z}}{5}v^5-\ldots...$

Cette équation aïant toujours lieu; elle a lieu en particulier entre les limites de ses membres; qui
sont $A\log.\ (1+v)$; et $v-\frac{1}{2}v^2+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5-\ldots..$
donc; $A\log.\ (1+v)=v-\frac{1}{2}v^2+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5-\ldots..$

$$A\log.\ 1-v=-v-\frac{1}{2}v^2-\frac{1}{3}v^3-\frac{1}{4}v^4-\frac{1}{5}v^5-\ldots..$$

$$A\log.\ \frac{1+v}{1-v}=2\ (v+\frac{1}{3}v^3+\frac{1}{5}v^5+\ldots..)$$

$$A\log.\sqrt{\frac{1+v}{1-v}}=v+\frac{1}{3}v^3+\frac{1}{5}v^5+\ldots.$$

Soit $1 + v = a$; la base du système
$A = (a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \frac{1}{4}(a-1)^4 + \frac{1}{5}(a-1)^5 - \ldots\ldots$
$= \frac{aa-1}{aa+1} + \frac{1}{3}\left(\frac{aa-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{aa-1}{aa+1}\right)^5 + \ldots\ldots$ en faisant $\frac{1+v}{1-v} = aa.$

Ce qui est la rélation par laquelle le module est determiné dans la base.

§ 7. Je ne m'arrête pas aux conséquences qui découlent de ces formules connues; mon seul but étant de montrer comment on peut les obtenir par les élémens. Je donnerai pour exemple de leur utilité, la facilité avec laquelle on obtient l'équation differentielle logarithmique; de laquelle réciproquement on a déduit le calcul des logarithmes.

Puisque $a^z = 1 + Az + \frac{A^2}{1.2}z^2 + \frac{A^3}{1.2.3}z^3 + \frac{A^4}{1.2..4}z^4 + \frac{A^5}{1.2..5}z^5 + \ldots.$
$\frac{d.a^z}{dz} = A\left(1 + Az + \frac{A^2}{1.2}z^2 + \frac{A^3}{1.2..3}z^3 + \frac{A^4}{1...4}z^4 + \ldots\right)$
$= A.a^z.$

de-là; $\frac{d^2.a^z}{dz^2} = A^2 a^z$

$\frac{d^3 a^z}{dz^3} = A^3 a^z$.

$\frac{d^4 a^z}{dz^4} = A^4. a^z$

Réciproquement. Puisque A log. $1 + v = v - \frac{1}{2}v^2 + \frac{1}{3}v^3 - \frac{1}{4}v^4 + \frac{1}{5}v^5 - \ldots\ldots$

$A\,\frac{d.\log.\,1+v}{dv} = 1 - v + v^2 - v^3 + v^4 - \ldots\ldots = \frac{1}{1+v}.$

SECONDE PARTIE. SUR LES SINUS, COSINUS, ET TANGENTES, DES ARCS DE CERCLE.

§ 8. *Lemmes connus.* 1. La différence des sinus de deux arcs est égale au double produit du cosinus de leur demi-somme par le sinus de leur demi-différence.

2. La différence du cosinus de deux arcs est égale au double produit du sinus de leur demi-somme et de leur demi-différence.

Symboliquement. 1. $\text{Sin. } a - \text{sin. } b = 2 \text{ cos. } \frac{a+b}{2} \text{ sin. } \frac{a-b}{2}$.

2. $\text{Cos. } b - \text{cos. } a = 2 \text{ sin. } \frac{a+b}{2} \text{ sin. } \frac{a-b}{2}$.

§ 9. Soient: Sin. a, sin. $2a$, sin. $3a$, sin. $4a$, sin. $5a$, sin. $6a$, ... les sinus d'arcs en progression arithmétique, croissant, *p. ex.* comme les nombres naturels. Soient prises les différences des ordres successifs de ces sinus; on obtient

Différences du premier ordre . $- 2 \text{ sin. } \frac{1}{2} a \left(\text{cos. } \frac{3}{2} a, \text{ cos. } \frac{5}{2} a, \text{ cos. } \frac{7}{2} a, \text{ cos. } \frac{9}{2} a, \text{ cos. } \frac{11}{2} a, \text{ cos. } \frac{13}{2} a \ldots\ldots\right)$

Différences du second ordre . . $- 2^2 \text{ sin.}^2 \frac{1}{2} a \left(\text{sin. } 2a, \text{ sin. } 3a, \text{ sin. } 4a, \text{ sin. } 5a, \text{ sin. } 6a, \text{ sin. } 7a \ldots\ldots\right)$

Différences du troisième ordre . $- 2^3 \text{ sin.}^3 \frac{1}{2} a \left(\text{cos. } \frac{5}{2} a, \text{ cos. } \frac{7}{2} a, \text{ cos. } \frac{9}{2} a, \text{ cos. } \frac{11}{2} a, \text{ cos. } \frac{13}{2} a, \text{ cos. } \frac{15}{2} a \ldots\ldots\right)$

Différences du quatrième ordre . . $+ 2^4 \text{ sin}^4 \frac{1}{2} a \left(\text{sin. } 3a, \text{ sin. } 4a, \text{ sin. } 5a, \text{ sin. } 6a, \text{ sin. } 7a, \text{ sin. } 8a \ldots\ldots\right)$

Différences du cinquième ordre . . $+ 2^5 \text{ sin.}^5 \frac{1}{2} a \left(\text{cos. } \frac{7}{2} a, \text{ cos. } \frac{9}{2} a, \text{ cos. } \frac{11}{2} a, \text{ cos. } \frac{13}{2} a, \text{ cos. } \frac{15}{2} a, \text{ cos. } \frac{17}{2} a \ldots\ldots\right)$

Différences du sixième ordre . . $- 2^6 \text{ sin.}^6 \frac{1}{2} a \left(\text{sin. } 4a, \text{ sin. } 5a, \text{ sin. } 6a, \text{ sin. } 7a, \text{ sin. } 8a, \text{ sin. } 9a \ldots\right)$

En général,

Différences du $2m^{me}$ ordre . . . $\pm 2^{2m}$ sin.$^{2m} \frac{1}{2} a$ (sin. $m + 1.a$, sin. $m + 2.a$, sin. $m + 3.a$, sin. $m + 4.a$, ...)

Différences du $2m + 1^{me}$ ordre . . $\pm 2^{2m+1}$ sin.$^{2m+1} \frac{1}{2} a$ (cos. $\frac{2m+3}{2} a$, cos. $\frac{2m+5}{2} a$, cos. $\frac{2m+7}{2} a$, cos. $\frac{2m+9}{2} a$, ...)

§ 10. De même; soient cos. a, cos. $2a$, cos. $3a$, cos. $4a$, cos. $5a$, cos. $6a$, ... les cosinus d'arcs en progression arithmétique croissant, *p. ex.* comme les nombres naturels. Soient prises les différences des ordres successifs de ces cosinus; on obtient

Différences du premier ordre . . $-$ 2 sin. $\frac{1}{2} a$ (sin. $\frac{3}{2} a$, sin. $\frac{5}{2} a$, sin. $\frac{7}{2} a$, sin. $\frac{9}{2} a$, sin. $\frac{11}{2} a$, sin. $\frac{13}{2} a$ . . .)

Différences du second ordre . . $- 2^2$ sin.$^2 \frac{1}{2} a$ (cos. $2a$, cos. $3a$, cos. $4a$, cos. $5a$, cos. $6a$, cos. $7a$ . . . .)

Différences du troisième ordre . . . $+ 2^3$ sin.$^3 \frac{1}{2} a$ (sin. $\frac{5}{2} a$, sin. $\frac{7}{2} a$, sin. $\frac{9}{2} a$, sin. $\frac{11}{2} a$, sin. $\frac{13}{2} a$, sin. $\frac{15}{2} a$ . . .)

Différences du quatrième ordre . . . $+ 2^4$ sin.$^4 \frac{1}{2} a$ (cos. $3a$, cos. $4a$, cos. $5a$, cos. $6a$, cos. $7a$, cos. $8a$, ...)

Différences du cinquième ordre . . . $- 2^5$ sin.$^5 \frac{1}{2} a$ (sin. $\frac{7}{2} a$, sin. $\frac{9}{2} a$, sin. $\frac{11}{2} a$, sin. $\frac{13}{2} a$, sin. $\frac{15}{2} a$, sin. $\frac{17}{2} a$ . . . . .)

Différences du sixième ordre . . . $- 2^6$ sin.$^6 \frac{1}{2} a$ (cos. $4a$, cos. $5a$, cos. $6a$, cos. $7a$, cos. $8a$, cos. $9a$ . . . . .)

En général,

Différences du $2m^{me}$ ordre . . . . . $\pm 2^{2m}$ sin.$^{2m} \frac{1}{2} a$ (cos. $m + 1.a$, cos. $m + 2.a$, cos. $m + 3.a$, cos. $m + 4.a$, cos. $m + 5.a$ . . . . .)

Différences du $2m + 1^{me}$ ordre . . . . $\pm 2^{2m+1}$ sin.$^{2m+1} \frac{1}{2} a$ (sin. $\frac{2m+3}{2} a$, sin. $\frac{2m+5}{2} a$, sin. $\frac{2m+7}{2} a$, sin. $\frac{2m+9}{2} a$, sin. $\frac{2m+11}{2} a$ . . .)

Remarque. En omettant le facteur $2^m \sin.^m \frac{1}{2} a$; ces suites (de même que la suite fondamentale) reviennent sur elles mêmes, ou sont toujours différentes, suivant que a est commensurable ou non avec la circonférence. Ces suites sont aussi celles des sinus ou cosinus d'arcs qui suivent la progression arithmétique des nombres naturels ; seulement, avec une origine différente.

§ 11. Comme sin. z est zéro, lorsque z est zéro ; et qu'outre celui le rapport d'égalité est la limite du rapport d'un arc à son sinus ; et que le sinus est plus petit que l'arc ; sin. z est une fonction de z de la forme, $z - Az^2 + Bz^3 + Cz^4 + Dz^5 + \ldots\ldots$ Et comme le cosinus d'un arc est l'unité, lorsque z est zéro ; cos. z est une fonction de z de la forme $1 - Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 + \ldots$.

§ 12. Soit donc ; $\sin. z = z - Az^2 + Bz^3 + Cz^4 + Dz^5 + Ez^6 + \ldots\ldots$

On aura aussi, $\sin. 2z = 2z - 2^2Az^2 + 2^3Bz^3 + 2^4Cz^4 + 2^5Dz^5 + 2^6Ez^6 + \ldots$

$\sin. 3z = 3z - 3^2Az^2 + 3^3Bz^3 + 3^4Cz^4 + 3^5Dz^5 + 3^6Ez^6 + \ldots$

$\sin. 4z = 4z - 4^2Az^2 + 4^3Bz^3 + 4^4Cz^4 + 4^5Dz^5 + 4^6Ez^6 + \ldots$

Soient prises les différences premières ; on obtient

$2 \sin. \frac{1}{2} z \cos. \frac{3}{2} z = z - (2^2 - 1^2)Az^2 + (2^3 - 1^3)Bz^3 + (2^4 - 1^4)Cz^4 + (2^5 - 1^5)Dz^5 + \ldots$

$2 \sin. \frac{1}{2} z \cos. \frac{5}{2} z = z - (3^2 - 2^2)Az^2 + (3^3 - 2^3)Bz^3 + (3^4 - 2^4)Cz^4 + (3^5 - 2^5)Dz^5 + \ldots$

$2 \sin. \frac{1}{2} z \cos. \frac{7}{2} z = z - (4^2 - 3^2)Az^2 + (4^3 - 3^3)Bz^3 + (4^4 - 3^4)Cz^4 + (4^5 - 3^5)Dz^5 + \ldots$

Réduisant en suites les facteurs du premier membre (§ 11), et exécutant les multiplications ; les premiers termes des produits sont $1z$; et le premier terme de chaque second membre est aussi $1z$; partant, nous apprenons seulement que le premier terme de l'expression du sinus est bien $1z$.

Soient prises les différences secondes ; on obtient

$$-2^2 \sin.^2 \tfrac{1}{2} z \sin. 2z = -\Delta^{ii}(3^2..1^2)Az^2 + \Delta^{ii}(3^3..1^3)Bz^3 + \Delta^{ii}(3^4...1^4)Cz^4 + \Delta^{ii}(3^5...1^5)Dz^5 + ..$$

$$-2^2 \sin.^2 \tfrac{1}{2} z \sin. 3z = -\Delta^{ii}(4^2...2^2)Az^2 + \Delta^{ii}(4^3...2^3)Bz^3 + \Delta^{ii}(4^4...2^4)Cz^4 + \Delta^{ii}(4^5...2^5)Dz^5 + ..$$

$$-2^2 \sin.^2 \tfrac{1}{2} z \sin. 4z = -\Delta^{ii}(5^2...3^2)Az^2 + \Delta^{ii}(5^3..3^3)Bz^3 + \Delta^{ii}(5^4...3^4)Cz^4 + \Delta^{ii}(5^5...3^5)Dz^5 + ...$$

Or ; le premier terme de chaque premier membre développé en suite conformement aux expressions du § 11^{me} est z^3 ; et les premiers membres ne contiennent pas les secondes puissances de z ; tandis que le coefficient du premier terme Az^2 des seconds membres, qui est la différence seconde des quarrés des nombres naturels, n'évanouit pas. Donc ; dans les seconds membres, le coefficient A de z^2 est zéro.

On démontre de la même manière ; que, dans la suite, $\sin. z = z - Az^2 + Bz^3 + Cz^4 + Dz^5 + Ez^6 + \ldots$, les coefficiens de toutes les puissances à exposans pairs évanouissent. Savoir ; aïant pris les différences de l'ordre pair zm qui sont $\pm z^{2m} \sin. z^{2m} \frac{1}{2} z \sin. pz$ (§ 10.) ; le premier terme du produit des facteurs développés en suites, contient la puissance impaire de z, z^{2m+1} ; de manière que dans ces produits, la puissance paire z^{2m} manque ; donc, elle doit aussi manquer dans les seconds membres : or, le premier terme de chaque second membre contient la puissance paire z^{2m}, avec deux facteurs dont l'un $\Delta^{2m} n^{2m}$ est la quantité constante $1.2\ldots 2m$ (§ 1.) et

partant n'évanouit pas ; donc, l'autre facteur de cette puissance évanouit. Partant ; le sinus d'un arc est une fonction de cet arc, de la forme sin. $z = z - Az^3 + Bz^5 + Cz^7 + Dz^9 + \ldots\ldots$ qui ne contient que les puissances impaires de z.

On a donc ; $\sin. z = z - Az^3 + Bz^5 + Cz^7 + Dz^9 + \ldots$

$$\sin. 2z = 2z - 2^3Az^3 + 2^5Bz^5 + 2^7Cz^7 + 2^9Dz^9 + \ldots$$

$$\sin. 3z = 3z - 3^3Az^3 + 3^5Bz^5 + 3^7Cz^7 + 3^9Dz^9 + \ldots$$

$$\sin. 4z = 4z - 4^3Az^3 + 4^5Bz^5 + 4^7Cz^7 + 4^9Dz^9 + \ldots$$

Soient prises les différences troisièmes ; on obtient

$$-2^3 \sin.^3 \tfrac{1}{2} z \cos. \tfrac{5}{2} z = -\Delta^{iii}(4^3 \ldots 1^3) Az^3 + \Delta^{iii}(4^5 \ldots 1^5) Bz^5 + \Delta^{iii}(4^7 \ldots 1^7) C^7 + \Delta^{iii}(4^9 \ldots 1^9) Dz^9 + \ldots$$

$$-2^3 \sin.^3 \tfrac{1}{2} z \cos. \tfrac{7}{2} z = -\Delta^{iii}(5^3 \ldots 2^3) Az^3 + \Delta^{iii}(5^5 \ldots 2^5) Bz^5 + \Delta^{iii}(5^7 \ldots 2^7) Cz^7 + \Delta^{iii}(5^9 \ldots 2^9) Dz^9 + \ldots$$

$$-2^3 \sin.^3 \tfrac{1}{2} z \cos. \tfrac{9}{2} z = -\Delta^{iii}(6^3 \ldots 3^3) Az^3 + \Delta^{iii}(6^5 \ldots 2^5) Bz^5 + \Delta^{iii}(6^7 \ldots 3^7) Cz^7 + \Delta^{iii}(6^9 \ldots 3^9) Dz^9 + \ldots$$

Réduisant en suites les facteurs des premiers membres conformement au § 11 ; les premiers termes de ces membres sont $-z^3$; et les premiers termes des seconds membres sont (§ 1.) $-1.2.3\, A^3$. Donc ; $1 = 1.2.3\, A$; et $A = \frac{1}{1.2.3}$.

Prenant successivement les différences quatrièmes et cinquièmes, on obtient

$$2^5 \sin.^5 \tfrac{1}{2} z \cos. \tfrac{7}{2} z = \Delta^{v}(6^5 \ldots 1^5) Bz^5 + \Delta^{v}(6^7 \ldots 1^7) Cz^7 + \Delta^{v}(6^9 \ldots 1^9) Dz^9 + \ldots$$

$$2^5 \sin.^5 \tfrac{1}{2} z \cos. \tfrac{9}{2} z = \Delta^{v}(7^5 \ldots 2^5) Bz^5 + \Delta^{v}(7^7 \ldots 2^7) Cz^7 + \Delta^{v}(7^9 \ldots 2^9) Dz^9 + \ldots$$

$$2^5 \sin.^5 \tfrac{1}{2} z \cos. \tfrac{11}{2} z = \Delta^{v}(8^5 \ldots 3^5) Bz^5 + \Delta^{v}(8^7 \ldots 3^7) Cz^7 + \Delta^{v}(8^9 \ldots 3^9) Dz^9 + \ldots$$

Réduisant en suites les facteurs des premiers membres (§ 11.) ; les premiers termes de ces membres sont z^5 ; et les

premiers termes des seconds membres sont $1.2\ldots 5Bz^5$. (§ 1.). Donc; $1 = 1.2\ldots 5B$; et $B = \frac{1}{1.2\ldots 5}$.

Prenant de même successivement les différences sixièmes et septièmes; on obtient

$$-2^7 \sin.^7 \tfrac{1}{2} z \cos. \tfrac{9}{2} z = \Delta^{vii}(8^7\ldots 1^7) Cz^7 + \Delta^{vii}(8^9\ldots 1^9) Dz^9 + \Delta^{vii}(8^{11}\ldots 1^{11}) Ez^{11} + \ldots$$

$$-2^7 \sin.^7 \tfrac{1}{2} z \cos. \tfrac{11}{2} z = \Delta^{vii}(9^7\ldots 2^7) Cz^7 + \Delta^{vii}(9^9\ldots 2^9) Dz^9 + \Delta^{vii}(9^{11}\ldots 2^{11}) Ez^{11} + \ldots$$

$$-2^7 \sin.^7 \tfrac{1}{2} z \cos. \tfrac{13}{2} z = \Delta^{vii}(10^7\ldots 3^7) Cz^7 + \Delta^{vii}(10^9\ldots 3^9) Dz^9 + \Delta^{vii}(10^{11}\ldots 3^{11}) Ez^{11} + \ldots$$

Réduisant en suites les facteurs des premiers membres; le premier terme de ces membres est z^7; et les premiers termes des seconds membres sont $1.2\ldots 7Cz^7$. Donc; $C = -\frac{1}{1.2\ldots 7}$.

Prenant de même les différences huitièmes et neuvièmes; on obtient, par les mêmes raisonnemens;

$D = +\frac{1}{1.2\ldots 9}$; puis $E = -\frac{1}{1.2\ldots 11}$; $F = +\frac{1}{1.2\ldots 13}$ . . . .

Donc, enfin; $\sin. z = z - \frac{1}{1.2.3} z^3 + \frac{1}{1.2..5} z^5 - \frac{1}{1.2\ldots 7} z^7 + \frac{1}{1.2..9} z^9 - \ldots\ldots$

§ 13. La recherche des cosinus se fait de la même manière.

Soit $\cos. z = 1 - Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 + \ldots$

et partant, $\cos. 2z = 1 - 2Az + 2^2 Bz^2 + 2^3 Cz^3 + 2^4 Dz^4 + 2^5 Ez^5 + \ldots$

$\cos. 3z = 1 - 3Az + 3^2 Bz^2 + 3^3 Cz^3 + 3^4 Dz^4 + 3^5 Ez^5 + \ldots$

$\cos. 4z = 1 - 4Az + 4^2 Bz^2 + 4^3 Cz^3 + 4^4 Dz^4 + 4^5 Ez^5 + \ldots$

Soient prises les différences premières ; on obtient

$$-2\sin.\tfrac{1}{2}z\sin.\tfrac{3}{2}z = -Az + (2^2-1^2)Bz^2 + (2^3-1^3)Cz^3 + (2^4-1^4)Dz^4 + \ldots$$

$$-2\sin.\tfrac{1}{2}z\sin.\tfrac{5}{2}z = -Az + (3^2-2^2)Bz^2 + (3^3-2^3)Cz^3 + (3^4-2^4)Dz^4 + \ldots$$

$$-2\sin.\tfrac{1}{2}z\sin.\tfrac{7}{2}z = -Az + (4^2-3^2)Bz^2 + (4^3-3^3)Cz^3 + (4^4-3^4)Dz^4 + \ldots$$

Développant en suites les facteurs des premiers membres ; les premiers termes de ces membres contiennent les secondes puissances z^2 de z ; et ces membres ne contiennent pas la première puissance de z. Donc ; dans les seconds membres, la seconde puissance de z doit aussi manquer ; donc, $A = 0$. On montre de la même manière, et conformement à ce qui est développé dans le § 12 ; que les puissances de z à exposans impairs manquent dans l'expression du cos. z ; de sorte que, le cosinus est une fonction de z de la forme ;

$$\cos.\ z = 1 - Az^2 + Bz^4 + Cz^6 + Dz^8 + \ldots.$$

et partant,
$$\cos.\ 2z = 1 - 2^2Az^2 + 2^4Bz^4 + 2^6Cz^6 + 2^8Dz^8 + \ldots.$$
$$\cos.\ 3z = 1 - 3^2Az^2 + 3^4Bz^4 + 3^6Cz^6 + 3^8Dz^8 + \ldots.$$
$$\cos.\ 4z = 1 - 4^4Az^2 + 4^4Bz^4 + 4^6Cz^6 + 4^8Dz^8 + \ldots.$$

Soient prises les différences secondes ; on obtient

$$-2^2\sin.^2\tfrac{1}{2}z\cos.\ 2z = -\Delta^{ii}(3^2\ldots.1^2)Az^2 + \Delta^{ii}(3^4\ldots1^4)Bz^4 + \Delta^{ii}(3^6\ldots1^6)Cz^6 + \Delta^{ii}(3^8\ldots,1^8)Dz^8 \ldots.$$

$$-2^2\sin.^2\tfrac{1}{2}z\cos.\ 3z = -\Delta^{ii}(4^2\ldots2^2)Az^2 + \Delta^{ii}(4^4\ldots2^4)Bz^4 + \Delta^{ii}(4^6\ldots2^6)Cz^6 + \Delta^{ii}(4^8\ldots2^8)Dz^8 \ldots.$$

$$-2^2\sin.^2\tfrac{1}{2}z\cos.\ 4z = -\Delta^{ii}(5^2\ldots3^2)Az^2 + \Delta^{ii}(5^4\ldots3^4)Bz^4 + \Delta^{ii}(5^6\ldots3^6)Cz^6 + \Delta^{ii}(5^8\ldots3^8)Dz^8 \ldots..$$

Réduisant en suites les premiers membres de toutes ces équations ; leur premier terme est $-z^2$. Mais, le premier

terme des seconds membres est — $1.2.Az^2$. Donc; $1 = 1.2.A$; et $A = \frac{1}{1.2}$.

Prenant successivement les différences troisièmes et quatrièmes; on obtient

$$2^4 \sin.^4 \tfrac{1}{2} z \cos. 3z = \Delta^{iv}(4^4 \ldots 1^4) Bz^4 + \Delta^{iv}(4^6 \ldots 1^6) Cz^6 + \Delta^{iv}(4^8 \ldots 1^8) Dz^8 + \Delta^{iv}(4^{10} \ldots 1^{10}) Ez^{10} \ldots.$$

$$2^4 \sin.^4 \tfrac{1}{2} z \cos. 4z = \Delta^{iv}(5^4 \ldots 2^4) Bz^4 + \Delta^{iv}(5^6 \ldots 2^6) Cz^6 + \Delta^{iv}(5^8 \ldots 2^8) Dz^8 + \Delta^{iv}(5^{10} \ldots 2^{10}) Ez^{10} \ldots$$

$$2^4 \sin.^4 \tfrac{1}{2} z \cos. 5z = \Delta^{iv}(6^4 \ldots 3^4) Bz^4 + \Delta^{iv}(6^6 \ldots 3^6) Cz^6 + \Delta^{iv}(6^8 \ldots 3^8) Dz^8 + \Delta^{iv}(6^{10} \ldots 3^{10}) Ez^{10} \ldots$$

D'où l'on a de même; $1. = 1.2 \ldots 4B$; et $B = \frac{1}{1.2 \ldots 4}$.

Soient prises successivement les différences cinquièmes et sixièmes; on obtient

$$-2^6 \sin.^6 \tfrac{1}{2} z \cos. 4z = \Delta^{vi}(6^6 \ldots 1^6) Cz^6 + \Delta^{vi}(6^8 \ldots 1^8) Dz^8 + \Delta^{vi}(6^{10} \ldots 1^{10}) Ez^{10} + \ldots$$

$$-2^6 \sin.^6 \tfrac{1}{2} z \cos. 5z = \Delta^{vi}(7^6 \ldots 2^6) Cz^6 + \Delta^{vi}(7^8 \ldots 2^8) Dz^8 + \Delta^{vi}(7^{10} \ldots 2^{10}) Ez^{10} + \ldots$$

$$-2^6 \sin^6 \tfrac{1}{2} z \cos. 6z = \Delta^{vi}(8^6 \ldots 3^6) Cz^6 + \Delta^{vi}(8^8 \ldots 3^8) Dz^8 + \Delta^{vi}(8^{10} \ldots 3^{10}) Ez^{10} + \ldots.$$

D'où l'on obtient $-1 = 1.2 \ldots 6C$; et $C = -\frac{1}{1.2 \ldots 6}$.

On obtient successivement .... $D = + \frac{1}{1.2 \ldots 8}$.

$E = - \frac{1}{1.2 \ldots 10}$.

Et partant; $\cos. z = 1 - \frac{1}{1.2} z^2 + \frac{1}{1.2..4} z^4 - \frac{1}{1.2 \ldots 6} z^6 + \frac{1}{1.2 \ldots 8} z^8 - \frac{1}{1.2 \ldots 10} z^{10} + \ldots\ldots$

§ 14. Puisque $\sin. z = z - \frac{1}{1.2.3} z^3 + \frac{1}{1.2 \ldots 5} z^5 - \frac{1}{1.2 \ldots 7} z^7 + \frac{1}{1.2..9} z^9 - \ldots\ldots$

et $\cos. z = 1 - \frac{1}{1.2} z^2 + \frac{1}{1.2..4} z^4 - \frac{1}{1.2 \ldots 6} z^6 + \frac{1}{1.2 \ldots 8} z^8 - \ldots$

$$\text{tang.}\, z \left(=\frac{\sin. z}{\cos. z}\right) = z \times \frac{1 - \frac{1}{1.2.3}z^2 + \frac{1}{1.2..5}z^4 - \frac{1}{1.2..7}z^6 + \frac{1}{1.2...9}z^8 - \ldots\ldots}{1 - \frac{1}{1.2}z^2 + \frac{1}{1.2..4}z^4 - \frac{1}{1.2..6}z^6 + \frac{1}{1.2..8}z^8 - \ldots\ldots}$$

§ 15. Après avoir exprimé le sinus, le cosinus, et la tangente d'un arc, dans cet arc ; on pourroit réciproquement par la méthode du retour des suites, exprimer l'arc dans ces fonctions de lui-même. La méthode suivante est plus élémentaire ; et plus conforme au procédé que j'ai suivi jusqu'à présent.

Il est connu ; que, $\cos. nz = \frac{(\cos. z + \sin. z\sqrt{-1})^n + (\cos. z - \sin. z\sqrt{-1})^n}{2}$

$$\sin. nz = \frac{(\cos. z + \sin. z\sqrt{-1})^n - (\cos. z - \sin. z\sqrt{-1})^n}{2\sqrt{-1}}$$

De-là ; $\cos. nz + \sin. nz\sqrt{-1} = (\cos. z + \sin. z\sqrt{-1})^n$;

$$(\cos. nz + \sin. nz\sqrt{-1})^{\frac{1}{n}} = \cos. z + \sin. z\sqrt{-1}$$

$$\cos. nz - \sin. nz\sqrt{-1} = (\cos. z - \sin. z\sqrt{-1})^n\,;$$

$$(\cos. nz - \sin. nz\sqrt{-1})^{\frac{1}{n}} = \cos. z - \sin. z\sqrt{-1}$$

Partant ; $\cos. z = \frac{(\cos. nz + \sin. nz\sqrt{-1})^{\frac{1}{n}} + (\cos. nz - \sin. nz\sqrt{-1})^{\frac{1}{n}}}{2}$.

$$\sin. z = \frac{(\cos. nz + \sin. nz\sqrt{-1})^{\frac{1}{n}} - (\cos. nz - \sin. nz\sqrt{-1})^{\frac{1}{n}}}{2\sqrt{-1}}.$$

De-là ; $n \sin. z = \cos. nz^{\frac{1}{n}} \times (\text{tang.}\, nz$ Partant aussi ; $n \sin. \frac{1}{n} z = \cos. z^{\frac{1}{n}}\,(\text{tang.}\, z$

$$- \frac{1-\frac{1}{n}}{1.2}.\ \frac{2-\frac{1}{n}}{3}\,\text{tang.}^3\, nz \qquad\qquad - \frac{1-\frac{1}{n}}{1.2}.\ \frac{2-\frac{1}{n}}{3}\,\text{tang.}^3 z$$

$$+ \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{4-\frac{1}{n}}{5}\,\text{tang.}^5\, nz \qquad\qquad + \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{4-\frac{1}{n}}{5}\,\text{tang.}^5 z$$

$$- \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{6-\frac{1}{n}}{7}\,\text{tang.}^7\, nz \qquad\qquad - \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{6-\frac{1}{n}}{7}\,\text{tang.}^7 z$$

$$+ \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{8-\frac{1}{n}}{9}\,\text{tang.}^9\, nz \qquad\qquad + \frac{1-\frac{1}{n}}{1.2}\ldots.\frac{8-\frac{1}{n}}{9}\,\text{tang.}^9 z$$

$$- \ldots\ldots\ldots\ldots \qquad\qquad - \ldots\ldots\ldots\ldots$$

$$+ \ldots\ldots\ldots\ldots \qquad\qquad + \ldots\ldots\ldots\ldots$$

Donc aussi ; les limites des deux membres de cette équation sont égales entr'elles. Mais, en augmentant n, les limites sont z, et tang. $z - \frac{1}{3}$ tang. $^3z + \frac{1}{5}$ tang. $^5z - \frac{1}{7}$ tang. $^7z + \frac{1}{9}$ tang. ^9z — . . . . . .

Donc ; $z = t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7 + \frac{1}{9}t^9 - \ldots\ldots$ (faisant t = tang. z).

Remarque. Je m'exprime dans ce mémoire d'une manière fort concise sur le passage des quantités variables susceptibles de limites à leurs limites. Je suppose connu que cette théorie peut être dégagée de toute idée de l'infini ; et rappellée à la méthode rigoureuse des anciens connue sous le nom de *Méthode d'Exhaustion.* D'après Newton, Maclaurin, Robins, et autres auteurs, j'ai taché de mettre cette théorie à l'abri de toute contestation, dans ma Prix sur l'Infini Mathématique, couronnée par l'Académie de Berlin en 1786. J'espère l'avoir fait de la manière la plus satisfaisante dans l'ouvrage qui s'imprime dans ce moment sous le titre : *Principiorum Calculi differentialis et integralis Expositio elementaris.*

§ 16. Des formules précédentes, on déduit aisément les formules différentielles des fonctions trigonométriques des arcs de cercle.

Puisque sin. $z = z - \frac{1}{1.2.3}z^3 + \frac{1}{1.2..5}z^5 - \frac{1}{1.2...7}z^7 + \frac{1}{1.2...9}z^9 - \ldots\ldots$

$$\frac{d.\,\text{sin.}\,z}{dz} = 1 - \frac{1}{1.2}z^2 + \frac{1}{1.2..4}z^4 - \frac{1}{1.2..6}z^6 + \frac{1}{1.2...8}z^8 - \ldots. = \text{cos.}\ z.$$

Puisque cos. $z = 1 - \frac{1}{1.2}z^2 + \frac{1}{1.2..4}z^4 - \frac{1}{1.2..6}z^6 + \frac{1}{1.2..8}z^8 - \ldots\ldots$

$$\frac{d.\cos.z}{dz} = -z + \frac{1}{1.2.3}z^3 - \frac{1}{1.2..5}z^5 + \frac{1}{1.2...7}z^7$$
$$- \ldots\ldots = -\sin.z.$$

Puisque $\tang.z = \frac{\sin.z}{\cos.z}$; $\frac{d.\tang.z}{dz} = \frac{\cos.z\ \frac{d.\sin.z}{dz} - \sin.z.\ \frac{d.\cos.z}{dz}}{\cos.^2z}$

$= \frac{\cos.^2z + \sin.^2z}{\cos.^2z} = \frac{1}{\cos.^2z} = \sec.^2z.$

C'est ce qu'on pourroit tirer de l'expression

$z = t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7 + \frac{1}{9}t^9$;

$\frac{dz}{dt} = 1 - tt + t^4 - t^6 + t^8 - \ldots = \frac{1}{1+tt} = \frac{1}{\sec.^2t}$; et partant $\frac{dt}{dz} = \sec.^2t.$

De-là encore, on déduit les rapports différentiels de tous les ordres successifs.

Savoir; $\frac{d.\sin.z}{dz} = \cos.z$ $\qquad \frac{d.\cos.z}{dz} = -\sin.z$

$\frac{d^2\sin.z}{dz^2} = -\sin.z$ $\qquad \frac{d^2\cos.z}{dz^2} = -\cos.z$

$\frac{d^3\sin.z}{dz^3} = -\cos.z$ $\qquad \frac{d^3\cos.z}{dz^3} = +\sin.z$

$\frac{d^4\sin.z}{dz^4} = +\sin.z$ $\qquad \frac{d^4\cos.z}{dz^4} = +\cos.z$

$\frac{d^5\sin.z}{dz^5} = +\cos.z$ $\qquad \frac{d^5\cos.z}{dz^5} = -\sin.z$

Troisième Partie. Sur l'Analogie entre les Logarithmes et les Fonctions trigonométriques des Arcs circulaires.

§ 17. La ressemblance qui règne entre le procédé par lequel j'ai obtenu les expressions des quantités exponentielles dans leurs exposans, et celui par lequel j'ai obtenu les expressions du sinus et du cosinus d'un arc dans cet arc, mè paroît ex-

pliquer de la manière la plus lumineuse l'analogie observée depuis longtems entre les quantités exponentielles et ces fonctions circulaires ; et rendre raison de la conformité des résultats de ces procédés.

Par le § 4. $\frac{e^{z}+e^{-z}}{2} = 1 + \frac{1}{1.2} z^2 + \frac{1}{1.2..4} z^4 + \frac{1}{1.2...6} z^6 + \frac{1}{1.2...8} z^8 + \frac{1}{1.2...10} z^{10} + \ldots$

Et (§ 13.) $\cos. z = 1 - \frac{1}{1.2} z^2 + \frac{1}{1.2..4} z^4 - \frac{1}{1.2...6} z^6 + \frac{1}{1.2...8} z^8 - \frac{1}{1.2...10} z^{10} + \ldots$

Ces expressions diffèrent seulement par les signes des termes alternatifs, qui contiennent des puissances impairement paires de z; partant, si dans la première on change le signe de zz, en substitant $-zz$ à zz, ou $z\sqrt{-1}$ à z, on obtiendra la seconde ; d'où l'on a été appellé à présenter cos. z sous la forme exponentielle imaginaire, $\cos. z = \frac{e^{+z\sqrt{-1}} + e^{-z\sqrt{-1}}}{2}$.

De même (§ 4.) ; $\frac{e^{z}-e^{-z}}{2} = z + \frac{1}{1.2..3} z^3 + \frac{1}{1.2..5} z^5 + \frac{1}{1.2...7} z^7 + \frac{1}{1.2...9} z^9 + \ldots\ldots$

Et $\sin. z = z - \frac{1}{1.2.3} z^3 + \frac{1}{1.2...5} z^5 - \frac{1}{1.2...7} z^7 + \frac{1}{1.2...9} z^9 - \ldots\ldots$

Si dans le second membre de la première équation on substitue $z\sqrt{-1}$ à z; et si on divise le résultat par $\sqrt{-1}$; on obtient le second membre de la seconde équation. De-là, on a été appellé à présenter sin. z sous la forme exponentielle imaginaire, $\sin. z = \frac{e^{+z\sqrt{-1}} - e^{-z\sqrt{-1}}}{2\sqrt{-1}}$.

De-là ; $\text{tang. } z = \frac{1}{\sqrt{-1}} \cdot \frac{e^{z\sqrt{-1}} - e^{-z\sqrt{-1}}}{e^{z\sqrt{-1}} + e^{-z\sqrt{-1}}}$; et $t\sqrt{-1} = \frac{e^{z\sqrt{-1}} - e^{-z\sqrt{-1}}}{e^{z\sqrt{-1}} + e^{-z\sqrt{-1}}}$.

Donc ; $e^{z\sqrt{-1}} : e^{-z\sqrt{-1}} = 1 + t\sqrt{-1} : 1 - t\sqrt{-1}$

$$e^{2z\sqrt{-1}} : 1 \qquad = 1 + t\sqrt{-1} : 1 - t\sqrt{-1};$$

$$e^{2z\sqrt{-1}} = \frac{1 + t\sqrt{-1}}{1 - t\sqrt{-1}};\ 2z\sqrt{-1} = \log.\frac{1 + t\sqrt{-1}}{1 - t\sqrt{-1}}.$$

$$z = \frac{1}{2\sqrt{-1}} \log.\frac{1 + t\sqrt{-1}}{1 - t\sqrt{-1}}.$$

Cette formule auroit pu aussi être déduite des deux expressions

$$\tfrac{1}{2}\log.\frac{1+v}{1-v} = v + \tfrac{1}{3}v^3 + \tfrac{1}{5}v^5 + \tfrac{1}{7}v^7 + \tfrac{1}{9}v^9 + \ldots.$$

$$z = t - \tfrac{1}{3}t^3 + \tfrac{1}{5}t^5 - \tfrac{1}{7}t^7 + \tfrac{1}{9}t^9 - \ldots\ldots$$

IX. *On the Method of observing the Changes that happen to the fixed Stars; with some Remarks on the Stability of the Light of our Sun. To which is added, a Catalogue of comparative Brightness, for ascertaining the Permanency of the Lustre of Stars. By* William Herschel, *LL.D. F.R.S.*

Read February 25, 1796.

THE earliest observers of the stars have taken notice of their different degrees of brilliancy, and, by way of expressing their ideas to others, have classed them into magnitudes. Brightness and size among the stars were taken as synonymous terms, and may still be used as such with sufficient truth, notwithstanding the latter, it seems, can only be looked upon as the consequence of the former. The brightest stars were called of the first magnitude; the next of the second; and those of an inferior lustre of the third, fourth, and fifth magnitudes; and so on.

Among the stars of the first two or three classes there seems to be some natural limit which confines them to a particular order. If we suppose the stars to be about the size of our sun, and at nearly an equal distance from us and from each other, those which form the first inclosure about us will appear brighter than the rest, and there can be only a small number of them. This hypothesis is nearly confirmed by observation, as may be seen by looking over a globe, and applying a pair of compasses opened to 60 degrees, which

should be the angle subtended by the stars of the first magnitude, if they were all scattered equally. For it will be found that the distances from Lyra to Arcturus; from Arcturus to Regulus; from Regulus to Sirius; from Sirius to β Navis; from Elgeuse to Canopus; from Canopus to α Centauri; from α Centauri to Achernar; from Achernar to α Crucis; from Procyon to Canopus; from Fomalhaut to Altair; and from Altair to Antares, agree sufficiently well with this hypothesis. It must also be remembered that a perfect equality in the mutual angular distribution of the stars that form the first inclosure, is a thing that is mathematically impossible, and therefore not to be looked for. This would authorize us to take in other intervals, such as from Arcturus to Antares; from Elgeuse to Regulus; from Achernar to Rigel; from Rigel to Capella; from Capella to Sirius; from Regulus to Spica; from Spica to α Crucis; and from Rigel to Castor; all which concur, in a great measure, to support the same hypothesis. But as the distribution and real magnitude of stars is not my present subject, what has been mentioned will be sufficient.

A second layer of stars will be more extensive; for the superficies of the celestial regions allotted for the situation of these successive stars exceeds the former in the ratio of 4 to 1. And on looking over the collection of stars which astronomers have pointed out as belonging to the second class, we find that their number is proportionally larger.

A similar way of considering the stars of the third order might be applied, if it did not already appear, from what has been said of the two former orders, when strictly compared with the state of the heavens, that such kind of limits can be of no real use in the classification of stars. The hypothesis

of an equality and an equal distribution of stars to which we have referred, is too far from being strictly true to be laid down as an unerring guide in this research. The stars of the first and second class, when scrupulously examined, evidently prove that if we would be accurate, we must admit them, in some degree at least, to be either of different sizes, or placed at different distances. Both varieties undoubtedly take place. This consideration alone is fully sufficient to shew, that how much truth soever there may be in the hypothesis of an equal distribution and equality of stars, when considered in a general view, it can be of no service in a case where great accuracy is required.

Since therefore it appears that in the classification of stars into magnitudes, there either is no natural standard at all, or at least none that can be satisfactory; it follows, that astronomers who have classed them thus, have referred their size or lustre to some imaginary idea of brightness. The great number of stars, indeed, which have been placed into every particular class, may assist us to form a kind of confused type in our minds, by which we may be enabled to arrange others; but how doubtful this must ever remain, we may see from the circumstance of the intermediate expressions that have been introduced.

1.2 m* for instance, denotes that a star so marked is between the first and second magnitude. 2.1 m signifies the same thing, with an intimation that the star so distinguished is nearly of the second magnitude, but partakes still something of the lustre of a star of the first order. With stars of the first, second, and third classes there may be some

* I use the letter m in a short way to express the magnitude of the stars.

necessity to introduce such subdivisions; but how very vague must be the expressions 5m, 5.6m, 6.5m, 6m! In vain have I endeavoured to find a criterion for a star of any one of these magnitudes. On looking over, for instance, the stars of the fifth order, I found that in the list of other stars which ought to be less bright because they were marked 5.6m, 6.5m, or 6m, there were many that exceeded the former in brightness, while among those that are put down 5.4m, 4.5m, or even 4m, which ought to be more bright, I found several of a lustre not equal to some of this fifth magnitude, which I was desirous to ascertain. I may therefore justly call the method that has been hitherto in use to point out the lustre of stars, a reference to an imaginary standard.

The inconvenience arising from this unknown, or at least ill ascertained type to which we are to refer, is such, that now our most careful observations labour under the greatest disadvantage. If any dependence could be placed upon the method of magnitudes, it would follow, that no less than eleven stars in the constellation of the Lion, namely, $\beta\,\sigma\,\pi\,\xi$ A *b c d* 54 48 72, had all undergone a change in their lustre since FLAMSTEED's time. For if the idea of magnitudes had been a clear one, our author, who marked β 1.2m, and γ 2m, ought to be understood to mean that β is larger than γ; but we now find that actually γ is larger than β. Every one of the eleven stars I have pointed out may be reduced to the same contradiction; and as the subject is of some consequence, I shall give a few other instances of them.

σ by FLAMSTEED is 4.5m, $\iota\,\phi\,\upsilon\,\lambda\,\kappa\,\pi\,\xi$ are all marked 4m, and therefore ought to be larger; but σ is larger than any of them.

π is marked 4m; *d* 6.5m, χ and *e* 4.5m, *c* and 72 5m; therefore π should be larger than all the former; but it is less.

ξ is marked 4m; but there are eleven stars, namely, σ *b* 54 A *d* χ *e* *c* 72 27 48 69, all marked in various manners less than that star, yet they all exceed it in magnitude.

Not to proceed any farther with particulars, we ought to account for this by allowing that FLAMSTEED did not compare the stars to each other, but referred each of them separately to its own imaginary standard of magnitude. This is the real source of all such contradictions, which therefore cannot be charged to our author. As we should, however, take it for granted, that the magnitudes were affixed to the stars with as much care as the nature of an unsettled standard would allow, a short inquiry into the extent of the confidence we may place upon the method of magnitudes will be of considerable use.

We have observed that in this method the brightness of stars is referred to unsettled standards; but admitting that a pretty general though coarse idea may be formed of these magnitudes, it may be granted that a mistake of a whole order in the first class cannot be supposed. The difference between a star of the first and second magnitude is so palpable that it excludes all suspicion of taking one for the other.

When subdivisions are introduced, the case becomes doubtful. 1.2m may easily pass for 2.1m. But though these two notations should not be sufficiently clear to be distinguished from each other, yet I am inclined to believe that the former may be precise enough to point out a difference from 2m, and the latter from 2.3m.

With the next order of stars the difference is much less striking; but yet 2m will convey an idea which may be pretty

well distinguished from 3m. 2.3m, however, cannot be sufficiently kept apart from 3.2m, or either of these expressions from 3m, or from 2m. Perhaps the former may be distinguished from 3.4, and the latter from 4m.

The following step from 3m to 4m, or indeed from 3.4m to 4.5m, is less decisive than from 2 to 3m.

Again, if a star had changed from 4m to 5m, or from 4.5m to 5.6m since FLAMSTEED'S time, we could hardly entertain more than a very slight suspicion of the alteration. From 4 to 5.6m, or from 4.5 to 6m, would be a pretty considerable step, and might serve as a foundation for an argument.

A change from 5m to 6m is such as no stress could be laid upon ; and such are the changes from 5.6 to 6.7m, and from 6 to 7m. In all these inferior orders less than an alteration of a magnitude and an half could hardly deserve attention.

Here we have supposed all references to be made to the same author; for when other astronomers are consulted the uncertainty is much increased. A star which in FLAMSTEED'S catalogue stands 1.2m, may be found 2m in another author : 2m in the former may be rated 2.3m, or even 3m by the latter. Of course 3m and 4m may be written for the magnitude of the same star by different persons. 4 and 5m as well as 5 and 6m are frequently interchanged, and no stress can be laid upon such nominal differences in different catalogues. We can hardly allow less than half a magnitude in the higher orders, and a whole one in the inferior classes, for this uncertainty.

To apply what has been said : suppose there should be some inducement to believe a certain star, such as β Leonis, to have changed its lustre. Now having no real, existing type of

comparison, we can only refer to the general, imaginary one; and here the rules we have laid down will be of considerable service. The magnitude of this star given by FLAMSTEED is 1.2m; but as we have shewn that there is some ground to admit that 1.2m, even in this coarse way of reference, may be distinguished from what the same author seems to have taken for 2m, we conclude that the star has probably lost some of its former brightness. Again, he gives β 1.2m, and γ 2m. This notation may be taken to imply, though indirectly, that β is larger than γ; which not being the case, we have an additional reason to suspect a change. DE LA CAILLE puts down β 2m. Now the difference between the notation 1.2m of FLAMSTEED and 2m of the latter author, can add nothing to the force of the argument for a change; as we have observed before, that a considerable allowance must be made for nominal varieties in different authors. Nor can we draw any support from the magnitude itself, because the star will pass very well for one of that order, when compared with other stars which are marked 2m by the same author. But when DE LA CAILLE marks β 2m, and γ 3m, we may then conclude that he estimated β to be larger than γ, though we do not know that he compared these two stars together; because a whole magnitude in the second class, as we have said, cannot well be mistaken, coarse as is the type to which the reference is made. Upon the whole, therefore, we conclude that β Leonis is now less brilliant than it was formerly.

In this manner, with proper circumspection, we may get at some certainty, even by the method of magnitudes; the imperfection of it, however, in other cases is very obvious. σ Leonis, for instance, being marked by FLAMSTEED 4.5m, the

star itself will in every respect pass for one of that magnitude, when compared to a mental standard taken from other stars of the same author. Nor can its being brighter than stars which have a magnitude of a superior lustre affixed to them, do more than raise a considerable suspicion of a change. But as this subject will occur again hereafter, and as it must be sufficiently apparent that the present method of expressing the brightness of the stars is very defective, we now proceed to propose a different one.

I place each star, instead of giving its magnitude, into a short series, constructed upon the order of brightness of the nearest proper stars. For instance, to express the lustre of D, I say CDE. By this short notation, instead of referring the star D to an imaginary uncertain standard, I refer it to a precise, and determined, existing one. C is a star that has a greater lustre than D; and E is another of less brightness than D. Both C and E are neighbouring stars, chosen in such a manner that I may see them at the same time with D, and therefore may be able to compare them properly. The lustre of C is in the same manner ascertained by BCD; that of B by ABC; and also the brightness of E by DEF; and that of F by EFG.

That this is the most natural, as well as the most effectual way to express the brightness of a star, and by that means to detect any change that may happen in its lustre, will appear, when we consider what is requisite to ascertain such a change. We can certainly not wish for a more decisive evidence, than to be assured, by actual inspection, that a certain star is now no longer more or less bright than such other stars to which it has been formerly compared; provided we are at the same

time assured that those other stars remain still in their former unaltered lustre. But if the star D will no longer stand in its former order CDE, it must have undergone a change; and if that order is now to be expressed by CED, the star has lost some part of its lustre; if on the contrary, it ought now to be denoted by DCE, its brightness must have had some addition. Then, if we should doubt the stability of C and E, we have recourse to the orders BCD, and DEF, which express their lustre; or even to ABC, and EFG, which continue the series both ways. Now having before us the series BCDEF, or if necessary even the more extended one ABCDEFG, it will be impossible to mistake a change of brightness in D, when every member of the series is found in its proper order, except D.

Here I have used the letters of the alphabet merely to explain my way of fixing the order of brightness of the stars. In the journal or catalogue itself, which gives this order of brightness, each star must bear its own proper name, or number. For instance, the brightness of the star δ Leonis may be expressed by $\beta\ \delta\ \varepsilon$ Leonis, or better by 94 — 68 — 17 Leonis; these being the numbers which the three above stars bear in the British catalogue of fixed stars.

Perhaps it may be thought that the known introduction of letters, added to the magnitudes of the stars, seems to be that very method which I now recommend, as different from what has already been used. And certainly if letters had been annexed to stars with a strict view to their order of brightness, they would now be of considerable service; but the intention of the astronomers who lettered the stars seems only to have been to give them a name, whereby to call them more readily, than by the descriptive method of pointing out their situation.

It was indeed natural enough to give the name α to the brightest star, on account of its being the most remarkable in a constellation; and we may admit that with a few of the most conspicuous stars the letters $\alpha\,\beta\,\gamma$ would present themselves in succession; but whoever compares all the letters of the Greek and English alphabet that have been used, with the numerical magnitudes annexed to the same stars, will immediately give up all thoughts of intended order. In the constellation of Andromeda, which happens to lie before me, I find the following arrangement: $\delta\,o\,\mu\,\varepsilon$, $\theta\,\pi\,\xi$, $\lambda\,\upsilon\,\upsilon\,\lambda$, and *d b c*. In that of Hercules $\varepsilon\,\delta$, $\xi\,\lambda\,\kappa$, $\pi\,\theta$, $\varrho\,\mu$, $\sigma\,\nu$, $\tau\,o$, and *h* A *e b k q c m* Z.

It will be needless to point out the irregularities which take place in every other constellation; they go indeed so far, that it would be wrong to call them irregularities, because certainly no order could be intended in the arrangement of the letters. A doubt has even arisen whether any succession of brightness might be argued from the very first, second, or third letters of the alphabet; and when we find them arranged thus: $\beta\,\alpha$ Cassiopeæ, $\beta\,\alpha$ Cancri, $\gamma\,\beta$ Aquilæ, $\beta\,\zeta$ Canis minoris, 41 γ Arietis, we can hardly think it safe to regard the order of letters as of the least consequence. To which may be added, that in many constellations $\alpha\,\beta\,\gamma$ are all marked to be of the same magnitude, in which case again the order of the letters can bring no information. And therefore, even in those cases where the order of the letters agrees with the different magnitudes assigned to them, the knowledge we can have of the former state of the heavens must be derived from the magnitudes, and cannot be from the letters.

It may in the next place be remarked, that if not the

letters, at least the numerical magnitudes affixed to the stars by astronomers, point out an order of brightness; and therefore contain my method already established. A succession of the marks 1, 2, 3, 4, 5, *&c.* and other intermediate notations, which are to be found in the British, and other catalogues, give us a long list of stars that are (or should be) in a regular order of brightness, from a star of the first magnitude down to one of the eighth or ninth.

That these marks, denoting the magnitudes of the stars, are of some use every astronomer will readily perceive; but if we would apply them to the purpose of detecting a change in the lustre of some suspected star, the defect of this method will easily appear, and has already been shewn in the instance of σ Leonis. It was hinted before that the subject would recur again, I shall therefore mention two other instances, in the first of which the common notation is sufficiently expressive. It will be so in all cases where a very considerable change takes place. Thus, β Persei being marked 2.3m, and ϱ of the same constellation 4m, there could be no doubt of a change in the light of Algol when it was found to be not brighter than ϱ. But let us in the next place take an observation recorded in my journal.

"May 12, 1782. β Lyræ is much less than γ."

Now, examining the British catalogue, we find β 3m, and γ 3m. Had the method of orders been adopted by FLAMSTEED, we should at once have pronounced this star to be changeable. For it would have been $\beta\,\gamma$ in his time, and $\gamma\,\beta$ at the time of observation; but since we have shewn that no inference can be drawn from the order of the letters, we have only the magnitudes to refer to. And here again the deviation

of β from its usual brightness not being so considerable, but that a star such as it appeared to be at the time of observation might pass for one of the third magnitude, we are left in the dark; notwithstanding which a few years after, this star was actually found to be not only changeable, but periodical.*

M. DE LA LANDE in mentioning the change of δ Ursæ majoris arranges the seven bright stars of that constellation as they appeared to him; and remarks that sometimes γ and ε should stand before β, and sometimes after it. Here we have something like an order of seven remarkable stars; but as it happens, the stars themselves are not favourable to the formation of a regular series. Mr. PIGOTT and Mr. GOODERICKE also compared the stars whose changes they were examining to other neighbouring stars that were proper to be estimated with them, and were in a manner forced to lay aside the method of magnitudes.† These instances contribute to support the arguments I have used, to shew that another method of ascertaining the lustre of the stars is required, while at the same time they sufficiently indicate that the comparative brightness of stars is the only safe one to which we can have recourse.

It will be necessary now to enter into a full display of my proposed method; for simple as it is in its principle, it is not only difficult but very laborious in its progress. I began to put it into execution about 14 years ago; but other very interesting astronomical pursuits have broken in upon the regular continuation of it. By relating the difficulties or inconveniences as they happened, it will appear that my present

* Phil. Trans. Vol. LXXVI. Part I. page 197.

† Phil. Trans. Vol. LXXV. Part I. page 127 and 154.

notation, as well as method of arranging the observations, are liable to the fewest objections.

The general disposition of the stars is in constellations. This order is to be preferred to that of right ascension, or polar distance, because the stars being to be compared to the nearest proper stars that can be found, the constellations themselves will generally answer that purpose better than other selections.

My first design was to draw each whole constellation into one series. Accordingly I began July 16, 1781, to arrange the stars in Ophiuchus thus:

"Order of the stars in Ophiuchus; $\alpha\,\beta\,\delta\,\zeta\,\eta\,\kappa\,\gamma\,\varepsilon$."

This way of placing the stars agrees so far with my present one, that any star, such as κ for instance, may be taken, and the expression of its lustre will be had by $\eta\,\kappa\,\gamma$. And as FLAMSTEED marks the magnitudes of these stars 3m 4m 3m, my arrangement does not agree with his. If we should now suspect κ to have changed its lustre, recourse may be had to another star on both sides, which gives $\zeta\,\eta\,\kappa\,\gamma\,\varepsilon$. The magnitudes of FLAMSTEED are 3m 3m 4m 3m 3.4m, where κ again seems to be placed in a situation to which it is not intitled.

A defect of this arrangement, which was not immediately perceived, is that in taking the stars of a constellation we have not always a proper connection of the steps of the series that may be formed of them: there being too much difference in the lustre of some of the stars, and too little in others.

Other inconveniences will also arise from the multiplicity of the members of a general series, and the trouble of arranging them when they are nearly equal. To get over these difficulties I marked the stars that differed much in lustre by

magnitudes or degrees of difference; in which I assumed three different sorts of each; namely, 1′ 1″ 1‴ 2′ 2″ 2‴, *&c.* For instance,

"May 12, 1783. Order of the stars in Bootes;

"$\alpha 1' \ \epsilon 2'' \ \eta 2''' \ \gamma \beta \delta 3' \ \varrho 3'' \ \zeta 3''' \ \pi 4$."

That this is not recurring to the old method of magnitudes, will appear when we consider that the stars are strictly compared. The series $\alpha \epsilon \eta \gamma \beta \delta \varrho \zeta \pi$ remains established, but the difference in the gradation of brightness between the members of the series is added to it. At first this seemed to answer the intended purpose; for $\alpha \epsilon \eta$ not being sufficiently distinguished, the addition 1′ to α, and 2″ to ϵ, shewed that α was very much brighter than ϵ, while 2‴ added to η denoted only a very small difference between this and ϵ. The difficulty which immediately after arose in the choice of the magnitudes, however, soon convinced me that the fallacy of them would still have some influence upon the arrangements. The same evening I marked the stars in Leo thus:

"Order of the stars in Leo;

"$\alpha 1''' \ \gamma 2' \ \beta 2' \ \delta \epsilon \ \zeta \theta \eta \ \mu o \ \varrho \nu \sigma$."

Here I parcelled them together in the order of brightness, but could not find a convenient way to denote the different degrees by using any derivation from magnitudes; therefore I contented myself with placing those close together that agreed nearly with each other, and kept a little distance between those that differed rather more. This might perhaps have answered the required end, if the confusion which would arise from the distance of letters had not proved a great objection. And that it would unavoidably bring on mistakes we

may see by the other constellations which were arranged that evening.

"Draco $\gamma\ \eta\ \beta\quad \delta\zeta\iota\quad \theta\lambda\alpha\varkappa\quad \xi$

"Cygnus $\alpha\quad \gamma\varepsilon\quad \beta\delta\zeta\quad \theta$

"Hercules $\beta\zeta\alpha\quad \delta\eta\pi\quad \gamma\varepsilon\mu$ r* changed."

August 16, 1783, being upon the same subject of assigning comparative magnitudes, I introduced lines to shew the intended distances of the letters, with a view to prevent mistakes that might be made in transcribing them, and expressed the order as follows:

"Order of the stars in Auriga;

"α —— $\gamma\beta$ —— $\iota\theta$ —— $\varepsilon\eta\zeta$ —— $\upsilon\pi\tau$"

The marks denoted that all the stars were in succession, but that the distance between those which are separated by lines was greater than that between the rest. When stars occurred that were nearly equal, I placed them under each other, thus:

"Order of the stars in Ursa Minor, $\alpha\beta$ —— γ —— $\begin{matrix}\delta\text{"}\\ \varepsilon\\ \zeta\end{matrix}$

But in this expression there is the inconvenience of its breaking in upon the lines above and below.

Another cause of disorder arose from the stars which are not lettered. For here we are obliged to use numbers in lieu of them; and these, unless properly separated, will run into one another, and occasion mistakes.

* I called it r changed, because this star, which in my edition of 1725 is marked 3 m, is only of the 5th magnitude. At that time I ascribed the difference to a change in the star; but I have since found that there is an error in the edition of 1725 which is not in that of 1712, where the star is marked as it ought to be, of the 5th magnitude.

In the next place, the letters themselves became troublesome; for a star cannot be found so readily in a catalogue or in an atlas by a letter, as it may be by a number.

The inconveniences attending the above different ways of notation having now been sufficiently pointed out, it remains only to lay down the method upon which, after many trials, I have fixed, in order to avoid them.

Setting aside the letters entirely I use only numbers in all my observations, and these numbers are such as I have added with red ink both to the edition of 1725 of the British catalogue, and to the Atlas Cœlestis taken from that catalogue, and printed in 1729. When I use other stars than what are contained in the British catalogue, the authors who have given them, and their numbers in the catalogues from whence they are taken, are particularly mentioned.

In the choice of the stars which are to express the lustre of any particular one, my first view is directed to a perfect equality. When two stars are perfectly alike in brightness, so that by looking often and a long while at them, I either cannot tell which is the brightest, or occasionally think one the largest, and sometimes, not long after, give the preference to the other, I put down their numbers together, only separated by a point. For instance, 30 . 24 Leonis. However, it can happen but very seldom that the equality in the lustre of two neighbouring stars is so perfect as not to leave an inclination to prefer one to the other; therefore I place that first which may probably be the largest, even though I do not particularly judge it to be so. But this preference is never to be understood to extend so far as to make it improper to change the order of the two stars; and the expression 24 . 30 Leonis will be equally good with the

former. When a third star is concerned, such as 30 . 24 . 77 Leonis, the order of them ought not to be changed ; notwithstanding an equality between each member of the series has been strictly ascertained. The reason of this is obvious. For by the order in which they are placed, it appears that 30 has been deemed equal to 24, and 24 equal to 77; but it is not affirmed that 30 has been compared to 77. There will be a great probability that these two last stars do not differ sensibly or materially ; but since actual comparison is what we are to go by, the order in which the stars are given must remain.

When two stars are so nearly alike in their lustre that they may be almost called equal, and even now and then leave us doubtful to which to give the preference ; but when upon a longer inspection of them we always return to decide it in favour of the same, I separate the numbers that denote these stars by a comma. For instance, 41 , 94 Leonis. This expression can certainly not be changed to 94 , 41 Leonis; much less can the order of three such stars, as 20 , 40 , 39 Libræ, admit of a different arrangement. If ever the state of the heavens should be such as to require a different order in these numbers, we need not hesitate a moment to declare a change in the brightness of one or more of the stars that are contained in the series to have taken place.

When two stars differ but very little in brightness, but so that even a doubt cannot arise to which the preference ought to be given, I separate the numbers by which they are to be found in the catalogue by a short line. For instance, 17 – 70 Leonis; or 68 – 17 – 70 Leonis. If, in the former instance, a breaking in upon the order is to be looked upon as a proof that at least one of the stars has undergone a change in its

lustre, much more must that change be evident in this case, where the stars are separated by lines instead of commas.

When two stars differ so much in brightness that one or two other stars might be put between them, and still leave sufficient room for distinction, they become partly unfit for standards by which the lustre of other stars can be ascertained. But as proper intermediate stars sometimes cannot conveniently be had, we are often obliged to retain them; and in that case I distinguish them by a line and comma –, or by two lines, as 32 – – 41 Leonis. A difference which exceeds those that are expressed by the above marks, I denote by a broken line, thus – – – for instance, 16 – – – 29 Bootis. It would be very easy to give a more extensive signification to lines by adding cross marks to them, such as, + ++ +++ ++++ *&c.;* but in estimations that are to ascertain the brightness of stars, such expressions would rather throw us back again to look for imaginary differences, resembling those which have been rejected in the old system of magnitudes. On the contrary, the marks I have introduced admit of so precise a definition, that they cannot possibly be mistaken: a point denoting equality of lustre: a comma indicating the least perceptible difference: a short line to mark a decided but small superiority: a line and comma, or double line, to express a considerable and striking excess of brightness; and a broken line to mark any other superiority which is to be looked upon as of no use in estimations that are intended for the purpose of detecting changes.

In a foregoing paragraph we have said that this method of ascertaining the lustre of the stars was difficult and laborious. The difficulty consists in avoiding the various causes of error

that may bias our judgment in assigning the comparative brightness of the stars: the different altitudes at which we view them: the state and situation of the moon: the time of the night with regard to twilight: the uncertainty of flying clouds: the twinkling and continual change of star-light, to whatever cause it may be owing; I mean such changes as last but few moments, or at most but a few minutes: a return into the dark after having been writing by candle-light: the zodiacal light: aurora borealis: and dew or damp upon the glasses or specula when a telescope is used. All these, it must be confessed, are real difficulties, which it requires much attention and perseverance to get the better of.

That the method is also laborious may be easily conceived; for each star must at least have two other stars to be compared with, and even these will often be found not to be sufficient. To look out for such proper objects, and then to make the necessary comparisons for every star in the heavens, can be no easy task, especially when we remember the difficulties I have enumerated, to which every single estimation of comparative brightness is subject. This ought, however, not to discourage us from a work which has in view the investigation of a point of great importance; and as I have already made a considerable progress, I shall give the result of my labour in small catalogues, of which I have joined one at the end of this paper.

That these investigations are of the importance we have ascribed to them, will appear when we call to our remembrance the great number of alterations of stars that we are certain have happened within the last two centuries, and the much greater number that we have reason to suspect to have

taken place. If we consider how little attention has formerly been paid to this subject, and that most of the observations we have are of a very late date, it would perhaps not appear extraordinary were we to admit the number of alterations that have probably happened to different stars to be a hundred; this compared with the number of stars that have been examined, with a view to ascertain their changes, which we can hardly rate at three thousand, will give us a proportion of 1 to 30. But we are very certain that had a number of observers applied themselves to the same subject, which is of such a nature as to require the attentive scrutiny of many diligent persons at the same time, many more discoveries might probably have been made of changeable and periodical stars, whose variations are too small to strike a general observer. In the application we shall make of this subject however, a proportion, such as 1 to 30, or even 1 to 300, is sufficiently striking to draw our attention.

By observations such as this paper has been calculated to promote and facilitate, we are enabled to resolve a problem not only of great consequence, but in which we are all immediately concerned. Who, for instance, would not wish to know what degree of permanency we ought to ascribe to the lustre of our sun? Not only the stability of our climates, but the very existence of the whole animal and vegetable creation itself is involved in the question. Where can we hope to receive information upon this subject but from astronomical observations? If it be allowed to admit the similarity of stars with our sun as a point established, how necessary will it be to take notice of the fate of our neighbouring *suns*, in order to guess at that of our own! That *star* which among the multi-

tude we have dignified by the name of *sun*, to-morrow may slowly begin to undergo a gradual decay of brightness, like β Leonis, α Ceti, α Draconis, δ Ursæ majoris, and many other diminishing stars that will be mentioned in my catalogues. It may suddenly increase, like the wonderful star in the back of Cassiopea's chair, and the no less remarkable one in the foot of Serpentarius; or gradually come on like β Geminorum, β Ceti, ζ Sagittarii, and many other increasing stars, for which I also refer to my catalogues. And lastly, it may turn into a periodical one of 25 days duration, as Algol is one of 3 days, δ Cephei of 5, β Lyræ of 6, η Antinoi of 7 days, and as many others are of various periods.

Now, if by a proper attention to this subject, and by frequently comparing the real state of the heavens with such catalogues of brightness as mine, it should be found that all, or many of the stars which we now have reason to suspect to be changeable, are indeed subject to an alteration in their lustre, it will much lessen the confidence we have hitherto placed upon the permanency of the equal emission of light of our sun. Many phænomena in natural history seem to point out some past changes in our climates. Perhaps the easiest way of accounting for them may be to surmise that our sun has been formerly sometimes more and sometimes less bright than it is at present. At all events, it will be highly presumptuous to lay any great stress upon the stability of the present order of things; and many hitherto unaccountable varieties that happen in our seasons, such as a general severity or mildness of uncommon winters or burning summers, may possibly meet with an easy solution in the real inequality of the sun's rays.

A method of ascertaining the quantity or intenseness of solar light might be contrived by some photometer or instrument properly constructed, which ought probably to be placed upon some high and insulated mountain, where the influence of various causes that affect heat and cold, though not entirely removed, would be considerably lessened. Perhaps the thermometer alone might be sufficient. For though the lustre of the sun should be the chief object of this research, yet, as the effect of light in producing expansion in mercury seems to be intimately connected with the quantity of the incident solar rays, it may be admitted that all conclusions drawn from their action upon the thermometer will apply to the investigation of the brilliancy of the sun. And here the forms laid down by Mr. MAYER, in his little treatise *De Variationibus Thermometri accuratius definiendis*,* may be of considerable service to distinguish the regular causes of the change of the thermometer from the adventitious ones, among which I place the probable instability of the sun's lustre.

Introductory Remarks and Explanations of the Arrangement and Characters used in the following Catalogue.

This catalogue contains nine constellations, which are arranged in alphabetical order. I have called the present collection the first catalogue. The rest of the constellations, which are pretty far advanced, will be given in successive small catalogues as soon as time will permit to complete them.

Each page is divided into four columns, the first of which gives the number of the stars in the British catalogue of Mr. FLAMSTEED, as they stand arranged in the edition of 1725.

* *Tobiæ Mayeri opera inedita,* I.

The second column contains the letters which have been affixed to the stars.

The third column gives the magnitude assigned to the stars by FLAMSTEED in the British catalogue; and

The fourth contains my determination of the comparative brightness of each star, by a reference to proper standards.

All numbers used in the fourth column refer to the stars of the same constellation in which they occur, except when they are marked by the name of some other constellation; and in that case the alteration so introduced extends only to the single number which is marked, and which then refers to the constellation affixed to the number.

The numbers at the head of the notes, which will be found at the end of the catalogue, refer to the stars in the same constellation to which the notes belong. They contain particulars which it will be useful to know for those who wish to review that constellation.

To each star which I could not find in the heavens, and which, upon examining FLAMSTEED's observations, appeared never to have been seen by him, I have put down "Does not exist." To such as I could not find in the heavens, but which nevertheless appeared to have been observed by FLAMSTEED, I have put down "Lost." This is to be understood only to mean that the star in question was not to be seen when I looked for it, but that possibly at some future time, if it be a changeable or periodical star, it may come to be visible again.

The observations in the notes, distinguished by marks of quotation, " " are taken from my own journals.

Errors in FLAMSTEED's catalogue, or in the Atlas Cœlestis,

are pointed out at the end of the constellations in which they occur, that they may be corrected.

Simple Characters.

‘ The least perceptible difference less bright.

. Equality.

, The least perceptible difference more bright.

- A very small difference more bright.

-, A small difference more bright.

- - A considerable difference more bright.

- - - Any great difference more bright in general.

Compound Characters, expressing the wavering of Star-light.

‘. From the least perceptible difference less bright to equality.

; From equality to the least perceptible difference more bright.

-, From a very small difference more bright, to the least perceptible difference.

=, From -, to - *&c.*

‘., The wavering expressed by the passing of the light from a state of the least perceptible difference less bright to equality, and to the least perceptible difference more bright.

-., The wavering expressed by the changes from - to , and to . or from . to , and to -

General Characters.

= Perfect equality.

< Less, but undetermined.

> Larger, but undetermined.

All the observations contained in this catalogue have been made in very fine nights, where no suspicion of any whitish haziness or thin clouds can be admitted that might have deceived me.

The compound expressions which occur in the catalogue are not such as have arisen from want of attention, but on the contrary from more than common and long inspection.

Whoever looks a long while at two stars which are equal, A and B for instance, will find that he is not always pleased with the expression A . B, but would incline rather to put them down A , B when A seems to have the preference, or A ‘ B when the advantage is on the side of B. Since, therefore, these three expressions A ‘ B A . B A , B seem equally to belong to the stars, my compound character A ‘; B is in that instance an useful one, which includes them all. This may seem to be a doubtful expression, but it is in fact a very positive one, amounting to A = B. For had the stars not been perfectly equal, the same causes which bring on these little waverings in the appearance of stars, whatever they are, would have operated so as perhaps to produce the comparative wavering lustres expressed by A ; B and A ,̲ B or A ;̲ B which denotes the union of the three expressions A . B and A , B and A – B. But if this had been the case, we could certainly not admit A = B.

Sometimes, when I was not willing to put down these compound marks, I have cast my eyes upon the ground, and after a few moments lifted them quickly up to the stars AB, and instantly decided which of the expressions ought to be used: this being repeated perhaps a dozen or more times, I took that expression for the most proper one which would occur oftener than any other in these transitory glances.

All observations upon stars of any considerable magnitude have been made with the naked eye. I was unwilling to introduce the fallacies, or at least the difficulties that occur in the use of a telescope, owing to various causes that need not be mentioned, where I could possibly do without it. In numberless instances, however, the telescope has been recurred to, notwithstanding the stars under examination were not so small but that I saw them very well with the naked eye; for in very fine nights, and in high situations, all the stars of the sixth, and most of the seventh magnitude, are sufficiently visible. But when small stars were situated very near each other, or very near brighter ones, it became necessary to remove the objection arising from the light of one star either overpowering or blending with that of the other.

Care has been taken in observations with the naked eye not to fix upon a star as a standard which has another close to it; for the united light of the two stars would certainly cause deceptions. And stars that stand in this predicament of course have been referred to others with the assistance of a telescope.

The largest stars, and in general all such as had no convenient stars in the same constellation to be compared with them, have their lustre ascertained by such as I could find in the neighbouring part of the heavens.

Whenever I use the expression of magnitude, which though not of so nice and critical distinction as would be required for the purpose of my catalogue, is still a very useful one for general purposes, I have endeavoured to conform my mental standard to the notation of FLAMSTEED.

The most remarkable expressions of brightness which are contradictory to FLAMSTEED's magnitudes, are pointed out in

the notes annexed to the constellations. They are pretty numerous, and with many stars so considerable, that we have great reason to suspect changes in their lustre since FLAMSTEED'S time. It is to be noticed, that in collating my observations of brightness with FLAMSTEED'S magnitudes, I have not only taken those which are in the British catalogue, but also those that are to be found in the *Observationes Fixarum.* The very extraordinary disagreement between the former and the latter ought not to pass unnoticed. Were it not for what FLAMSTEED says in his *Prolegomena,* when he mentions the arrangement of the catalogue, "Undecima columna indicat "cujus magnitudinis stellam esse arbitratus sum quando eam "observatam habui," I should entirely reject the magnitudes of the catalogue as being without authority to support them. Nor can I conceive how such a remarkable disagreement could escape the author's notice, or remain unperceived by astronomers till this time, if the lustre of the stars in general had not been looked upon as a thing of no material consequence.

To shew what the difference is to which I allude, let us cast an eye upon the 9 constellations which are contained in the following catalogue of brightness.

In Aquarius there are 108 stars. To 49 of these no magnitudes can be found in FLAMSTEED'S observations; of 38 the magnitudes annexed to them agree with those of the catalogue; and of 21 they disagree with them.

In Aquila there are 71 stars. 39 are not observed; 16 agree; 16 disagree.

In Capricornus are 51 stars. 22 not observed; 17 agree; 12 disagree.

In Cygnus are 81 stars. 47 not observed; 21 agree; 13 disagree.

In Delphinus are 18 stars. 11 are not observed; 3 agree, and 4 disagree.

In Equuleus are 10 stars. 5 are not observed; 3 agree, and 2 disagree.

In Hercules are 113 stars. 10 are not observed; 54 agree, and 49 disagree.

In Pegasus are 89 stars. 22 are not observed; 37 agree, and 30 disagree.

In Sagitta are 18 stars. 3 are not observed; 13 agree, and 2 disagree.

To this may be added, that the disagreement in several stars is so considerable as to amount to two magnitudes; in many to one and an half, and in still more to one magnitude: not only with stars of a small size, but with some of the brightest in the constellation. I do not include α Cygni, which is marked 2m in the catalogue, and in the observation 7m, as that must certainly be a mistake; but cannot help regretting that a work to which every astronomer has been taught to look up as the first authority, should have been sent to the press with so many errors, that we hardly know how far to give our confidence to what is laid down in it.

I. *Catalogue of the comparative Brightness of the Stars.*

Lustre of the stars in Aquarius.			
1		6	70 Aquilæ , 1
2	ε	5:4	2 – 23 2 – – 6
3		5	3 – 5
4		6	5 , 4
5		6	3 – 5 , 4
6	μ	4.5	13 , 6 – 7 6 , 18 2 – – 6 – 7
7		6	6 – 7 – – 8 18 , 7
8		6.7	7 – – 8 , 9
9		6	8 , 9
10		6	11 , 10
11		6	12 , 11 , 10
12		6	12 , 11
13	ν	5	23 . 13 , 6
14		6	17 – 14
15		6	21 . 15 , 16
16		6	15 , 16 , 20
17		6	19 , 17 – 14
18		6	6 , 18 , 7
19		6	19 , 17
20		6	16 , 20
21		6	21 . 15
22	β	3	34 ; 22 , 49 Capricorni
23	ξ	6	2 – 23 . 13
24		6	26 – 24
25	d	6	25 . 27
26		6	27 , 26 – 24
27		6	25 . 27 , 26
28		6	32 , 28 28 , 30

Lustre of the stars in Aquarius.			
29		6	35, 29
30		6	46, 30 30 – 60 28, 30 30 – – 36
31	o	5	31 – 32
32		6	31 – 32, 28
33	ι	4	73, 33, 57 33 . 23 Capricorni
34	α	3	34 . 88 Pegasi 34 ⁝ 22
35		6.5	41, 35, 29
36		6	30 – – 36 37, 36
37		6	45 . 37 37, 36
38	e	6	38, 42
39		6	42 . 39, 45
40		7.8	45 . 40 40, 61
41		6	47, 41, 35 41 – 49
42		7	38, 42, 45 42 . 39 42, 53
43	θ	4	71, 43, 57
44		6	51, 44
45		6	42, 45 39, 45 . 40 45 . 37 45 . 50
46	ϱ	5.6	46, 30
47		5.6	47, 41 59, 47 . 68
48	γ	3	62 . 48 – 52
49		5	41 – 49
50		6	45 . 50, 56
51		6	63, 51, 44
52	π	5	48 – 52
53		6	42, 53
54		6	58 . 54
55	ζ	4	76 . 55, 62
56		6	50, 56, 61
57	σ	5	33, 57 43, 57
58		6	58 . 54 74 – 58 – 64
59	υ	5	66 ⁏ 59, 47
60		6	30 – 60
61		6	56, 61 40, 61

Lustre of the stars in Aquarius.			
62	η	4	55 , 62 . 48
63	κ	5	63 , 51
64		6	58 – 64 . 65
65		6	64 . 65 . 75
66	g^1	6	66 ‾, 59
67		6	67 . 78
68	g^2	6	47 . 68
69	τ^1	5	7 –, 69 69 ‾, 77
70		6	70 . 74
71	τ^2	6 . 5	73 , 71 , 43 71 –, 69
72		6	Does not exist.
73	λ	4	73 , 33 73 , 71 73 , 88
74		6	70 . 74 – 58
75		7	65 . 75
76	δ	3	76 . 55
77		6	69 ‾, 77
78		6	67 . 78 81 , 78 80 – 78 . 80
79		2 . 1	8 Pegasi , 79 , 44 Pegasi
80		7	80 – 78 . 80
81		7	82 , 81 , 78
82		7	82 , 81
83	b^1	6	83 , 92
84	b^2	7	87 – 84
85	b^3	6	92 – – 85 , 87
86	c^1	6	18 Piscis aust . 86 99 , 86 . 89
87	b^4	6	85 , 87 – 84
88	c^2	4	88 – 18 Piscis aust 73 , 88 ‾, 98
89	c^3	5 . 6	86 . 89 . 101 89 , 104
90	ϕ	5	93 . 90 – 92 91 , 90 . 93
91	ψ^1	5	91 , 90
92	χ	6	90 – 92 , 96 83 , 92 – – 85
93	ψ^2	5	93 . 90 90 . 93
94		6	94 , 95

Lustre of the stars in Aquarius.

95	ψ^3	5	94 , 95 , 97
96		6.7	92 , 96
97		6	95 , 97
98	b^1	5	88 -, 98 , 99
99	b^2	5	98 , 99 , 86
100	b^3	5	101 – 100
101	b^4	5	89 . 101 – 100
102	ω^1	5	105 , 102
103	A^1	5	104 . 103 . 106
104	A^2		89 , 104 . 103
105	ω^2	5	105 , 102
106	A^3	5	103 . 106 . 107
107	A^4	6	106 . 107 . 108
108	A^5	6	107 . 108

Lustre of the stars in Aquila.

1	*m*	4	16 , 1 , 12
2	*o*	5	2 . 3
3	*n*	5	2 . 3 , 9
4		5	9 , 4 , 5
5		6	4 , 5
6	*l*	4	6 . 12 6 . 63 Serpentis
7		6	8 , 7
8			8 , 7
9	*k*	5.4	3 , 9 12 , 9 , 14 9 , 4
10		6	11 , 10
11		6	18 -, 11 , 10
12	*i*	4	1 , 12 6 . 12 , 9 12 . 63 Serpentis
13	ε	3.4	13 – 18
14	*g*	6	9 , 14 , 15
15	*h*	6	14 , 15
16	λ	3	16 , 30 16 , 65 16 , 1
17	ζ	3	50 , 17 – 65

Lustre of the stars in Aquila.			
18		6	13 – 18 , 11 18 , 19 18 – 11
19		6	18 , 19 , 22 21 ; 19
20		5 . 6	26 . 20 37 . 20
21		5	21 ; 19 23 . 21
22		6	19 , 22
23		7	23 . 24 23 . 21 23 – 24
24		7	23 . 24 23 – 24
25	ω^1	6	28 . 25 25 –, 29
26	*f*	6	39 , 26 . 20
27	*d*	6	32 – 27 27 – 35
28	A	6	31 , 28 . 25
29	ω^2	7	25 –, 29
30	δ	3	65 , 30 , 55 16 , 30 65 ; 30
31	*b*	6	31 , 28
32	ν	5	41 – 32 – 27 38 , 32 , 44
33		6	Does not exist.
34		6	Does not exist.
35	*c*	6	27 – 35
36	*e*	6	36 , 42 36 – 45
37	*k*	6	37 . 20 39 , 37 , 51
38	μ	4	41 – 38 . 44 38 , 32 38 –, 59 67 ; 38
39	κ	3 . 4	39 , 26 39 , 37
40		6	Does not exist.
41	ι	3 . 4	55 – 41 – 38 41 – 32 41 –, 71 41 . 55
42		6	36 , 42 , 45 62 , 42 . 66 58 , 42
43		6	Does not exist.
44	σ	5	38 . 44 32 , 44 59 , 44 . 54
45		6	42 , 45 36 – 45
46		6	61 . 46 47 , 46 , 48
47	χ	6	47 . 52 47 , 46
48	ψ	6	46 , 48
49	υ	6	63 . 49
50	γ	3	53 – – – 50 , 17 50 , 34 Sagittarii

Lustre of the stars in Aquila.			
51		5	37 , 51 56 . 51
52	π	6	47 . 52 . 61
53	α	1 . 2	53 – – – 50 21 Scorpii – – 53 – – 50 Cygni
54	ο	6 . 5	44 . 54 54 , 63 59 – 54
55	η	3 . 4	30 , 55 . 60 55 – 41 41 , 55
56		5	57 – – 56 . 51
57		6	57 – – 56
58		6	62 , 58 , 42 58 – 66
59	ξ	5	38 ; 59 , 44 59 – 54
60	β	3 . 4	30 , 60 . 55
61	φ	6	52 . 61 , 46
62		6	62 ; 58 66 . 62 , 64
63	τ	6	54 , 63 . 49
64		6	66 , 64 62 , 64
65	θ	3	17 – 65 , 30 16 , 65 65 ; 30
66		5 . 6	42 . 66 66 , 64 66 . 62 58 – 66
67	ρ	5	67 ; 38
68		6	69 , 68 69 – 68
69		5	70 , 69 , 68 70 , 69 – 68
70		5	71 – 70 , 69 70 , 1 Aquarii
71		4	71 – 70 41 ; 71
Lustre of the stars in Capricornus.			
1		6	2 , 1 . 3
2	ξ	6	2 , 1
3		6	1 . 3
4		6	7 , 4
5	α^1	4	6 – 5 –, 8
6	α^2	3	6 – 5
7	σ	obs.	10 , 7 , 12 15 . 7 , 4
8	ν	6	5 –, 8 , 11
9	β	3	49 – 9 – – 40
10	π	obs.	11 , 10 , 12 10 , 7 10 , 15
11	ρ	6	8 , 11 , 10

Lustre of the stars in Capricornus.			
12	ο	obs.	10 , 12 7 , 12 15 , 12
13		6	15 – 13 14 =¯, 13
14	τ	6	14 , 15 14 =¯, 13
15	ν	6	10 , 15 , 12 15 . 7 14 . 15 – 13
16	ψ	5	16 , 18
17		6	24 ¯, 17
18	ω	6	16 , 18 ; 24
19		6.7	23 – – 19 , 21
20		6.7	20 . 21 22 – 20 . 25
21		6	19 , 21 20 . 21
22	η	5	23 –, 22 – 20 22 – 24 .
23	θ	5	23 – – 19 23 –, 22 33 Aquarii . 23
24	A	6	18 ; 24 ¯, 17 22 – 24 , 25
25	χ¹	6	20 . 25 –, 26 24 , 25
26	χ²	6	25 –, 26 . 27
27	χ³	6	26 . 27
28	φ	6	36 – 28 , 33
29		6	32 – 29 – 30
30		6	32 – – 30 – 31 29 – 30
31		7	30 – 31
32	ι	5	32 – – 30 32 – 29
33		6	36 , 33 , 35 41 , 33 28 , 33 , 35 34 Sagittarii . 33
34	ζ	5	34 , 39 34 – – 36
35		6	33 , 35
36	b	6	39 – 36 , 43 36 , 33 34 – – 36 – 28
37		6	43 –, 37 , 38
38		6	37 , 38
39	ε	4	34 , 39 – 36
40	γ	4	9 – – 40
41		6	41 , 33
42	d¹	6	42 – 44 42 –, 44 42 ¯, 51 42 . 48 51 ; 42 ¯, 48

Lustre of the stars in Capricornus.

43	κ	5	36 , 43 -, 37
44	d^2	6	42 - 44 . 45 42 -, 44 , 45 44 ; 45
45		6	44 ; 45 44 , 45 44 . 45
46	c^1	6	46 -, 47
47	c^2	6	46 -, 47
48	λ	5	42 . 48 , 51 48 - - 50 42 -, 48
49	δ	3	49 - 9 22 Aquarii , 49
50		6	48 - - 50
51	μ	5	42 ; 51 48 , 51 51 ; 42

Lustre of the stars in Cygnus.

1	κ	4	1 . 10 1 -, 10 10 -; 1 10 . 1 1 - 32
2		5	12 ; 2 -, 9
3		6	8 Vulpeculæ . 3 3 Vulpeculæ - - 3
4		6	14 . 4 12 , 4 15 , 4 - 11 8 - 4 , 11
5			Does not exist. A small star near the place 9 - - 5
6	β	3.4	53 , 6 , 18 18 ; 6 53 -, 6 6 -. 14 Lyræ
7		6	16 , 7
8		6	17 . 8 8 ; 15 8 , 12 8 , 14 8 - 4 21 - 8 - 17
9			12 -, 9 12 - 9
10	ι	6	1 . 10 10 . 1 10 -; 1 65 -; 10
11		6	4 , 11
12	φ	5	12 ; 2 8 , 12 , 4 12 - 9
13	θ	4	32 , 13 33 , 13 , 20 33 , 13 , 23
14		6	8 , 14 . 4
15		6	15 , 22 8 ; 15 . 4 15 , 25
16	c^1	6	16 , 7
17	χ	5	8 - 17 21 - - 17 . 8 21 =, 17
18	δ	3.4	6 , 18 , 64 53 , 18 53 -, 18 - 64 53 , 18 , 64 8 ; 6 18 -, 64
19		6	22 , 19 25 , 19

Lustre of the stars in Cygnus.			
20		5.6	24 . 20 . 26 13 , 20
21	η	6	21 . 41 21 - - 17 21 = ; 17 21 - 8
22		6	25 . 22 22 , 25 15 , 22 , 19
23		6	13 , 23
24	ψ	5	24 . 20
25		6	15 , 25 . 22 22 , 25 , 19
26	c^2	6	20 . 26
27	b^1	5	36 - 27
28	b^2	5	34 ; 28 , 36 28 . 35 28 . 34
29	b^3	6	29 . 34 34 . 29
30	o^1	4	32 - 30 32 -, 30
31	o^2	5	31 , 32
32		5.6	32 . 33 32 , 13 31 , 32 - 30 1 - 32 31 - 32 -, 30
33			32 . 33 , 13
34		6	29 . 34 ; 28 28 . 34 34 . 29 34 , 36 34 , 40.
35	*m*	6	28 35 39 - 35
36		6	28 , 36 - 27 34 . 36
37	γ	3	37 , 53 37 - 53 37 -, 53 37 ; 8 Peg. 37 - 8 Peg. 37 ; 8 Pegasi 5 Cephei -, 37
38			Does not exist, or is lost.
39	*h*	6	41 - 39 - 35 47 - 39 - 35
40		6	34 , 40 40 , 42
41	*i*	4	41 , 52 21 . 41 41 - 39
42		6	40 , 42 , 44
43	w^1	5	46 , 43
44		6	42 , 44
45	w^2	5	45 , 46
46	w^3	5	45 , 46 , 43
47	*l*	6	47 - 39
48		6	49 - 48 ; 48 49 -, 48 ; 48 49 - - 48 , 48
49		6	49 - 48 ; 48 49 -, 48 ; 48 49 - - 48 , 48

Lustre of the stars in Cygnus.

50	α	2	50 - - - 37 53 Aquilæ ⸪ 50 53 Aquilæ - - 50
51		6	56 , 51
52	k	6	41 , 52
53	ε	3	37 , 53 , 6 37 - 53 , 18 37 - 53 -, 18 37 -, 53 ⸪ 6
54	λ	4	67 . 54
55		6	56 . 55 - 59 55 . 63 63 . 55 , 59
56		6	57 , 56 . 55 57 , 56 , 51
57		6	57 , 56
58	ν	4	62 . 58 . 67
59	f^1	5.6	55 - 59 , 60 63 , 59 55 , 59 59 , 68
60		6	59 , 60
61		6	70 . 61 , 69
62	ξ	4	65 , 62 , 58
63	f^2	6	55 . 63 , 59 63 . 55 68 , 63
64	ζ	3	18 , 64 18 ⸪ 64
65	τ	4	65 , 62 65 - 66 65 ⸪ 10
66	υ	5	66 , 78 65 - 66 , 67
67	σ	4	58 . 67 . 54 67 ; 78 66 , 67
68	A	6	59 , 68 , 63
69		6	69 . 70 70 - 69 79 , 69 61 , 69
70		6	69 . 70 72 , 70 - 69 70 . 61
71	g	6	80 . 71
72		6	72 , 70 74 , 72
73	ϱ	4	73 - 81
74		6	74 , 72 74 - 77 75 . 74 , 72
75		6	75 . 74
76		6	77 , 76
77		6	74 - 77 , 76
78		3.4	66 , 78 67 ; 78 - 14 Pegasi
79		6	79 , 69
80	π^1	4	81 - 80 . 71
81	π^2	5	73 - 81 - 80

Lustre of the stars in Delphinus.

1		6	8 , 1 , 10
2	ε	3	12 , 2 , 11
3	η	6	5 . 3 , 8
4	ζ	5	. 11 – 4 . 7
5	ι	6	7 , 5 . 3
6	β	3	6 , 9
7	κ	6	4 . 7 , 5
8	θ	6	3 , 8 , 1
9	α	3	6 , 9 . 12
10		6	1 , 10
11	δ	3 . 4	2 , 11 – 4
12	γ	3	9 . 12 , 2
13		5	13 – 14 1 Equulei – 13
14		6	13 – 14
15		6	18 , 15
16		6	17 , 16 , 18
17		6	17 , 16
18		6	16 , 18 , 15

Lustre of the stars in Equuleus.

1		5	10 , 1 – 13 Delphini
2		6	4 , 2
3		6	3 . 4
4		6	3 . 4 , 2
5	γ	4	8 – – 5 . 7 5 – 6
6		6	5 – 6
7	δ	4	5 . 7 , 10
8	α	4	8 – – 5
9		6	10 – 9
10	β	4	7 , 10 , 1 10 – 9

Lustre of the stars in Hercules.

1	χ	6	1 –, 4 1 , 30 30 . 1
2		6	4 , 2 –, 14
3		5	3 – 9
4		6	1 –, 4 , 2
5	r	5	7 – – 5 – 16
6	υ	6	35 , 6 – – 14 35 – 6 , 30 6 , 52 11 , 6 11 ⁻, 6
7	κ	5	7 – – 5
8	q	5.6	16 , 8
9		6	3 – 9 , 43 Serpentis
10		5	10 – 17 10 – 19
11	φ	6	11 . 35 – 6 11 – – – 14 35 . 11 , 6 11 ⁻, 6 22 , 11
12		6	21 – – 12 – 15
13		5.6	15 . 13
14		7	6 – – 14 11 – – – 14 2 –, 14
15		6	12 – 15 . 13
16		6	5 – 16 , 8
17		6	10 – 17 , 18 19 , 17 , 18
18		7	17 , 18
19		6	10 – 19 , 17
20	γ	3	64 –, 20 22 , 20 . 58 22 – 20
21	ο	6	21 – – 12 28 – 21
22	τ	4	44 , 22 , 20 22 , 85 44 – 22 – 20 22 , 11
23		5	20 Coronæ ∸ 23 23 – 26
24	ω	6	24 ⁻, 29 24 , 60 24 – 29 24 –, 29
25		5	30 – 25 30 –, 25 59 , 25
26		7.6	23 – 26 26 – 31
27	β	3	27 –, 64 40 – 27 27 ; 40 27 – – 64 27 ; 40 27 , 40
28		6	28 – 21
29	b	4	24 ⁻, 29 60 ; 29 24 – 29 24 –, 29

Lustre of the stars in Hercules.			
30	*g*	5	1 , 30 30 – 25 6 , 30 52 , 30 42 , 30 , 34 30 . 1 30 –, 25
31		7	26 – 31
32		6	48 , 32
33		6	41 , 33
34		6	30 , 34
35	σ	4	58 , 35 85 , 35 11 , 35 , 6 35 . 11
36	*m*	6	37 – – 36 38 – 36
37	*m*	6	37 – – 36 37 – 38 37 , 45
38		6	37 – 38 41 , 38 – 36
39		5	39 – 50
40	ξ	3	40 – 27 27 ; 40 27 ; 40 27 , 40 27 ; 40 37 Serpentis – 40 –, 64
41		6	45 – – 41 47 , 41 , 38 41 , 33
42		5	52 , 42 , 30
43	*i*	5 . 6	45 , 43 , 47
44	η	3	65 , 44 , 22 44 , 86 44 – 22 65 – 44 – 58
45	*e*	5	45 – – 41 37 , 45 , 43
46		7	48 – 46
47	*k*	5	43 , 47 , 41
48		6	50 – 48 , 32 48 – 46
49		6	60 – – 49
50		5	39 – 50 – 48 53 – – 50
51		5	53 . 51 – – 56
52		5 . 6	6 , 52 , 30 52 , 42
53		5	53 – – 50 53 . 51
54		5	One of these two does not exist.
55		5	60 , 54 or 55
56		6	51 – – 56 . 57
57		6	56 . 57
58	ε	3	20 . 58 , 35 58 , 91 44 – 58 103 – 58 – 76
59	*d*	6	68 , 59 , 61 59 , 25

Lustre of the stars in Hercules.			
60		6	24, 6 ; 29 60 - - 49 60, 66 60, 54 or 55
61	c	6	59, 61
62		6	71 - - 62
63		6	72 . 63 63, 78
64	α	3	64 – 27 Ophiu 27 Ophiu – 64 64, 65 65 –, 64 27 –, 64 –, 20 64 ⁏ 67
65	δ	4	65 – 64 67, 65 64, 65 – 44
66	ω	6	60, 66, 37 Ophiuchi
67	π	3.4	67, 65 67, 27 Ophiuchi
68	u	5	70 . 68, 72 68, 90 68, 59 69 – 68
69	e	4.5	76 – 69 69 – 68 94, 69 – 99
70		4	70 - - 62 70 . 68 70 – 73
71		5	Does not exist.
72	w	6	68, 72 . 63 90, 72
73		6	70 – 73
74		6	77, 74 . 88
75	ρ	4	75, 76 91, 75 75, 94
76	λ	4.5	75, 76 – 69 58 – 76
77	κ	6	82 . 77, 74
78		6.7	63,78 78, 93
79		6	79 – 83 89, 79
80		4	Does not exist.
81		4	Does not exist.
82	y	6	82 . 77
83		7	79 – 83 . 84
84		7	83 . 84
85	ι	4	22, 85, 35 14 Lyræ, 85
86	μ	4	44, 86, 92 86 – 92
87		6	87 – 89
88	Z	6	74 . 88
89		6	87 – 89, 79
90	f	6	68, 90, 72

Lustre of the stars in Hercules.			
91	θ	4	58 , 91 , 75 92 . 91
92	ξ	4	86 , 92 . 103 103 . 92 . 91 92 . 109 92 , 109 86 – 92 ; 103
93		5	78 , 93
94	ν	5	75 , 94 , 69 103 – 94 . 100 109 , 94
95		4	102 , 95 , 101 95 . 98
96		5	101 , 96 . 97 101 . 96 – 98
97		5.6	96 . 97 98 , 97
98		5	95 . 98 96 – 98 , 97
99	b	5	69 – 99 – 104 106 . 99 100 , 99 107 , 99 . 108
100	i	6	94 . 100 , 99
101		5	95 , 101 . 96 95 , 101 . 96
102		4.5	102 , 95
103	o	4	109 . 103 92 . 103 – 94 92 ; 103 – 58
104	A	4.5	99 – 104
105		5	106 – 105
106		5.6	106 . 99
107	t	6	107 , 99
108		6	99 . 108
109		4	92 . 109 . 103 92 , 109 , 94 109 , 111
110		4.5	111 , 110 , 113
111		4	109 , 111 , 110
112		5	113 , 112
113		5	110 , 113 , 112

Lustre of the stars in Pegasus.			
1	e	4	24 – 1 , 10 1 . 9
2	f	4.5	10 – 2 , 16 9 – 2 . 13 2 – 13
3		6	3 , 4
4		6	3 , 4 . 7
5		6.7	13 – 5 12 , 5 13 , 5
6		6	16 , 6 – 11

Lustre of the stars in Pegasus.

7		6	4 · 7
8	ε	3	8 – 54 37 Cygni – 8 21 Andr ⌐ 8 , 54 8 , 53 8 ; 16 Ceti 8 , 24 Pisc austr
9	g	4 · 5	1 . 9 – 2
10	κ	4	1 , 10 – 2
11		6	6 – 11
12		6	16 , 12 , 5
13		6	2 . 13 – 5 13 , 17 13 , 5 2 – 13
14		6	16 , 14 , 15 78 Cygni – 14
15		6	14 , 15
16		6	2 , 16 16 , 6 16 , 12 16 , 4
17		6	13 , 17 . 21 17 , 21
18		5	22 – 18 . 19 19 , 18
19		6	18 . 19 19 , 18 22 – 19
20		6	21 , 20
21		5	17 . 21 , 20 17 , 21 , 20
22	ν	5	46 , 22 – 18 22 , 35 22 – 19 31 , 22
23		6	38 , 23
24	ι	4	24 – 1
25		6 . 7	25 – 28
26	θ	4	42 , 26 – 46
27		5	29 – 27
28		6 . 7	25 – 28
29	π	4 · 5	29 – 27
30		6	35 , 30 31 , 30 – 36 31 =' 30
31		4 · 5	31 , 30 31 =' 30 31 , 22 50 · 31 , 49
32		6	43 , 32 , 38 43 · 32
33		6 . 7	39 · 33
34		6	37 , 34
35		6	22 , 35 , 37 35 , 30
36		6 . 7	30 – 36
37		6	35 , 37 , 34
38		6	32 , 38 , 23

Lustre of the stars in Pegasus.			
39		6.7	41 , 39 , 45 39 · 33
40		6	40 , 41
41		6.7	40 , 41 , 39
42	ζ	3	48 , 42 , 26
43	ο	5	47 – 43 , 32 56 , 43 , 32
44	η	3	53 ,̄ 44 ,̄ 48 53 , 44 , 88 54 , 44 ; 88 16 Ceti ; 44
45		6.7	39 , 45
46	ξ	5	26 – 46 , 22 46 – 50
47	λ	4	48 , 47 – 43 47 , 68 47 –, 68 48 , 47 – 68
48	μ	4	44 ,̄ 48 , 47 88 – – 48 , 42
49	σ	6	50 , 49 , 52 31 , 49
50	ρ	6	46 – 50 , 49 50 · 31
51		6	56 – 51 , 60
52		6	49 , 52
53	β	2	54 , 53 ,̄ 44 54 · 53 , 44 8 , 53 · 54
54	α	2	8 – 54 54 , 53 8 , 54 · 53 53 · 54 , 44
55	*l*	5	55 , 59
56		5.6	62 – 56 – 51 56 , 64 56 . 72 56 , 71 62 – 56 56 · 43 78 ; 56
57	*m*	6	58 , 57
58	*n*	6	59 , 58 , 57
59	*p*	6.5	55 , 59 , 58
60		6	51 , 60 , 61
61		6	60 , 61
62	τ	6	68 , 62 – 56 62 , 78 84 , 62 62 – 56
63		6	67 – 63 · 73
64		6	56 , 64 · 67
65		6	69 , 65
66		6	70 – 66 , 86
67		6.7	64 · 67 – 63
68	υ	6	47 , 68 , 62 68 , 70 47 –, 68 47 – 68
69		6	71 , 69 , 65

Lustre of the stars in Pegasus.			
70	q	5.6	68, 70 - 66 70, 82
71	y	6	56, 71, 69 71, 85
72		6	56 . 72
73		6	63 . 73
74		7	75, 74, 76 75 -, 74, 76
75	S	6	81, 75, 74 75 -, 74
76		6	74, 76
77		6	82, 77 - 80
78		5.6	62, 78 - 79 62, 78; 56
79		6	78 - 79
80		6	77 - 80
81	φ	6	89, 81, 75
82		6	70, 82, 77
83	r	6	87, 83 23 Piscium - 83
84	ψ	6	84 . 62 84, 89
85		6	71, 85 87 . 85
86		5.6	66, 86
87	u	6	87 . 85 87, 83
88	γ	2	44, 88 44; 88 - - 48 34 Aquarii . 88 88; 6 Arietis
89	χ	6	84, 89, 81
Lustre of the stars in Sagitta.			
1		6	1 Vulpeculæ - 1 2 Vulpeculæ, 1
2		6	2, 3
3		6	2, 3
4	ε	5	6 -, 4
5	α	4	5 . 6 7 -, 5 . 6 5 - 9 Vulpeculæ
6	β	4	5 . 6 -, 4 5 . 6, 8
7	δ	4.5	7 -, 5 12 - 7
8	ζ	6	6, 8 - 9 8, 16 9 Vulpec. 8 12 Vulp. 8
9		6	8 - 9

Lustre of the stars in Sagitta.			
10		6	11, 10 . 15
11		6	11, 10
12	γ	4	12 – 7
13	χ	6	15 . 13
14	y	6	14, 15
15	Z	6	14, 15 . 13 10 . 15
16	η	6	8 . 16 –, 17
17	θ	6	16 –, 17 – 18
18		6	17 – 18

Notes to Aquarius.

2 "August 2, 1788. 20-feet reflector 2 (ε) 4.3m FL. 5.4m." The difference amounts to one whole magnitude. In FLAMSTEED's observations no magnitude is mentioned.

6 Is less than 13, and very little brighter than 18. The former is contrary to the catalogue, and the latter inconsistent with the magnitude assigned to 18. None of these stars have any magnitude in FLAMSTEED's observations.

8 Is larger than 9, contrary to the catalogue. In the observations 8 is 6m, but 9 has no magnitude.

13 Is less than 23, and is larger than 6; both are contrary to the catalogue. There are no magnitudes of either of these stars in FLAMSTEED's observations.

23 Is larger than 13, contrary to the catalogue, and from the expression 2 – 23 (see 2) it appears that 23 is undervalued by FLAMSTEED, or has changed its lustre. FLAMSTEED's observations give no magnitude of 23.

34 Is equal to 88 Pegasi, which the catalogue has 2m. See 88 Pegasi.

35 Is less than 41, contrary to the catalogue, "Oct. 13, 1786, 5.6m." There is no magnitude of 35 in FLAMSTEED's observations.

40 Is larger than 61, contrary to the catalogue. This is a considerable deviation, amounting to $1\frac{1}{2}$m. In the observations 40 is 7m, 61 6m.

41 Is larger than 49 and 35, contrary to the catalogue. It is also contrary to the observations. "Oct. 13, 1786, 41 6.5m."

42 Is larger than 45, 39 and 53, contrary to the catalogue. The observations give 6m to 53.

43 Is less than 71. See 71. There is no magnitude to either of these stars in FLAMSTEED's observations.

48 Is less than 62, contrary to the catalogue; and is now probably less bright than it was formerly. 48 being but little brighter than 52 confirms the same. There is no observation of 48, but 62 is 5m.

59 Is less bright than 66, contrary to the catalogue. The observations give 59 6m.

71 Is brighter than 69, contrary to the catalogue. These stars are so near each other that a change must be evident, unless FLAMSTEED should have made a mistake in writing down their magnitudes. 71, 43 confirms the same conclusion. In the observations neither 69 nor 71 has a magnitude assigned.

72. There is no observation of FLAMSTEED upon this star.

78 Is less than 81, contrary to the catalogue; and in the observations it is 5m.

79 Is less than 8 Pegasi, contrary to the catalogue. The difference between 2.1m and 3m would be striking, if the lowness of the situation of 79 did not render its real magnitude very uncertain. In my estimation no allowance is made for that low situation. In the observations there is no magnitude to either of these stars.

80 There are two stars, the smallest of which agrees best with the place of 80 in Atlas, but neither of them seems to accord completely in relative situation with 81 and 82. In one of my sweeps a star, supposed to be 80, was taken with the following deduction; "Sept. 12, 1785. This star requires a correction of — 1′ 13″ in time of RA, and — 6′ in PD."

84 Is larger than 87, contrary to the catalogue. In the observations they are both 8m.

85 Is much less than 92, which does not agree with the magnitudes of the catalogue. In the observations it is marked 8m.

86 Is larger than 89, contrary to the catalogue. There is no magnitude to either of these stars in the observations.

88. "Oct. 13, 1786, 20-feet reflector, 4.3m." Flamsteed's observations give it 4m.

89 Is larger than 101 and 104, contrary to the catalogue. In the observations 104 is 6m.

94 Is larger than 95, contrary to the catalogue. In the observations they are both 5m.

96. "Sept. 12, 1785, 6m." In Flamsteed's observations it is also marked 6m.

Catalogue.

80 Requires — 18′ in RA, and — 6′ in PD.

The PD of 96 requires — 8′.

Atlas.

The RA of 30 requires + 1°.

72 must be out.

80 requires — 18′ in RA, and — 6′ in PD.

84 requires — 10′ in RA, and — 12′ in PD.

85 requires — 28′ in RA.

96 requires — 8′ in PD.

Notes to Aquila.

"July 23, 1781. Order of magnitude $\alpha\,\gamma\,\zeta\,\theta\,\delta\,\eta\,\beta\,\varepsilon$."

3 Is larger than 9, contrary to the catalogue. In FLAMSTEED's observations both are marked 6m.

6 and 12 are both larger than 63 Serpentis; but that star is placed among the changeable ones. See Phil. Trans. Vol. LXXVI. page 211. FLAMSTEED's observations give 3m. to 6.

13. "Sept. 3, 1784; 20-feet reflector, 13 (ε) 5.6 FL. 3.4m, but strong twilight." It is not much larger than either 11, 18, or 19, so that we may be pretty certain it must have lost some of its lustre since the time of FLAMSTEED. In his observations it is marked 4m.

20 Is less than 26 and 37, contrary to the catalogue. In the observations 20 is marked 5m.

21 Is less than 23, contrary to the catalogue. But in the observations 23 is marked 5m. The error therefore is probably in the catalogue.

24 The star I estimate is one of two small ones.

33 There is no observation of this star in FLAMSTEED's work.

34 This star was never observed by FLAMSTEED.

37 Is larger than 20 and 51, contrary to the catalogue. The latter is marked 6m in the observations.

38 Is less than 67, contrary to the catalogue.

39 Is not much larger than 26 and 37, which will not agree with 3.4m of the catalogue; but in FLAMSTEED's observations it is put down only 5m.

40 There is no observation of this star in FLAMSTEED's work.

43 There is no observation of this star in FLAMSTEED's work.

55 This star is periodical. The time of its period as given by Mr. PIGOTT, the discoverer, is $7^{d}\ 4^{h}\ 15'$. See Phil. Trans. Vol. LXXV. page 127.

56 Is much less than 57, contrary to the catalogue; but in the observations 56 is only marked 6m.

66 Is less than 42 and 58, contrary to the catalogue, and it is moreover marked 5m in the observations.

Atlas.

The RA of 23 requires — 20′.

29 Should be 30′ from 25, and 48′ from 28, on the south following side of the two stars.

The stars 33, 34, 40, and 43 should be out.

Notes to Capricornus.

"Sept. 27, 1782. Order of magnitude $\delta \beta \alpha \gamma$."

6 in 1780 was less than $\beta \gamma \delta \zeta \eta \theta \iota$.

13 Is not equal to 14 as the catalogue gives it. In FLAMSTEED's observations 14 is without magnitude assigned to it.

19 and 20 are larger than 21, contrary to the catalogue. In the observations 19 is marked 6m.

34 Is larger than 39, contrary to the catalogue. Neither of them has any magnitude given with them in FLAMSTEED's observations.

36 Is larger than 43, contrary to the catalogue. It has either been under-rated, or gained additional lustre since FLAMSTEED's time. Neither of the stars has any magnitude in his observations.

42 Is larger than 48, contrary to the catalogue. The latter has no magnitude in the observations, and the former is marked once of the 5th and once of the 6th, which may be put down 5 . 6m.

Catalogue.

The letter *e* should be added to 26. FLAMSTEED has used it in his observations, page 75.

Atlas.

31 requires about + 22′ in RA.

Notes to Cygnus.

"May 12, 1783. Order of magnitude $\alpha \quad \gamma \varepsilon \beta \delta \quad \zeta \quad \theta$."

5. There is no observation of this star by FLAMSTEED.

Page 67 a star was observed without time, but by page 71 and 122 it appears that the defective observation belongs to 2. There is a star 8 or 9m, about 50′ from 2, 1° 20′ from 9, and and 1° 30′ from 6; and calling that star 5, its brightness may be expressed by 9 - - 5.

10. "Sept. 15, 1783. 10 is at least 4m. It is larger than 13." If the authority of the catalogue be good, there can be no doubt of a change since FLAMSTEED's time; but in his observations there is no magnitude to this star.

12 Is less than 8, contrary to the catalogue. "Sept. 7, 1784, 12 (φ) 6m." In FLAMSTEED's observations there is no magnitude to either of the stars.

13 Is less than 32, contrary to the catalogue. But in the observations 13 has no magnitude.

17 Is less than 21, contrary to the catalogue. But in the observations neither of the stars have any magnitude.

18 Is larger than 64, contrary to the catalogue. But in FLAMSTEED's observations neither of the stars have any magnitude.

21 Is larger than 41, contrary to the catalogue. But 21 is without magnitude in the observations.

23. The expression 13, 23 does not agree with the catalogue. But 13 has no magnitude in the observations.

27 Is less than 36, contrary to the catalogue; but in FLAMSTEED's observations are no magnitudes of these stars.

30 Is less than 32, contrary to the catalogue; it is also contrary to the observations, which give 30 5m and 32 6.5m.

31 Is larger than 30, contrary to the catalogue. It is also contrary to the magnitudes given in the observations "Sept.

27, 1788; 20-feet reflector 30 (1st. o) 5m, Fl. 4m. 31 (2d. o) 4m, Fl. 5m."

34 Is a changeable star. Its period perhaps is about 18 years. See Phil. Trans. Vol. LXXVI. page 201.

38. In Flamsteed's observations, page 75, a star was taken without RA, marked "quæ præcedit ω." The time of this observation however is sufficiently determined by the 37 before it, and 45 and 46 just after; but there is no star visible in the space pointed out that can possibly be taken for 38. "Sept. 22, 1783, 38 lost. There is not a star of the 7, 8, 9, or 10th magnitude near the place." It therefore does not exist, or rather is lost.

41 Is less than 21, and not much larger than 52, which is contrary to the catalogue. "It is less than 4m." In Flamsteed's observations it is marked 4m, but 21 and 52 are without magnitudes.

48. "Sept. 5, 1784; I could not see this star, but instead of it found in the neighbourhood 2 stars of the 7th magnitude within 5 or 6′ of each other." "Nov. 15, 1795. If one of the stars be 48, its magnitude is over-rated, and must be about 7.8m. That of the two which is nearest to 49 is the largest."

59 Is less than 55, 56 and 63, contrary to the catalogue. It is also contrary to the magnitudes given in the observations: 63 is without magnitude.

66 Is larger than 78, contrary to the catalogue. See 78. "Sept. 13, 1784; 20-feet reflector 66 (*v*) 4m, Fl. 5m. It is larger than 54 (λ), contrary to the catalogue." Neither 66 nor 54 have any magnitude in Flamsteed's observations.

71 Is equal to 80, contrary to the catalogue. Neither of them has any magnitude given with them in the observations.

78 Is less than 66 and 67. "It is much too small for 3.4m." In FLAMSTEED'S observations I find it marked 6m.

81 Is larger than 80, contrary to the catalogue. But in the observations there is no magnitude to either of the stars. "Sept. 27, 1788; 20-feet reflector, 81 ($2^d \pi$) 3.4m, FL. 5m." It is either undervalued in the catalogue, or grown brighter since FLAMSTEED'S time.

P. The changeable star in the neck of the swan. Its period is 396 days 21 hours. See Phil. Trans. Vol. LXXVI. page 200. Its present lustre is 17 -- P.

Atlas.

14 requires + 1° in PD.

5 should be out.

Notes to Delphinus.

"Aug. 14, 1781. Order of magnitude $\beta\alpha \begin{matrix} \\ \gamma \end{matrix} \delta \begin{matrix} \\ \varepsilon \end{matrix}$"

9. In the catalogue it is marked 3m; in FLAMSTEED'S observations it is 6m. My expression 6, 9.12 agrees best with the catalogue.

13 "Aug. 7, 1785, 6m." In FLAMSTEED'S observations it is also marked 6m.

Atlas.

12 Should be placed about 52′ more south on plate 23. It is right on plate 25.

Notes to Equuleus.

"Aug. 13, 1781. Order of magnitude $\alpha \cdot \begin{smallmatrix} \gamma \\ \delta \end{smallmatrix} \beta$"

6. In the catalogue we have 4m; in the observations FLAMSTEED has once marked it 6m, and once 8m. If there

be any accuracy in these various notations, the star must certainly be changeable.

Notes to Hercules.

"May 12, 1783. Order of magnitude $\beta\,\zeta\,\alpha \quad \delta\,\eta\,\pi \quad \gamma\,\varepsilon\,\mu$."

5 Is much less than 7. My edition has this star 3m; that of 1712 has it 5m. FLAMSTEED's observations give 6m, which agrees best with 7 - - 5 as I give its present lustre.

8 Is less than 16, contrary to the catalogue. But in the observations this star has been marked twice 7m, twice 6m, and once 5m.

11 Is larger than 6 and 35, contrary to the catalogue. But in the observations we have this star given twice 4m, and once 3m. It is therefore undervalued in the catalogue, or is subject to changes in its lustre.

13 Is less than 15, contrary to the catalogue. The observations give them both 6m. "May 25, 1795, 13 and 15 are both smaller than FL. gives them, and are about 7.8m."

20 Is less than 22, contrary to the catalogue. In the observations they are both 4m.

22 "May 12, 1787. 22 (τ) 3m, FL. 4m."

23 Is not much larger than 26, contrary to the catalogue. The observations give 23 6m, and 26 7m.

25. "May 16, 1787. 25 7.6m, FL. 5m." In the observations this star is also 5m.

27. By my observations the light of this star seems to be subject to change. FLAMSTEED's observations give it twice 3m, and once 2m.

29 Is less than 24 and 60, contrary to the catalogue. In the observations 24 is marked 6 and 5m; 60 is given 5m, 6m, and

4m; and 29 is put down five times 5m, once 6m, and once 4m. Very possibly this star may be changeable.

30 Is larger than 1 and 52, contrary to the catalogue. In the observations 30 is given three times 5m; 1 twice 5m, and twice 4m; and 52 twice 5m.

37 Is larger than 45, contrary to the catalogue. But in the observations we have 37 twice 6m, once 5m; and 45 twice 6m, and twice 5m.

40. From the expressions I have given of the brightness of this star, we have great reason to suppose it to be changeable. FLAMSTEED'S observations give it 3m.

47 Is less than 43, contrary to the catalogue. The observations, however, give 47 three times 6m, and only once 5m.

52 Is larger than 42, contrary to the catalogue. In the observations both are twice marked 5m.

54 or 55. FLAMSTEED observed but one of these stars, once 4m, once 5m, and once 6m.

58 Is less than 103, and not much larger than 76, contrary to the catalogue. It is also contrary to the magnitudes of the observations.

62 "Is less than it is marked. I suppose it to be 7 or 7.8m." FLAMSTEED'S observations give it 6m.

64. From my expression of brightness it appears that this star is changeable, and I may venture to announce it periodical. A series of observations upon it will be given when the period of the changes shall have been more fully ascertained. FLAMSTEED has but one observation of its magnitude, which is 5m.

65. This star is probably changeable, but its connected re-

ference to neighbouring changeable stars has hitherto rendered it difficult to come at the truth. In FLAMSTEED's observations it is three times 3m, and twice 4m.

67. This star is probably changeable. FLAMSTEED's observations give it twice 3m.

69 Is less than 94, contrary to the catalogue; but the magnitudes in the observations are favourable to my notation.

78 Is larger than 93, contrary to the catalogue. The latter has no magnitude in the observations, and the former is marked 6m.

95 Is less than 102, contrary to the catalogue. The observations give 95 twice 4m, and 102 once 4m, and once 5m.

99 Is less than 100, 106 and 107, contrary to the catalogue; and also to the magnitudes of the observations. It is larger than 104, which is doubly inconsistent with the catalogue, and yet the observations also give to 104 a larger magnitude.

105 Is less than 106, contrary to the catalogue. The observations give once 3m, and once 6m. "July 17, 1785; 20-feet reflector, 105 7.6m FL. 5m is visibly less than 106."

Catalogue.

In the edition of 1725, 5 (*r*) should be 5m.

Atlas.

The PD of 2 requires + 34′.
The RA of 4 requires + 16′ and the PD — 34′.
The RA of 110, 111, 112 and 113 requires + 5°.
55, 71, 80 and 81 should be out.

Notes to Pegasus.

2. The expressions 2 , 16 and 2 . 13 shew that this star is over-rated in the catalogue. In FLAMSTEED'S observations it stands 6m.

8 Is larger than 53 and 54, contrary to the catalogue. In the observations are no magnitudes of these stars.

18 Is not sufficiently distinguished from 19 to agree with the magnitudes of the catalogue. In FLAMSTEED'S observations 18 is marked once 5m, once 6m, and once 7m; and 19 is 7m.

20. "Oct. 19, 1784, 7m." In FLAMSTEED'S observations it stands 6m.

21 Is less than 17, contrary to the catalogue. It is also contrary to the magnitudes given in the observations. If there be any accuracy in the magnitudes of the catalogue and of the observations, we ought to look upon this star as changeable; for the latter give it once 3m, and once 6m, while the former has 5m.

27. "Sept. 6, 1784, 6m." In FLAMSTEED'S observations there is no magnitude of this star.

31 Is no larger than 50, contrary to the catalogue, "Sept. 5, 1784, 6m," and "Oct. 19, 1784, 5.6m." FLAMSTEED'S observations give it twice 5m, and 50 also 5m.

32. "Sept. 8, 1784, 6 . 5m." In FLAMSTEED'S observations it stands once 4m, and once 5m.

42 Is less than 48, contrary to the catalogue. But in the observations there is no magnitude to 48. "Sept. 19, 1784, 48 (μ) 4.3m."

43 Is less than 56, contrary to the catalogue. It is also

contrary to the magnitudes in the observations, where 43 is 4m, 56 5m.

47. "Sept. 19, 1784, 4.3m." FLAMSTEED's observations give no magnitude.

62 Is larger than 56 and 78, contrary to the catalogue. In the observations 56 is 5m.

63 Is less than 67, contrary to the catalogue. The observations give no magnitude of these stars.

68 Is larger than 70, contrary to the catalogue. In the observations 68 is without magnitude, and 70 is 5m and 6m.

76 Is less than 74, contrary to the catalogue. In the observations both stars are marked 6m.

79. "Sept. 8, 1784, 6.7m." The observations give no magnitude.

86 Is less than 66, contrary to the catalogue, and contrary to the observations, where the former is marked 5m, the latter 5m, and twice 6m.

88 Is less than 44, contrary to the catalogue. There are no magnitudes of these stars in the observations. It is also less than 34 Aquarii, which FLAMSTEED has observed 3m, and hardly larger than 6 Arietis, which he has also observed 3m. Therefore, if the catalogue may be trusted where this star is 2m, it must have lost some of its former lustre. But I rather suppose that this star, as well as 53 and 54, have been overvalued in the catalogue.

Catalogue and Atlas.

The letter *t*, which FLAMSTEED has annexed to 31 in his observations page 57 and 130 should be added.

Notes to Sagitta.

" Sept. 7, 1781. Order of magnitude $\gamma\,\delta\,{}^{\alpha}_{\beta}$ "

7 Is larger than 5 and 6, contrary to the catalogue. By the order of magnitude, it appears that 14 years ago it was also larger. In FLAMSTEED'S observations 5, 6 and 7 are marked 4m.

WM. HERSCHEL.

Slough, near Windsor,
Jan. 1, 1796.

X. *Experiments and Observations on the Inflection, Reflection, and Colours of Light. By* Henry Brougham *Jun. Esq. Communicated by Sir* Charles Blagden, *Knt. Sec. R. S.*

Read January 28, 1796.

It has always appeared wonderful to me, since nature seems to delight in those close analogies which enable her to preserve simplicity and even uniformity in variety, that there should be no dispositions in the parts of light, with respect to inflection and reflection, analogous or similar to their different refrangibility. In order to ascertain the existence of such properties, I began a course of experiments and observations, a short account of which forms the substance of this paper. For the sake of perspicuity I shall begin with the analytical branch of the subject, comprehending my observations under two parts: *flexion,* or the bending of the rays in their passage by bodies, and *reflection.* And I shall conclude by applying the principles there established to the explanation of phænomena, in the way of synthesis.

As in every experimental inquiry much depends on the attention paid to the minutest circumstances, in justice to myself I ought to mention, that each experiment was set down as particularly as possible immediately after it was made; that they were all repeated every favourable day for nearly a year, and before various persons; and as any thing like a preconceived

opinion, with respect to matter of theory that is in dispute, will, it is more than probable, influence us in the manner of drawing our conclusions, and even in the manner of recording the experiments that lead to these, I have endeavoured as much as possible to keep in view the saying of the Brahmin: "that he who obstinately adheres to any set of opinions, may "bring himself at last to believe that the fresh *sandal wood* is "a flame of fire."*

Part I. *Of Flexion.*

In order to fix our ideas on a subject which has never been treated of with mathematical precision, we shall suppose, for the present, that all the parts of light are equally acted upon in their passage by bodies; and deduce several of the most important propositions which occur, without mentioning the demonstrations.

Def. 1. If a ray passes within a certain distance of any body, it is bent inwards; this we shall call Inflection. 2. If it passes at a still greater distance it is turned away; this may be termed Deflection. 3. The angle of inflection is that which the inflected ray makes with the line drawn parallel to the edge of the inflecting body, and the angle of incidence is that made by the ray before inflection, at the point where it meets the parallel. And so of the angle of deflection.

Proposition I. The force by which bodies inflect and deflect the rays acts in lines perpendicular to their surfaces.

Prop. II. The sines of inflection and deflection are each of them to the sine of incidence in a given ratio; (and what this ratio is we shall afterwards shew).

* Asiatic Researches, Vol. I. p. 224.

Prop. III. The bending force is to the propelling force of light, as the sine of the difference between the angles of inflection (or deflection) and incidence, to the cosine of the angle of inflection (or deflection).

Prop. IV. The rays of light may be made to revolve round a centre in a spiral orbit.

Prop. V. If the inflecting surface be of considerable extent, and a plane, then the curve described may be found by help of the 41. Prop. Book I. *Principia;* provided only, the proportion of the force to the distance be given. Thus, if the bending force be inversely as the distance, the curve cannot be found; for in order to obtain its equation, a curvilinear area must be squared, which in this case is a conic hyberbola; the relation, however, between its ordinates and abscissæ may be obtained in fluxions, thus; $y\dot{y} + by = a^2\dot{x}^2$.

If the force (which is most probable) be inversely as the square of the distance, the curve to be squared is the cubic hyperbola; Species LXV. genus III. of NEWTON's Enumeration; and this being quadrable, the curve described by the light will be the *parabola campaniformis pura;* Species LXIX. of NEWTON.

If the force be inversely as the cube of the distance, the curve is a circular arch, and that of deflection is a conic hyperbola.* If the inflecting body be a globe or cylinder, and the force be inversely as the square of the distance from the surface, then by Prop. 71. Book I. *Principia,* the attraction to the centre is inversely as the square of the distance from that centre; and therefore, by Prop. 11. and 13. of the same book, the ray moves in an ellipse by the inflecting, and an hyperbola

* *Principia,* Lib. I. Prop. 8.

by the deflecting force, each having one focus in the centre of the body. The truth of these things mathematicians will easily determine.

Prop. VI. If a ray fall on a specular surface, it will be bent before incidence into a curve, having two points of contrary flexure, and then will be bent back the contrary way into an equal and similar curve; as in fig. 1. (Tab. VII.)

Corollary to these propositions. If a pencil of rays fall *converging* on an interposed body, the shadow will be less than the body by twice the sine of inflection.

And if a pencil fall *diverging* on the body, the shadow will be greater than the body by twice the sine of inflection; but less than it should be, if the rays had passed without bending, by twice the sine of the difference between the angles of inflection and incidence.—The sine or angle of incidence is greater than the sine or angle of inflection, when the incident rays make an acute angle with the body; but when they make an obtuse or right angle, then the sine or angle of inflection is less than that of incidence. The sine of incidence is greater than that of deflection, if the angle made by the incident ray with the body is obtuse, but less, if that angle be acute or right.—If a globe or circle be held in a beam of light the rays may be made to converge to a focus.

Hitherto it has been supposed, that the parts of which light consists have all the same disposition to be acted upon by bodies which inflect and deflect them; but we shall now see that this is by no means the case.

Obs. 1. Into my darkened chamber I let a beam of the sun's light, through a hole in a metal plate (fixed in the window-shut) of $\frac{1}{40}$th of an inch diameter; and all other light being absorbed

by black cloth hung before the window, and in the room, at the hole I placed a prism of glass, whose refracting angle was 45 degrees, and which was covered all over with black paper, except a small part on each side, which was free from impurities, and through which the light was refracted, so as to form a distinct and tolerably homogeneous spectrum on a chart at six feet from the window. In the rays, at two feet from the prism, I placed a black unpolished pin (whose diameter was every where one-tenth of an inch) parallel to the chart, and in a vertical position. Its shadow was formed in the spectrum on the chart, and had a considerable penumbra, especially in the brightest red, for it was by no means of the same thickness in all its parts; that in violet was broadest and most distinct; that in the red narrowest and most confused, and that in the intermediate colours was of an intermediate thickness and degree of distinctness. It was not bounded by straight, but by curvilinear sides, convex towards the axis to which they approached as to an asymptote, and that, nearest in the least refrangible rays, as is represented in fig. 2. where AB is the axis, IKLMNA and HGFEDA the two outlines. Nor could this be owing to any irregularity in the pin, for the same thing happened in all sorts of bodies that were used; and also if the prism was moved on its axis, so that the colours might ascend and descend on these bodies, still wherever the red fell it made the least, and the violet the greatest shadow.

Obs. 2. In the place of the pin, I fixed a screen, having in it a large hole on which was a brass plate, pierced with a small hole $\frac{1}{42}$ of an inch in diameter; then causing an assistant to move the prism slowly on its axis, I observed the round image made by the different rays passing through the hole to the chart; that

made by the red was greatest, by the violet, least, and by the intermediate rays, of an intermediate size. Also when at the back of the hole I held a sharp blade of a knife, so as to produce the fringes mentioned by GRIMALDO and NEWTON; those fringes in the red were broadest, and most moved inwards to the shadow, and most dilated when the knife was moved over the hole; and the hole itself on the chart was more dilated during the motion when illuminated by the red than when illuminated by any other of the rays, and least of all when illuminated by the violet. Now in Obs. 1. the angle of incidence of the red rays was equal to that of the violet and all the rest, and yet the angle of inflection was greatest, and least in the violet; and indeed the difference between the two was greater than appears at first from the experiment; for that part of the shadow which was formed by the violet fell at a greater distance from the point of incidence, than did that part which was formed by the red, from the divergency of the different rays upwards by the refraction, as appears in fig. 3. where DE is the window, FG the beam propagated through the hole F, refracted by the prism KIH, and painting on the chart OP *qs*; the spectrum *vr* being separated into L*r* the red rays incident on the pin CD at C, and M*v* the violet incident at D; the shadow of DC being formed in *vr*, so that *v* being farther from D than *r* is from C, therefore (by the propositions formerly laid down) the shadow in *v* should be considerably less than that in *r*, if the rays were equally inflected. Lastly, in Obs. 2. the angle of the red's incidence was nearly equal to that of the violet's, by the motion of the prism, and the consequent motion of the colours; only that, if there was any difference, it was on the side of the violet;

and yet the violet was least inflected, and the red most inflected; and so of the second inflection by the knife blade: wherefore I conclude that the rays of the sun's light differ in degree of inflexibility, and that those which are *least refrangible* are *most inflexible.*

Obs. 3. My room being darkened as before, and a conical beam propagated through the small hole in the window-shut; at this hole I placed a hollow prism, made of broken plates of mirror, and of such an angle, that when filled with distilled water, it cast a spectrum on an horizontal table, and was there received on a chart seven feet from the window. I then placed on the same table, and in the rays between the chart and the prism, at three inches from the chart, two sharp knife-blades with even edges, and fixed to a board with wax, so as to make an angle with one another; moving them nearer and nearer, till I saw the fringes appear in the red light on the chart, and then in the orange and other colours successively. I then withdrew one, and the fringes became faint and narrow, and not all within the shadow of the remaining knife, but at its edge, and even in the light of the spectrum. Lastly, when I slowly approached the other, they moved into the shadow, and became broader, and farther separated one from another, there being the like fringes in both shadows; this I repeated in all the rays, and plainly saw that at the approach of the knife, the fringes became broader, and farther removed from one another, and from the light, in the red than in the violet, or any of the other rays.

Obs. 4. In repeating the foregoing experiment, I observed a very curious phænomenon. When the angle of the knife-blades was so held in any of the rays as to make the hyper-

bolic fringes described by NEWTON,* and these being always of the colour in which they were held, moving the angle a little, so as to make the fringes out of the light that went to the top of any one division of the spectrum and also out of that which went near the bottom of the next, the fringes were made of two colours; one part was of the highest colour, and the other of the lowest, but *both* were on the ground of the highest. Thus if held on the confine of the green and blue, the upper half of each fringe was blue, the under green, but both parts in the blue division of the spectrum; and trying the same in all the rays, it was evident that the red was moved farther into the orange, and the orange into the yellow, than the blue was into the indigo, or the indigo into the violet. Now, in Obs. 3. the fringes were formed by the *inflection* of one knife, and were moved into its shadow, and separated and dilated by the *deflection* of the other; and this most in the red and least in the violet: likewise in Obs. 4. the fringes of one colour were deflected into the region of the next, and this most in the red, and least in the violet; although in both observations the violet, from the position of the chart, was farthest from the angle, and consequently had the rays been equally deflected, the violet should have been farthest moved, and most dilated by the deflection; but seeing that at equal angles of incidence in the third, and at less in the fourth observation, the red was most and the violet less deflected, it is evident that the most *inflexible* rays are also most *deflexible*.

Having thus found that the parts of light differ in *flexibility*, I wished next to learn two things; in what proportion the angle of *inflection* is to that of *deflection* at equal incidences;

* Optics, Book III. Obs. 8.

and secondly, what proportion the different *flexibilities* of the different rays bear to one another. But the nature of the coloured fringes must first be understood, so that I defer this inquiry till after I have made use of the principles now laid down, for the explanation of natural phænomena, and proceed in the mean time to

PART II. *Of Reflection.*

That bodies reflect light by a repulsive power, extending to some distance from their surfaces, has never been denied since the time of Sir ISAAC NEWTON.* Now this power extends to a distance much greater than that of apparent contact, at which an attraction again begins, still at a distance, though less than that at which before there was a repulsion; as will appear by the following demonstration which occurs to me, and which is general with respect to the theory of BOSCOVICH.† In fig. 4. let the body A have for P an attraction, which, at the distance of AP, is proportional to PM; then let P move towards A so as to come to the situation P′, and let the attraction here be P′M′; as it is continual during the motion of P to P′, MM′ is a curve line. Now in the case of the attraction of bodies for light, and for one another, PM is less than P′M′, and consequently MM′ does not ever return into itself, and therefore it must go, *ad infinitum*, having its arc between AB and AC, to which it approaches as asymptotes; the abscissa always representing the distance, and the ordinate the attraction at that distance: let P′ now continue its motion to P″, and M′ will move to M″, and if P″ meets A, or the

* Optics, Book II. Part III. prop. 8.

† *Nova Theoria Philosophiæ Naturalis.*

bodies come into perfect contact, P″ M″, will be infinite; so that the attraction being changed into cohesion, will be infinite, and the bodies inseparable, contrary to universal experience; so that P can never come nearer to A than a given distance. In the case of gravity, PM is inversely as the square of AP, so that the curve NMM‴ is the cubic hyperbola; but the demonstration holds, whatever be the proportion of the force to the distance. It appears then that flexion, refraction, and reflection, are performed by a force acting at a definite distance; and it is reasonable to think even *a priori*, that as this same force, in other circumstances, is exerted to a different degree on the different parts of light, in refracting, inflecting, and deflecting them, it should also be exercised with the like variations in reflecting them. Let us attend to the proof, which enables us to change conjecture into conviction.

Obs. 1. The sun shining into my darkened chamber through a small hole $\frac{1}{40}$th of an inch in diameter, I placed a pin of $\frac{1}{30}$th of an inch diameter in the cone of light (one-half inch from the hole) inclined to the rays at an angle of about 45°, and its shadow was received on a chart parallel to it, at the distance of two feet. The shadow was surrounded by the three fringes on each side, discovered by GRIMALDO; beyond these there were two streaks of white light diverging from the shadow, and mottled with bright colours, very irregularly scattered up and down; but on using another pin, whose surface was well polished, and placing it nearer the hole than before, the colours in the streaks became much brighter (and the streaks themselves narrower), being extended from one side to the other, so that, except in a very few points here and there, no

white was now to be seen ; and on moving the pin, the colours moved also. But they disappeared if the pin was deprived of its polish, by being held in the flame of a candle, or if a roll of paper was used instead of the pin ; also, they were much brighter in direct than in reflected light, and in the light of the sun at the focus of a lens, than in his direct unrefracted light. Placing a piece of paper round the hole in the window-shut, I observed the colours continued there ; and inclining the chart to the point where they left off, I saw them continued on it, and then proceed as before to the shadow. If the pin was held horizontally, or nearly so, they were seen of a great size on the floor, the walls, and roof of the room, forming a large circle ; and if the chart was laid horizontally, and the pin held between the hole and it, in a vertical position, the circle was seen on the chart, and became an oval, by inclining the pin a little to the horizon.

Obs. 2. Having produced a clear set of colours, as in the last observation, I viewed them as attentively as possible, and found that they were divided into sets, sometimes separated by a gleam of white light, sometimes by a line of shadow, and sometimes contiguous, or even running a little into one another. They were spectra, or images of the sun, for they varied with the luminous body by whose rays they were formed, and with the size of the beam in which the pin was held; and when, by placing it between my eye and the candle, a little to one side, I let the colours fall on my retina, I plainly saw that they resembled the candle, in shape and size (though a little distended), and also in motion, since if the flame was blown upon, they had the like agitation. The colours therefore which fell on the chart were images of the

sun; they had parallel sides pretty distinctly defined, but the ends were confused and semicircular, like those of the prismatic spectrum. Like it too, they were oblong, and in some the length exceeded the breadth six, even eight times; the breadth was, as I found by measurement, exactly equal to that of the sun's image received on a chart, as far from the pin as the image was, and the length was always to the breadth at all distances, in the same ratio, but not in all positions of the pin; for if it was moved on its axis, the images moved towards the shadow on one side, and from it on the other, becoming longer and longer (the breadth remaining the same) the nearer they came to the shadow on the one side, and shorter in the same proportion, the farther they went from it on the other.

Obs. 3. Having picked out an image that appeared very bright and well defined, I let it through a hole with moveable sides, in the upper part of a sort of desk, which moved to any opening by hinges, and had a chart for its under side, on which the image fell, and I shut the hole so close as to prevent any of the others from coming through; I then had a full opportunity of examining it, in all respects, and I counted in it distinctly the seven prismatic colours; the red was farthest from the shadow of the pin, and from the pin itself; then the orange; then the yellow, green, blue, and indigo, and the violet nearest of all; in short, it was exactly similar to a prismatic spectrum, much diminished in length and breadth, and turned horizontally on the wall opposite to the prism, with the red farthest away. In fig. 5. *se* is the pin, reflecting the rays CP and CO, which pass through PO, the hole in the desk ED, to the chart or bottom of the desk RTSD, and form there the

spectrum IK divided into its colours, I being violet, and K red. On moving the hole in the desk, and letting through other images, the colours were not in all arranged the same way, but I moved the pin on its axis, and observed those where the order was inverted to move, not only with respect to the pin, but also with respect to the contiguous images; and I was surprised to see them assume the order of colours first mentioned, namely, the red outermost, and the violet innermost. In like manner the images, which before the motion were regular, on moving into the places left by the others had always the order of their colours inverted, so that the thing must be owing to some irregularities in the pin's surface; for those which were made by a small glass tube filled with quicksilver, and freed from scratches by a blow-pipe, preserved during the motion the proper order of colours. Another irregularity in the arrangement was also observable even in the glass tube; for two contiguous images, by mixing one with another for two or three successions, appeared each to have outermost a dull colour, between red and violet, and innermost a green; but here, unless the succession continued through all the images, the outermost of all was red, and the innermost image had universally violet in the inside.

Obs. 4. I placed at a hole in the window-shut a prism, to refract the rays, and received the spectrum at the distance of six feet from the window, on a chart; then, at the distance of two feet, I placed a screen with a hole in the middle of it, through which I let pass successively the different rays. At the distance of one inch from the hole, between it and the chart, I placed the reflecting cylindrical body; the images were found on the chart and walls of the room round to the sides

of the hole on the screen, and were always wholly of the colour in which they were formed, except in the confines of the green, where a small quantity of white light fell, and made them of all the seven colours; but this was almost wholly prevented by using a prism with a greater refracting angle, and holding the pin and screen farther from it. I then removed the screen, and left the reflector in its place, so as it might reach through the rays; and thus there were formed images, having in them, from top to bottom, the seven colours, one after another, the lowest division being red, the highest violet. They were inclined considerably towards their tops, and were much broader at the bottom or red parts than at the tops or violet parts. And lastly, the reflector being moved so that the images might be disturbed (as in the former experiment made in the white light), the red was most, the violet least dilated. In case these effects might be owing to any peculiarities in the shape or position of the reflector, I placed at three feet from the prism a lens of four inches breadth, to collect the rays to a focus, six feet beyond which I held a chart, and there received the spectrum inverted, the red being uppermost, and the violet undermost; holding the reflector at two feet from the focus, and four from the chart, the images were formed just as before, only inverted, inclining towards the violet, of greater breadth towards the red, and more distended towards the same quarter when the reflector was moved.

Obs. 5. Things remaining as in the last part of the last experiment, at the focus of the lens I placed a second prism, which refracted the rays into a white beam,* and this I

* Optics, Book II. Part II. Prop. 2.

received on a screen with a hole in the middle, through which a small part of it passed, and falling on the reflector placed behind, was formed by it into images, after the manner of the first experiment, each having in regular order the seven prismatic colours. One of the brightest and most distinct I let pass through a hole in a second screen, and it fell on the chart. I then caused an assistant to intercept the red rays between the first prism and the lens, and immediately the red part of the image vanished ; and when the violet was intercepted, the violet of the image vanished ; and if the green was intercepted, the green was wanting in the image. In short, whatever colours were stopped, the same were missing in the image. In fig. 6, the rays passing through the hole C of the window AB, are refracted by the prism PMN, and separated into DV, DG, and DR, violet, green, and red ; which being collected into a focus F by the lens L, are there again refracted by a prism P′M′N′, and formed into a white beam *abmn*, part of which is intercepted by the screen SS′, and part passes through the hole *b*, as *b*H to H on the chart XYZW, and part is reflected by the body *o q* into a set of images which are received on a screen TU, and one of them, *rgv*, let pass to WXYZ ; but when an obstacle E stops DR, *r* the red vanishes ; and if DG be stopped, *g* the green vanishes ; and if DV be stopped, *v* disappears. Lastly, if DR and DG be stopped, *g* and *r* vanish.

Obs. 6. Having produced a set of bright images, I let one pass through the desk described in the third experiment, and received it on a small lens $\frac{1}{2}$ inch broad, to collect them into a focus, which I received on the chart, by moving it a little on its hinge ; and by all the observations I could make, and all the

tests I could think of, it was white inclining to yellow, and of the same nature and constitution with the sun's direct light; but if any ray was stopped before coming to the lens, the focus was a mixture of the remaining rays; and the chart being moved a little farther round, the image was formed on it, the colours being in an inverted order. At the focus I held a reflector, and there were formed images of all the seven colours, as in the sun's direct light (Exp. 1.); if the light was sufficiently strong, and the desk near the window-shut hole, one of these could even be collected by a second lens into a white focus. This experiment is rendered more uniform by substituting for the lens a concave metallic mirror, and placing at the focus another mirror to reduce the rays into a beam, which may be made of any composition we please, by stopping one or more of the colours at the hole in the desk. I observed in the course of these experiments a phænomenon worth mentioning; if a comb (as in NEWTON'S experiment*) be very swiftly moved before one of the images, or more, a sensation of white is produced; but this is still more evident, if the pin be swiftly moved round its axis, for then the images move also, and running into one another, cause a sensation of perfect whiteness.

Obs. 7. I let an image through the hole in the desk, and viewed it through a glass prism, holding its axis parallel to the sides of the image, and its refracting angle upwards; I found that, if the image was bright and free from white light, the colours were not changed by the refraction; but, if it was mixed and diluted with white, the prism, decompounding the white, caused the image to appear violet at one side, and red

* Optics, Book I. Part II. Prop. 5.

at the other; yet still this only confused the colours of the image, without changing them. Farther, if the prism was moved on its axis, the violet was lifted higher than the red or any of the other colours. Nor was the constitution of the colours at all changed by reflection from a pin or mirror, except in so far as they were mixed by a concave one, as mentioned in the last experiment. If a pin was held behind the hole to reflect the colours, it formed other images of the colour in which it was held, and, as far as I could judge, threw the red to the greatest distance, and breadth, and inclination. Nor were the colours of the image changed by reflection from natural bodies, for these were all of the colours in which they were held, but brightest in that which they were disposed to reflect most copiously. Likewise the rings of colours made by thin plates were broadest in the red, and narrowest in the violet; and the like happened to the fringes that surround the shadows of bodies. Lastly, the shadows of bodies were themselves broadest in the violet, and narrowest in the red.

Obs. 8. I filled with water a glass tube, whose diameter was $\frac{1}{4}$th of an inch, and consequently the radius of curvature $\frac{1}{8}$th, and whose sides were $\frac{1}{30}$th of an inch thick; then standing at four feet from a candle, I held the tube $\frac{1}{4}$th of an inch from my eye, so that the light of the candle might be refracted through it, and moved my eyelids close enough to prevent the extraneous scattered light from entering along with that which was regularly refracted. I saw several images of the candle all highly coloured, and the colours were in order, from the candle outwards, red, orange, and so on to violet; I then filled the tube with clear diluted sulphuric acid, and dropped a small piece of chalk to the bottom, when immediately an

effervescence took place, by the escape of fixed air, which rose in bubbles through the tube; and looking at the candle through one of these, I saw the images formed with the colours still in the same order, but a little larger than before.

We are now to see to what conclusions these experiments lead us.—The first experiment shows, that all sorts of light, whether direct, or reflected, or refracted, produces colours by reflection from a curve surface. From the second we learn, that these colours are distinct images or spectra of the luminous body, much dilated in length, but not at all in breadth; and that the angle of incidence being changed, the dilatation of the images is also changed: and from the third experiment it appears, that each full image is composed of seven colours; red, orange, yellow, green, blue, indigo, and violet; and that the proper order is red outermost, and violet innermost, the rest being in their order. The fourth experiment shows, that these images are produced, not by any accidental or new modification impressed on the rays, but by the white light being decomposed by reflection; that the mean rays, or those at the confine of the green and blue, are reflected at an angle equal to that of incidence, and the red at a less, the violet at a greater angle. Experiments 5th and 6th prove, beyond a doubt, the decomposition and separation of the rays by reflection; for in both we see that the colours in the images are those, and those only, which were mixed in the ray by reflection or refraction, before and at incidence, whilst the 6th is (in addition) a proof that all the rays of any one image, if mixed together, compound a beam exactly similar to the beam that was at first decompounded. The 7th experiment shows, that the colours into which the rays are separated by reflection are homogeneous

and unchangeable ; that they differ in flexibility and refrangibility ; that they bear the same part in forming images by reflection, and fringes by flexion, and colours from thin plates, which the rays separated by the prism do : and in the 8th experiment we see, that when the rays are placed in the same situation with respect to refraction, whether out of a rarer into a denser or a denser into a rarer medium, in which they before were with respect to reflection, the position of the colours produced is diametrically opposite in the two cases. Seeing then that in all sorts of light, direct, refracted, reflected, simple, and homogeneous, or heterogeneous and compounded, and in whatever way the separation and mixture may have been made, some of the rays at equal or the same incidences are constantly reflected nearer the perpendicular than the mean rays, and others not so near ; and seeing that by such reflection the compound ray, of whatever kind, is separated into parts so simple that they can never more be changed ; and considering the different places to which these parts are reflected ; it is evident, that the sun's light consists of parts different in reflexibility, and that those which are least refrangible are most reflexible. By reflexibility, I here mean a disposition to be reflected near to the perpendicular in any degree.

Although I have given what I take to be sufficient proof of this property of light, yet I am aware that something more is requisite. It will be asked, why does neither a plain, a common convex, nor a common concave mirror separate the rays by reflection ? This is what has always hindered us from even suspecting such a thing as different reflexibility. I shall, however, take an opportunity of removing this obstacle, in the second part of the plan, when I come to explain the reason of

the colours made by the reflecting body, and the manner of their formation. At present I shall only caution those who may wish to repeat the above experiments, that the hole in the window-shut must be small, the room quite dark, the pin well polished, and the desk, chart, &c. placed at a distance from the pin not greater than three feet, otherwise the images will be dilute and dim; nor, on the other hand, less than six inches, otherwise they will be too short, and the colours not far enough separated one from another.

My next object of inquiry was the different degrees of reflexibility belonging to each ray. It appears, not only from mathematical considerations sufficiently obvious, but also from the experiments I have related, that though the different rays have at the same or equal incidences different angles of reflection, yet each ray is constant to itself in degree of reflexibility, and that its sine of reflection bears always the same ratio to its sine of incidence. The question then is, what are the sines of reflection of the different rays, the sine of incidence being the same to all?

Obs. 9. In summer, at noon, when the sun's light was exceedingly strong, and there was not the vestige of a cloud in the sky, I produced an uncommonly fine set of images, by fixing at an inch from the small hole $\frac{1}{50}$th of an inch diameter, a pin $\frac{1}{25}$th of an inch diameter. One of the brightest of these I let pass through the desk to the chart below at $2\frac{1}{2}$ feet from the pin, and the image was 3 inches from the shadow in a straight line. I delineated it carefully, by drawing two parallel lines for the sides, and marking the semicircular ends. Then with the point of a small needle I marked the confines of the contiguous colours on one of the parallel sides, and

afterwards drew across the image parallel lines; this operation I repeated with the same and different images, at many distances from the pin, and on different days, with various sorts of pins, and sizes of holes, &c. &c. and all these repetitions were made before I once examined the result of any one measurement, that I might be unprejudiced in trying the thing over again. I then compared the sketches of divided images, which I thus obtained, and found sufficient reason to conclude, that the differences between the sines of reflection in the different rays were in the harmonical order. For the divisions were nearly as $\frac{1}{9}$; $\frac{1}{18}$; $\frac{1}{12}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{3}{80}$, $\frac{1}{16}$; which, when compounded with the scale, give 1, $\frac{15}{16}$, $\frac{9}{10}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{18}$, $\frac{1}{2}$; and these are exactly the change of the notes in an octave, obtained by taking the sums of the octave, and a second major, a third major, a fourth, a fifth, a sixth major, a seventh major, and an eighth, instead of the difference between a double octave, and a second major, a third major, and so on. Thus the spectrum by reflection is divided exactly as the spectrum by refraction, only that the former is inverted, and the different rays have reflexibilities that are inversely as their refrangibilities.

Having settled this (I flatter myself) curious and important point, I proceeded next to inquire into the absolute reflexibility of the extreme colours; for if this be known, the angle of incidence being given, the angle of reflection of all the different rays may be found. For obtaining a solution of this problem I made the following experiment.

Obs. 10. The sun shining strongly through the small hole in the window-shut, and the rays diverging into a cone, whose base fell on an horizontal chart $2\frac{1}{2}$ feet from the hole, between the hole and chart I placed a screen, which had a plate

and small hole in it; the rays passing through this, fell on a small pin, so placed that the images formed might be at right angles to the shadow; one of these I measured, together with its distance from the shadow, the distance of the shadow from the hole, the breadth of the shadow, and the diameter of the pin; these measures were as follows. In fig. 7. C is the centre, and B*en* the circumference of the pin, GM the chart, and GD a line in it, being the axis of all the images, at right angles to CD, the distance of C from D the centre of the shadow, and also to the shadow itself; GE is the parallel side of the image, G being red, E violet, and F the confine of the green and blue; C*e* is a radius parallel to ED, and CA another drawn through B, the point where OB is incident, at the angle OBA, to which (by what was before shown) ABF is equal. By measurement GE is $\frac{1}{4}$th of an inch, CB $\frac{1}{80}$th, CD $4\frac{1}{2}$; now the shadow being lessened by a penumbra, this added to half the shadow, and their sum to the distance between the penumbra and the violet, gave ED $\frac{417}{400}$th of an inch. From whence it is easy to calculate, that the angle of incidence being 77° 20′, the angle of the red's reflection ABG is 75° 50′, and that of the violet's 78° 51′. Now the natural sines of 77° 20′, 75° 50′, and 78° 51′, are as 9756, 9695, and 9811; or as 250, 248, and 251; which are very nearly as $77\frac{1}{2}$, 77, and 78; and making an allowance for the omissions made in the reductions, the errors in the operations and measurements, they may be accounted as accurately in the above proportion. Now these extremes, 77 and 78, are the very proportions of the red's refrangibility to the violet's.* So that the reflexibility of the red is to that of the violet as the re-

* Optics, Book I. Part I. Prop. 7.

frangibilities inversely.' But it is obvious that the sine of incidence is not the same in the two cases; for in the one it is equal to that of the mean ray's reflection, while in the other none of the rays are refracted at an angle equal to that of incidence, otherwise they would not be refracted at all. This, however, being a consequence of the essential distinction in the circumstances, does not impair the beautiful analogy which we have seen is preserved in the two operations, and which proves them to be different exertions of the same power. Now we may find, from the data obtained, the sines of all the rays in the spectrum, by adding to 77 the lengths of the spaces into which it is divided, and which are without any sensible error as the differences of those sines. The sines of the *red* will be from 77 to $77\frac{1}{8}$; the *orange* from $77\frac{1}{8}$ to $77\frac{1}{5}$; the *yellow* from $77\frac{1}{5}$ to $77\frac{1}{3}$; the *green* from $77\frac{1}{3}$ to $77\frac{1}{2}$; the *blue* from $77\frac{1}{2}$ to $77\frac{2}{3}$; the *indigo* from $77\frac{2}{3}$ to $77\frac{7}{9}$; the *violet* from $77\frac{7}{9}$ to 78. So that the sine of incidence being given, that of the reflection of all the different rays may be found; and the angle of incidence being 50° 48′, the angles of reflection are as follows: of the extreme red 50° 21′; of the orange 50° 27′; of the yellow 50° 32′; of the green 50° 39; of the blue 50° 48′; of the indigo 50° 57′; of the violet 51° 3′; and of the extreme violet 51° 15′.

I shall conclude this part of the subject with a few remarks on the physical cause of reflexibility. As light is reflected by a power extending to some distance from the reflecting surface, the different reflexibility of its parts arises from a constitutional disposition of these to be acted upon differently by the power. And as these parts are of different sizes, those which are largest will be acted upon most strongly. I shall not hesitate to go a

step farther. In fig. 8. let EC be the reflecting surface, DH the perpendicular, and AB a ray incident at B, and produced to F, and reflected into GB; draw GH parallel to FB, and GF to HB. Then HB : (HG :) BF : : sin. HGB : sin. HBG, or : : sin. GBF : sin. HBG. But GBF is the supplement of GBA, the sum of the angles of reflection and incidence; wherefore HB : BF : : the sine of the sum of the angles of reflection and incidence, to the sine of the angle of reflection; so that if I be the angle of incidence, R that of reflection, V the velocity of light, and F the reflecting force; $F = \frac{V \times \text{sin. } (R+I)}{\text{sin. } R.}$. By accommodating this formula to the different cases, we obtain F in all the rays; and the ratio of F in one set to F in another being required, we have (by striking out V, which is constant) $F : F' : : \frac{\text{sin. } (R+I)}{\text{sin. } R} : \frac{\text{sin. } (R'+I')}{\text{sin. } R'}$. Suppose we would know F and F′ in the red and violet respectively; $I = 50° \; 48'$ — $R = 50° \; 21'$, and $R' = 51° \; 15'$; then $F : F' : : \frac{\text{sin. } 101° \; 9'}{\text{sin. } 50° \; 21'} : \frac{\text{sin. } 102° \; 3'}{\text{sin. } 51° \; 15'}$. Performing the division in each by logarithms, and finding the natural sines corresponding to the quotients; F : F′ : : 1275 : 1253. But the force exerted on the red is to that exerted on the violet, as the size of the red to the size of the violet (by hypothesis); therefore, the red particles are to the violet as 1275 to 1253. This may be extended to all the other colours, by similar calculations; their sizes lying between 1275 and 1253, which are the extreme red and extreme violet; thus the red will be from 1275 to 1272½; the orange from 1272½ to 1270; the yellow from 1270 to 1267; the green from 1267 to 1264; the blue from 1264 to 1260; the indigo from 1260 to 1258; and the violet, from 1258 to 1253.

All this follows mathematically, on the supposition that the parts of light are acted upon in proportion to their sizes; and to say the truth, I see no other proportion in which we can reasonably suppose them to be influenced; for such an action is not only conformable to the universal laws of attraction and repulsion, but also to the following arguments. If the action be not in the simple ratio, it must either be in a lower or in a higher; let it be in a lower, as that of the square root, then the size of the red would be to the size of the violet as the squares of the forces; that is, as 1625625 to 1572009: a difference evidently too great; and, *a fortiori,* of the cube or any other root. On the other hand, if the action were in a higher ratio, as that of the square, then the particles would be as the square roots of the forces, or nearly as 35.70 to 35.39, a difference evidently too small; for if the size of the red particles were only $\frac{3}{10}$ths greater than that of the violet, and the velocity of both were equal, the momentum, and consequently the intensity of the red, could not so much exceed that of the violet as we find it does, and as seems to me to be proved by the experiment of BUFFON (on accidental colours), who found, that after looking at a white object, when he shut his eyes, it first became violet, then blue, or a mixture of blue and the other colours, and last of all red; so in the impression of the white, compounded of the impressions of all the other rays mixed together, the violet was first obliterated or weakest, and the red last or strongest. To this reasoning on the intensity of the particles as owing to their size, I see only two objections that can be made. The one is, that the intensity is increased when the rays are thrown into a focus; but we must recollect that the rays in this case are mixed, and their

particles so blended as to be increased in size; for the number of separate rays thrown into one place will not increase their intensity sensibly. The other objection is, that passage in NEWTON, where he says "that the orange and yellow are the "most luminous of all the colours, affecting the senses most "strongly."* Now, besides that this is an assertion opposed by the positive experiment just now quoted, I think an answer may be thus made to it; the whole light, from which the spectrum is never free, which inclines to yellow, and which is composed also of red, abounds in the yellow and orange of the spectrum; so that both of these colours derive their superior lustre rather than intensity from this circumstance; or if they have any degree of the latter more than the red, it is in fact owing to their mixture with the red and the other rays, which are all in the white.

Having endeavoured to unfold the property of flexibility, as varied in inflection, deflection, and reflection; and also the physical cause of this property; and having indulged in a speculation depending on this cause, I flatter myself neither altogether useless nor unimportant, I hasten now to the natural phænomena, the explanation of which depends on the property, whose existence and nature we have just now been investigating; and that we may treat this part of the subject with conciseness and order, we shall rank the phænomena under a division similar to that under which we laid down the principles, beginning with those appearances which are explicable on the principles of flexion.

1. It is observable, that when a body is exposed in the sun's light, so as to cast a shadow, and another body is ap-

* Optics, Book I. Part I. Prop. 7.

proached to it, either between the sun and it, or the shadow and it, or on the same line with it, the shadow of the one body comes out a considerable way, and meets that of the other. Now it is evident, that when the bodies are held at a sufficient distance from one another, a penumbra is formed round the shadow of each, making it less than it should be were there no inflection; but when the bodies are brought so close to one another that the edge of the one is within the sphere of the other's inflection, the light being already bent by this last, the former can have none to bend, and consequently no penumbra in the part of the shadow corresponding to that part of the body which is within the other's sphere of inflection; and the rest of the shadow having a penumbra, this part that has none will be larger than it, and increase as the bodies approach, till at last it meets the other shadow; the like appearance happening when the shadows are thrown on the eye. Mr. MELVILL has endeavoured to show that it belongs simply to a case of vision;* however, we have now seen that it has no reference to the structure or position of the eye, but only to the common nature of all shadows.

Obs. 11. If we shut out all the light coming into a room from external objects, except what may pass through a small hole of $\frac{1}{2}$ or $\frac{1}{4}$th of an inch in diameter, the images of the external objects, as clouds, houses, trees, will be painted on the opposite wall, by the rays of light crossing at the hole; but if a piece of rough glass, or of very fine paper, be held so as to cover it all over, the light does not pass through; then if the paper be wetted with oil, or the glass with water, so as to give either a small degree of transparency, the first rays that come

* Edinburgh Literary Essays, Vol. II.

through are those from red and orange objects, and last from blue and violet. Now it is evident that transparency in general, and this particular fact, are explicable by what was before laid down. It was found by Newton, that a body transmits the light incident on it more or less, according to the continuity of its particles, and that a strong reflection takes place on the confines of a vacuum.* How does this happen? The initial velocity of light is sufficient to carry it through the first surface or set of particles, but it is so much diminished, that it is reflected by the repulsive power of the backside of these particles, unless there be others behind at a certain distance, namely, that at which inflection or attraction acts, that is, apparent contact; this attraction renews the impetus of light, and transmits it to another set, and so on. Now this action being strongest on the largest and red particles, and weakest on the blue and violet, if the continuity be diminished, the former will be transmitted, and not the latter; which is conformable to the experiment just now mentioned.

3. The doctrine of flexibility furnishes an easy and satisfactory explanation of the different colours which are assumed by flame. Whether we suppose the light to come from the burning body, or the oxygenous gaz, the largest or red particles have the strongest attraction for bodies, the violet the weakest; when therefore the gaz and the body combine, the precipitation of light must be in the reverse order of the affinity between the particles of light and those of the bodies. If then the combination take place slowly, the violet and blue particles will be first emitted, and last of all the red; and this is consistent with fact; for any inflammable body whatever,

* Optics, Book II. Part III. Prop. 3.

on being lighted, burns at first with a blue or violet flame, and afterwards has its flame of two or three distinct colours, blue, white, red, &c. as is seen remarkably in the case of a candle. Nay, I have observed in the flame of a blow-pipe all the seven primary colours at once. When, indeed, a body is burnt in pure oxygenous gaz, the combination is so rapid, that white light alone is precipitated undecomposed; but in common air, where the azotic gaz impedes the combustion, the above phænomena are obvious.

4. A curious phænomenon has often surprised philosophers, namely, blue shadows. These I have observed at all times, when the paper on which I received them was illuminated by the sky, and any other light; and the reason of them I take to be this, that the shadow made by one light is illuminated by the blue rays from the sky; for I have often observed purple, and even reddish ones, when the sky or clouds happened to be of those colours; and this account of the matter is confirmed by an experiment. Having received the coloured spectrum made by a prism with a large refracting angle, on a sheet of rough white paper, and held above it another sheet, I stopped all the rays that illuminated the first except the blue, and violet, and red; and if I held a body between the blue and the second paper, its shadow was red; and if I held a body between the red and the paper, its shadow was blue; and so of other colours. This I take to amount to a demonstration of the thing.*

* Since writing the above, I find the same explanation of the matter given by Mr. Melvill, and some of the French academicians, particularly Messieurs Buffon and Beguelin; also Count Rumford; but I have thought fit to keep it in, on account of the experiment that occurred to me in illustration of it.

5. Passing over other phænomena of less note, I come now to one that has divided opticians more than any other; I mean the coloured fringes that surround the shadows of bodies. I made several observations on these, which enable me to conclude that each fringe is an image of the luminous body; for holding between my eye and a candle two knife blades, as I approached the one to the other, the edge of the candle seemed multiplied, and soon became coloured, coming wholly away from the candle, and as the knives approached still nearer, became distinct dilated images, highly tinctured with the prismatic colours; and just before the knives met, the candle, whose edges had been all along coloured with red and yellow, became much distended, till at last it was divided in the middle, one half seeming to be drawn away by each knife, and then it wholly disappeared. I have observed three kinds of these images; two without and one within the shadow; the first had its colours in the order from the shadow, red outermost, and violet innermost; the second and third had the colours in the contrary order, but the second was so very faint that I could never perceive it unless when let fall on my eye. All this is easily explained by the different flexibility of the rays. In fig. 9. let AD be a body, by which the rays SDT and S′D′T′ pass; and let SD be within AD's sphere of inflection, and S′D′ within its sphere of deflection; then SD will be bent into DG, but because of the different inflexibility of its parts, the red will be bent into DR, and the violet into DV, and the intermediate rays will fall between R and V, the whole forming an image RGV, separated into the seven primary colours; and in like manner, by the different deflexibility of the parts whereof S′D′ consists, an image without the

shadow, as V′G′R′ will be formed, similar to VGR, R′ being red and V′ violet, all which is both theory and experience; and the same explanation may be extended to the other cases. Now, in all these, the bending power stretching to a very small definite distance, and being of different degrees of strength at different distances from the body, several pencils or small beams, passing through different parts of the spheres, will be acted upon by the power in its different states of strength; and each beam will be disposed into an image in the way before described; of these images I have sometimes observed four, and even, by using great care, the faint lineaments of a fifth. In forming them, the power acts strongest at the smallest distances, and of consequence bends the mean flexible rays, that pass near, farther inwards or outwards than those that pass farther off; so that the extreme rays will in the former case be more separated from the mean than in the latter; and the nearer image will always be the largest and most highly coloured, which is consistent with fact. This explains fully the celebrated experiment of Sir Isaac Newton with the knives, and the explanation is confirmed by the experiments which I related above on flexibility, where the bending force acted most strongly on those images formed out of red light, and least strongly on those formed out of violet and blue light. A number of other phænomena are explicable on the same principles, being only particular cases as it were of the coloured fringes or images; I shall here mention a few of the most remarkable.

6. When making some of the experiments which I have related in the course of this paper, I observed that when the sun was surrounded, but not covered, by clear white clouds,

the white image on the chart (the hole being $1\frac{1}{2}$ inch in diameter) was surrounded by two rainbows, pretty broad and bright; in the colours were red on the outside, and violet next the white of the image. These bows must not be confounded with one which sometimes appears wholly of a dull red and yellow, when the sun or moon shines through a cloud, and which is owing to the direct transmission of the red rays and reflection of the others; for not only are the colours different in species, in brightness, and in number, in the phænomena under discussion, but likewise they are formed by the hole in the window, as I knew by altering its shape into an oblong; and the colours now were not disposed in circles, but in broad lines of the same breadth as the bows had been, running along the shadow of the hole's sides, and in the same position of colours as before. It is evident that their cause is the inflection of the light which comes from the clouds by the sides of the hole (for if the sky have no clouds the colours do not appear), which separate the white light into the parts of which it is composed.

7. It is observable, that when we look at any luminous body, at a distance greater than one or two feet, its flame appears surrounded by two bows of faint colours, the innermost of them terminating in a white which continues to the flame; and the colours are red outermost, and green and blue innermost: the appearance is most remarkable if we look at a small hole in the window-shut, the room being otherwise dark; and if the eye be pressed upon, and then opened, the colours are more lively than before, as DES CARTES observed;* from which both he and NEWTON concluded, that the appearance

* *De Meteoribus.*

was owing entirely to wrinkles formed on the surface of the eye by the pressure.* But this could neither form the bows with the regularity in which they always appear, nor could the colours be in the order above mentioned from the different refrangibility of the rays; it will also be obvious to any one who tries the thing, that the pressure only increases the brightness and breadth of the bows, but does not form them. The true solution of the difficulty seems to be this: the rays which enter the pupil, are inflected in their passage through the fibres, which extend over the cornea, and which are very minute, but opaque; by these they are decompounded into fringes, having the red outermost, and the violet innermost; and the fringes formed by each fibre being joined together, form the bow. How then does the pressure enlarge and vivify them? The fibres are naturally extended over the surface of a spherical segment; when this surface is compressed into a plane circle, they are condensed into a much less space, and consequently brought nearer to one another, the rays are therefore more inflected and separated than before. If this explanation be true, it will follow, that the like bows may be produced by small hairs, like fibres, placed near one another; and this I found perfectly consistent with fact; the bows are in this case brighter than in the other; and the small hairs on a hat, or the hand, made them brighter than any other I have tried: a circumstance which I observed in both cases, seems to show clearly the identity of the causes; the white space, which reached from the interior bow to the flame, was speckled or mottled, in a manner which cannot be easily described, but which any one will perceive upon trying the experiment.

* *Lect. Opticæ, Sect. III. ad finem.*

8. The last of these phænomena, which I shall mention, is the celebrated one observed by Sir ISAAC NEWTON, namely, the rings of colours with which the focus of a concave glass mirror is surrounded. Sir ISAAC made several most ingenious and accurate experiments to investigate their nature;* and finding their breadth to be in the inverse subduplicate ratio of the mirror's thickness, he concluded that they were of the same nature and original with those of thin plates, described by him.† The Duc de CHAULNES pursued these experiments with considerable success; he found that the rings were brighter the nearer to the perpendicular the rays were incident; and that if, instead of a concave glass mirror, a metal one was used, with a small piece of fine cambric, or reticulated silver wire stretched before it, the colours were no longer disposed in rings, but in streaks, of the same shape with the intervals between the threads; hence he concludes that they are owing to inflection; that in passing through the first surface, they are inflected and condensed by the second.‡ I am not, I own, quite satisfied with this account of the matter: that they are produced by inflection, the Duke's experiments put beyond doubt; but that they should be formed in passing through the first surface, and reflected by the second, is quite inconsistent with the ratio observed by their breadth, this being greater in the thinnest glass, and also with the order of the colours. Besides, all the coloured images which fall on the backside of the mirror, will be (by what we before found when speaking of flexibility §) reflected into a white focus; so

* Optics, Book II. Part IV.

† Book II. Parts I. and II.

‡ *Mém. de l'Académie, pour l'année* 1755.

§ Part II. Obs. 6 of this paper.

that, upon the whole, there appears every reason to believe that the rings are formed by the first surface, out of the light which, after reflection from the second surface, is scattered, and passes on to the chart. It will follow, 1. that a plane mirror makes them not, for the regularly reflected light, not being thrown to a focus, mixes with the decompounded scattered light, and dilutes it. 2. That the nearer to the perpendicular the rays are incident, the more light will be reflected to the focus, and consequently the less will dilute and weaken the rings. 3. That the thinner the mirror is, or the nearer the two surfaces are, the broader will the rings be. 4. That the rings farther from the focus will be broader. And lastly, that when homogeneous light is reflected, the fringes or images will be larger, and farther from one another, in red than in any other primary colour. All which is perfectly consistent with the experiments of Newton and Chaulnes. There is only one difficulty that may be started to this explanation: how happens it that the colours (made by the mirror) are always circular? We answer, it is owing to the manner of polishing the concave mirror, which is laid between a convex and concave plate, and then turned round (with putty or melted pitch) in the very direction in which the rings are. If it should be asked, why does the thickness of the mirror influence the breadth of the rings exactly in the inverse subduplicate ratio? We answer, that to a certain distance from the point of incidence (and the rays are never scattered far from it) this is demonstrable, to hold as a property of mathematical lines in general.

Having found that the fringes by flexion are images of the

luminous body,* I thought that, from this consideration, a method of determining the different degrees of flexibility of the different rays might be deduced, similar to that which I had formerly used for determining their degrees of reflexibility.† I therefore made the following experiment.

Obs. 12. Having let into my darkened chamber a strong beam of the sun's light, through a hole $\frac{1}{40}$th of an inch in diameter, I held a hair at four feet from the hole, and receiving the shadow at two feet from the hair, I drew a line across the middle of the coloured images, and pointed off in each the divisions of the colours, as nearly as I could observe; and repeating the observation several times and at different distances, I found, by the same way I had formerly done in my experiment on reflexibility, that the axis, or line, drawn through the middle of each, was divided inversely, according to the intervals of the cords which sound the notes in an octave, *ut, re, mi, sol, la, fa, si, ut.* But as the measures in these experiments were very minute, and the operations of consequence liable to inaccuracy, I thought proper to try the thing by another test.

Obs. 13. The sun shining into the room as before, I placed at the hole an hollow prism made of fine plate-glass, and filled with pure water, its refracting angle being 55°; the spectrum was thrown on an horizontal chart eight feet from the window, and at four feet from the prism there was placed, in the rays, a rough black pin $\frac{1}{20}$th of an inch in diameter. The shadow in the spectrum was bounded by hyperbolic sides, as before described; and drawing a line, which might be the axis

* Page 256. † Page 247.

of the shadow, and pass precisely through its middle, I marked on one side 6 or 8 points of the shadow's outline, in each set of rays; and this being often repeated, at different distances and in different shadows, the position of the axis remaining the same, the curves formed by joining the points were all parallel; which shows that each sine of inflection taken apart has a given ratio to the sine of incidence. I afterwards divided the axis according to the musical intervals, and thus found where each colour of the spectrum had terminated, in what colour each part of the shadows had been, and by what rays formed. Then I joined the parts that I had marked, and obtained a curve, which I took to be, either nearly or accurately, an hyperbola of the 4th order. I next measured the ordinates (the axis of the spectrum and shadow being the axis of the curve) at the confines of each colour; first, the ordinate at the extremity of the rectilinear red, then that at the confine of the red and orange, and so on to that at the extreme rectilinear violet; to each of these ordinates I added the greatest one, or that in the violet, which (in fig. 10.) is VV'; that is, I produced vV to V', so that vV' is equal to vV; and through V' I drew $V'R'$ parallel to the axis VR, and produced gG to G', and rR to R'; then from V' I set off $V'g'$ equal to $G'g$, and $V'r'$ equal to $R'r$, and the other ordinates in like manner; and I found, according to the method before described,* that VV' was divided inversely, after the manner of the musical intervals. It is therefore evident that the inflexibilities of the rays are directly as their deflexibilities, and reflexibilities, but inversely as their refrangibilities. The same may be proved, by measuring and dividing the images made in the inside

* Page 247.

of the shadows; these I have found to be, at equal incidences and distances, equal to the images on the outside, both in breadth, in distance from the edge of the shadow, and in the relation which their divisions bear to one another; wherefore, whatever be the ratio of the angle of inflection to that of incidence, the same is the ratio of the angle of deflection to that of incidence; so that the angle of deflection is equal to the angle of inflection. If farther proof of this proposition be desired, the following experiment and observations, which from the importance of the thing I do not scruple to add, may be sufficient.

Obs. 14. When two knife blades were placed by one another in a beam of light which entered the dark room, so that the one might form and the other distend the images, I made in one of the blades (with a file) a small dent, which, on the chart, cast an elliptic or semicircular outline; then I observed that the images of both blades were disturbed by it, and wound round the edges of the semicircle; and they were all affected in precisely the same manner and degree. So then the first knife deflected the images formed by the second, in precisely the same degree that it inflected those images which itself formed, and so of the other knife; otherwise the effect of the dent would have been different upon the two sets of images. We may therefore conclude, that the angles or sines of inflection and deflection, bear the same ratio to the angle or sine of incidence, and that they are equal to one another. My next object was to determine this ratio in one of these cases, and consequently in both; and it was very agreeable to find data for the solution of this problem in NEWTON's measurements of the images and shadow; since this philosopher's well

known accuracy in such matters, besides the singular ingenuity of the methods he employed, made me more satisfied with these than any experiment I could make on the subject. In fig. 11. CS is the line perpendicular to the chart SU, and passing through the centre of the body, whose half is CD or SE; EB is parallel to CS, and AI a ray incident at D; ADB or EDI is the angle of incidence; EDR that of the red's deflection; EDV that of the violet's; and EDG that of the intermediate's. According to NEWTON,* CD was $\frac{1}{500}$th of an inch, DE 6 inches, SI $\frac{1}{108}$th of an inch, RV $\frac{1}{170}$th, and consequently RG $\frac{1}{340}$th; GS was $\frac{1}{76}$; whence the angles IDE, EDV, EDG, and EDR, will be found to be 4′, 30″; 5′; 7′, and 9′, respectively. Now the natural sines of 4′, 30″; 5′; 7′, and 9′, are as the numbers 1309, 1454, 2035½, and 2617, which are as the sines of incidence, deflection, and inflection of the violet, green, and red. Thus the angles of flexion of the extreme and mean rays being given, those of the other rays are found by dividing the difference between 1454 and 2617 in the harmonical ratio: for then the red will be equal to $145\frac{3}{8}$; the orange $87\frac{9}{40}$; the yellow $155\frac{1}{15}$; the green $193\frac{5}{6}$; the blue $193\frac{5}{6}$; the indigo $129\frac{2}{9}$; and the violet $258\frac{4}{9}$; and by adding to the number 1454 the violet, and to their sum the indigo, and so on, we get the flexibility of the red, from 2617 to $2471\frac{5}{8}$; of the orange, from $2471\frac{5}{8}$ to $2384\frac{2}{5}$; of the yellow, from $2384\frac{2}{5}$ to $2229\frac{1}{3}$; of the green, from $2229\frac{1}{3}$ to $2035\frac{1}{2}$; of the blue, from $2035\frac{1}{2}$ to $1841\frac{2}{3}$; of the indigo, from $1841\frac{2}{3}$ to $1712\frac{4}{9}$; and of the violet, from $1712\frac{4}{9}$ to 1454: the common sine of incidence being 1309. It is therefore evident, that the flexibility of the red is not to that of the violet as the refrangi-

* Optics, Book III. Obs. 3.

bility of the violet to that of the red; and a little attention will convince us that we had no reason to expect the analogy should be kept up in this respect; for the refrangibility of the rays depends on the species of the refracting medium, and follows no general rule; whereas our calculation has been made concerning the action of the bending power at a certain distance, greater than that whereat the particles of media act on the rays in refracting them. It was observed, in the mathematical propositions prefixed to this paper, that the angle of flexion is less than that of incidence, when, in the case of inflection, the angle made by the ray and the body is acute, and when, in the case of deflection, that angle is obtuse; and when the ray is perpendicular or parallel, the angle of incidence vanishes in both cases. It is evident, therefore, that in both these situations of things the ratio of 1309 to 2036, being that of a less to a greater, will not enable us to find the angle of flexion, although it serves very well when the ray before inflection makes an obtuse, and before the deflection, an acute angle. I have therefore mentioned the angle made by the bent ray with the incident, which gives a general formula; for let the angle of incidence be I, and that which the bent ray makes with the incident B, then F being the angle of flexion, we have $F = B \pm I$; so that if $I = O$; $F = B$; or if the incident makes an obtuse angle with the body, in the case of deflection, and an acute in that of inflection, then $F = I - B$, and in the remaining case $F = I + B$.

These observations enable us to give a very short summary of optical science. When the particles of light pass at a certain distance from any body, a repulsive power drives them off; at a distance a little less, this power becomes attractive;

at a still less distance, it again becomes repulsive; and at the least distance, it becomes attractive as before; always acting in the same direction. These things hold whatever be the direction of the particles; but if, when produced, it passes through the body, then the nearest repulsive force drives the particles back, and the nearest attractive force either transmits them, or turns them out of their course during transmission. Farther, the particles differ in their dispositions to be acted upon by this power, in all these varieties of exertion; and those which are most strongly affected by its exertion in one case, are also most strongly affected by that exertion when varied; except in the cases of refraction, of which we before spoke; and these dispositions of the parts are in all the cases in the same harmonical ratio. Lastly, the cause of these different dispositions is the magnitude of the particles being various.

All that remains now to be done on this part of the subject is to explain one or two phænomena relating to reflexibility.

1. It has been remarked, that if we look at a candle, or other luminous body, with our eyes almost shut, bright streaks seem to dart upwards and downwards from it. NEWTON* explains this by refraction through the humours adhering to the eyelids. ROHAULT† and Mr. YOUNG‡ ascribe them to reflections. DES CARTES makes them arise from wrinkles on the eye's surface. DE LA HIRE from refraction through the moisture on the eyelids, as through a concave lens; and PRIESTLEY|| from inflection through the lashes. The truth of Sir ISAAC's explanation is obvious, because the streaks which dart from the top of the luminous body are formed

* *Lect. Opt. Sect. III. ad finem.*
† *Physica*, p. 249. CLARK's ed.
‡ Phil. Trans. 1793.
|| On Vision, Vol. II.

by the under eyelid, or at least by the moisture adhering to the under ciliary process, and those which appear from the bottom of the body, by the upper eyelid ; which could not be, either if they were formed by reflection from the processes, or by inflection through the lashes.

I have, however, observed another kind of streaks, mottled with broken colours of all kinds, and formed by reflection from the moisture on the processes; in these the under streak corresponds to the under process, and *vice versa:* they may be formed by any polished body held in the proper position between the pupil and luminous body. The colours are very beautiful when made by the sun, and resemble, in form and irregularity of arrangement, some of the streaks made by large half-polished bodies, as described in Part II. of this paper.

2. The next object of attention is one of the greatest importance to our theory, namely, the formation of images by reflection: three things here require explanation, the number of the images, their colours, and their variations in point of size.

Obs. 15. I have uniformly found that no reflecting surface forms them, except it be curve, and (its surface) of a structure somewhat fibrous. A plain mirror, nor a concave, nor a convex one do not make them, unless they are of that structure; and, for the same reason, quicksilver, when held so as to reflect the light incident upon it, forms them not, but by *triturating* it, so as to divide it into small particles, and by placing these in the beam of the sun's light, each particle formed an image, with the colours in the regular order and very bright: on holding a cylinder in the rays, and observing the lengths of the images, I found that if the curvature was increased, the

images were also increased in size, being more distended, and highly coloured. These things immediately suggest the explanation. Each of the small fibres forms an image, which, from the different reflexibility of the rays, is divided into the seven primary colours. But why does not a plain mirror form *one* of these upon the same principles? In fig. 12. let AE be the curve surface of a very convex mirror, that is of a small fibre; GC a ray reflected by the small surface DC; it will be separated into CI red, and CK violet, by the unequal action of FC on its parts. But if DC is continued to L in a straight line, then LC's sphere of reflection extending a little way beyond it, to KC, the part nearest to C, and not to IC, will drive KC and also the indigo and part of the blue nearer to the perpendicular; then IC being within LC's sphere of inflection, will, together with the orange, yellow, and part of the green, be brought nearer to KC; so that IC and KC will both be brought to an angle equal to that of incidence, and will be reflected in a parallel white beam. If LC is removed a little, or the surface becomes more convex, IC is attracted, and KC repelled, but not so much as to reduce them to parallelism and whiteness, an image being formed narrower and less coloured than when LC is moved so far round that KC is attracted, and IC deflected or repelled. If LC is moved round so that the mirror is concave, then KC is repelled, and IC attracted, as before, unless the curvature be considerable; and then KC and IC are both repelled, and an image formed in the *caustic* by reflection. In Obs. 3. we found that certain irregularities in the surface of the reflector caused the images to be in the inverted order of colours. How does this happen? In fig. 13. let *gf*, *fe*, *er*, *ri*, and *ib*, represent the sections of the con-

vex fibres on the surface of the reflector, and let the ray AB be reflected from *ef*, separated into B*r* red, and B*v* violet; then if AB was so inclined to *ef*, that B*r* and B*v* fell upon *er*, the side of the fibre next to *ef*, and a little larger than *ef*, it is evident that B*v* will be reflected into *v*V, and B*r* into *r*R, and an image VR will be formed, having the violet outermost and the red innermost, the intermediate colours being in their order, from V to R. Lastly, it is evident that the greater the angle of incidence is, the longer will be the image, and the farther separated its colours; for which reason the farther the images are from the shadow, the less dilated and coloured will they be. Nor will they have the same appearance at all distances from the point of incidence; very near it, they will be all in the form of fringes across the streak, the breadth being greater than the length (if I may use the expression), but as we recede from it, they will become distended, as before described, the length increasing faster than the breadth, and at one point or distance they will be just as long as broad; all which agrees with experiment; and it is needless to show by particular demonstration, the manner in which one image is divided from another, the reason obviously being the manner in which the fibres on the reflecting surface are arranged and inclined to one another.

3. A number of phænomena, involved in that of the images, are explicable by what has been said on them. If a piece of metal be scratched, and then exposed in the sunshine, a number of broken colours will be formed by the scratches, as may be seen either by letting them fall on the eye, or by receiving them on a white object. This is evidently owing to the different reflexibility of the rays incident on the scratches, which

are so many irregular specula, of great curvature; the images are therefore distorted and broken, just as a candle, &c. appears broken and coloured when viewed through a piece of irregular crystal, such as the bottom of a wine glass. If we look attentively at any object exposed in the light of the sun, provided it be not polished, we shall see its surface mottled with various points of colours, from the specular nature of its minute particles. If we look towards the sun, with a hat on our head, held down, so that the sun's direct light may not fall on our eyes, but on the hairs of the hat, and be reflected, we shall see a variety of lively colours darting in all directions from those hairs; and we may easily satisfy ourselves that they are not the consequence of flexion, by trying the same thing with unpolished threads, in which case they do not appear, provided the threads be not very small. In the same manner we may account for the colours of spider webs, of different cloths which change their colours when their position is altered, and of some fossils which appear of different streaks of colours when held in the light, such as the fire marble of Saxony, &c. All these bodies having surfaces of a fibrous structure, each fibre reflects and decompounds the rays.

4. The consideration of the foregoing phænomena inclined me to think, that upon the principles which have been laid down, the colours of natural bodies may be explained. The celebrated discovery of Newton, that these depend on the thickness of their parts, is degraded by a comparison with his hypothesis of the fits of rays and waves of æther. Delighted and astonished by the former, we gladly turn from the latter; and unwilling to involve in the smoke of unintelligible theory so fair a fabric, founded on strict induction, we wish to find

some continuation of experiments and observations which may relieve us from the necessity of the supposition. My speculations on this subject have by no means been completed, as I have not yet finished the demonstrations and experiments into which it has engaged me to enter; but, in order to complete my plan, I shall offer a few hints on the subject. The parts of light are affirmed, in Prop. III. Book I. Part I. of the Optics, to be different in reflexibility; that is, according to the author's definition, in disposition to be turned back, and not transmitted at the confines of two transparent media. That the demonstration involves a logical error appears pretty evident. When the rays, by refraction through the base of the prism used in the experiment, are separated into their parts, these become divergent, the violet and red emerging at very different angles, and these were also incident on the base at different angles, from the refraction of the side at which they entered; when, therefore, the prism is moved round on its axis, as described in the proposition, the base is nearest the violet, from the position of the rays by refraction, and meets it first; so that the violet being reflected as soon as it meets the base, it is reflected before any of the other rays, not from a different disposition to be so, but merely from its different refrangibility; although then this experiment is a complete proof of the different refrangibility of the rays, it proves nothing else; and indeed an experiment will convince us, that the rays all have the same disposition to be reflected, provided the angle of incidence be the same. For I held a prism vertically, and let the spectrum of another prism be reflected by the base of the former, so that the rays had all the same angle of incidence; then turning round the vertical prism on its axis, when one sort of rays

was transmitted or reflected, all were transmitted or reflected. We cannot therefore apply the different reflexibility of light, to the explanation of the colours of bodies, since this property has no existence. But we have shewn that the rays differ in *reflexibility*, taking the word in the new sense, as explained above; let us see whether this principle will not solve the important problem. It is evident that the particles of bodies are specular. Now I take the colours of bodies to depend, not on the size, but on the position of these particles, or at least on only the size in as far as it influences their position; an idea perfectly familiar to mathematicians.

Obs. 16. In making some of the experiments, which I related above on the reflexibility of light, I observed, among the regular images made by most of the pins which I used, one or two all of the same colour, as red, blue, *&c.* and when the pin was moved these moved also, becoming of other colours in regular order, like the rest; which shows plainly that their being of one colour at first was owing to some fibre in the surface jutting out, or rather to several of these, which stopped the red and all the rest but the blue of several images, or the blue and all the rest but the red. Farther, I produced several regular images by two or three very small pins, and with considerable trouble I at last contrived to place them in such a position as that one blue colour of considerable size might be produced, then a red, and so on, by altering the posture of the pins; now, whether the posture or the size be altered it matters not, for the one affects the other. Is it not evident that this experiment, and the conclusion to which it evidently leads, may be transferred to the colours of natural bodies as seen by reflection? for the parts being specular and

spherical, each will form an image of the luminous body; and by the position of the sides of the neighbouring ones, any six of the colours may be stopped, while the seventh emerges; and if this happens in one part, it will happen in all, since that the texture and size of the parts is the same throughout, has never been called in question. But it will be asked, how are the particles to reflect a mixture of different colours? We answer, that a particle having its sides concave, and front convex, will produce the effect; for the colours will be thus mixed in a proportion determined by the position of the others. How can whiteness and blackness be produced? If the particles be large, then the whole light incident on each will be reflected and separated, and all the images being compounded and mixed together, a confused sensation, or a sensation of white, will be the result. For the parts being transparent, and the images formed by the convex surface of the second row of particles, these will be larger in proportion to the thickness of the particles, or plates through which they have to pass before they meet with obstruction, and consequently will not be stopped by other particles; and in like manner the colour will be red if the particles are a little less, and so on. If the particles be very small, the light will be separated into images also small, with which, and with one another, the particles interfering, the light by many reflections and obstructions will be totally lost. How do bodies appear of their proper colours though no luminous body be shining, whose image may be formed by a reflection? They reflect images of the clouds, which reflect the sun's white light; for if we hold between our eye and a hole in the window, illuminated by the light of the clouds, a reflecting body, as a pin, &c. coloured images are

formed of the hole distended like those of the sun, as I have often found; and the same holds of inflection. Why does cutting a body to pieces not alter its colours? This only changes the position of masses of particles, not of the particles themselves; but if by bruising them we change their magnitude and position, we change also their colour; thus the leaves of vegetables bruised in a mortar, many paints powdered, &c. Why do many bodies change colours when viewed in different positions? Because they reflect two colours, or more, of each image to different quarters; and it matters not whether their position with respect to us or our position with respect to them be changed. How do bodies appear coloured by transmitted light? Because the foregoing reasonings apply also to the flexion of the rays in their passage through the parts of bodies. These observations appear to me to furnish a very simple solution of the problem. I shall endeavour, in a future communication, to confirm what has been said, by other remarks and experiments; for it would be tedious, and perhaps superfluous, to illustrate what has been said by figures and demonstrations.*

Pursuant to these remarks, it will not be difficult to account for the rings of colours of thin plates by reflection, as we before did those of thick plates by flexion; † indeed those formed in the experiment of the two lenses, supposed by NEWTON to

* It is obvious that the different refrangibility of the rays will not account for the bright and distinct colours of bodies: if the refracting angle of a prism be continually diminished, till, for example, it is equal to one of a minute, the refraction will produce no sensible colours; indeed almost every piece of plane glass has its sides in a small degree inclined to one another, and yet no colours are formed; much less then will refraction through the infinitely smaller parts of bodies, produce separation of the rays.

† Page 260 of this paper.

be owing to the plates of air between them, appear to have a different cause, as may be without much reasoning gathered from the curious experiments of the Abbé MAZEAS,* and even from one or two of Sir ISAAC'S own, in which he supposes some medium more subtile than air to be between the glasses.† But at present I forbear entering into the subject any farther: this paper has been already extended to a greater length than was at first intended. And I hasten to conclude, by a short summary of Propositions, containing the principal things which have been demonstrated in the course of it.

Prop. I. The angles of inflection and deflection are equal, at equal incidences.

Prop. II. The sine of inflection is to that of incidence in a given ratio (which is determined in the paper.)

Prop. III. The sun's light consists of parts which differ in degree of inflexibility and deflexibility, those which are most refrangible being least flexible.

Prop. IV. The flexibilities of the rays are inversely as their refrangibilities; and the spectrum by flexion is divided by the harmonical ratio, like the spectrum by refraction.

Prop. V. The angle of reflection is not equal to that of incidence, except in particular (though common) combinations of circumstances, and in the mean rays of the spectrum.

Prop. VI. The rays which are most refrangible are least reflexible, or make the least angle of reflection.

Prop. VII. The reflexibilities of the different rays are inversely as their refrangibilities, and the spectrum by reflection is divided in the harmonical ratio, like that by refraction.

* *Mém. de l'Académie pour l'année* 1738.

† Optics, Book II. Part I. Obs. 10 and 11.

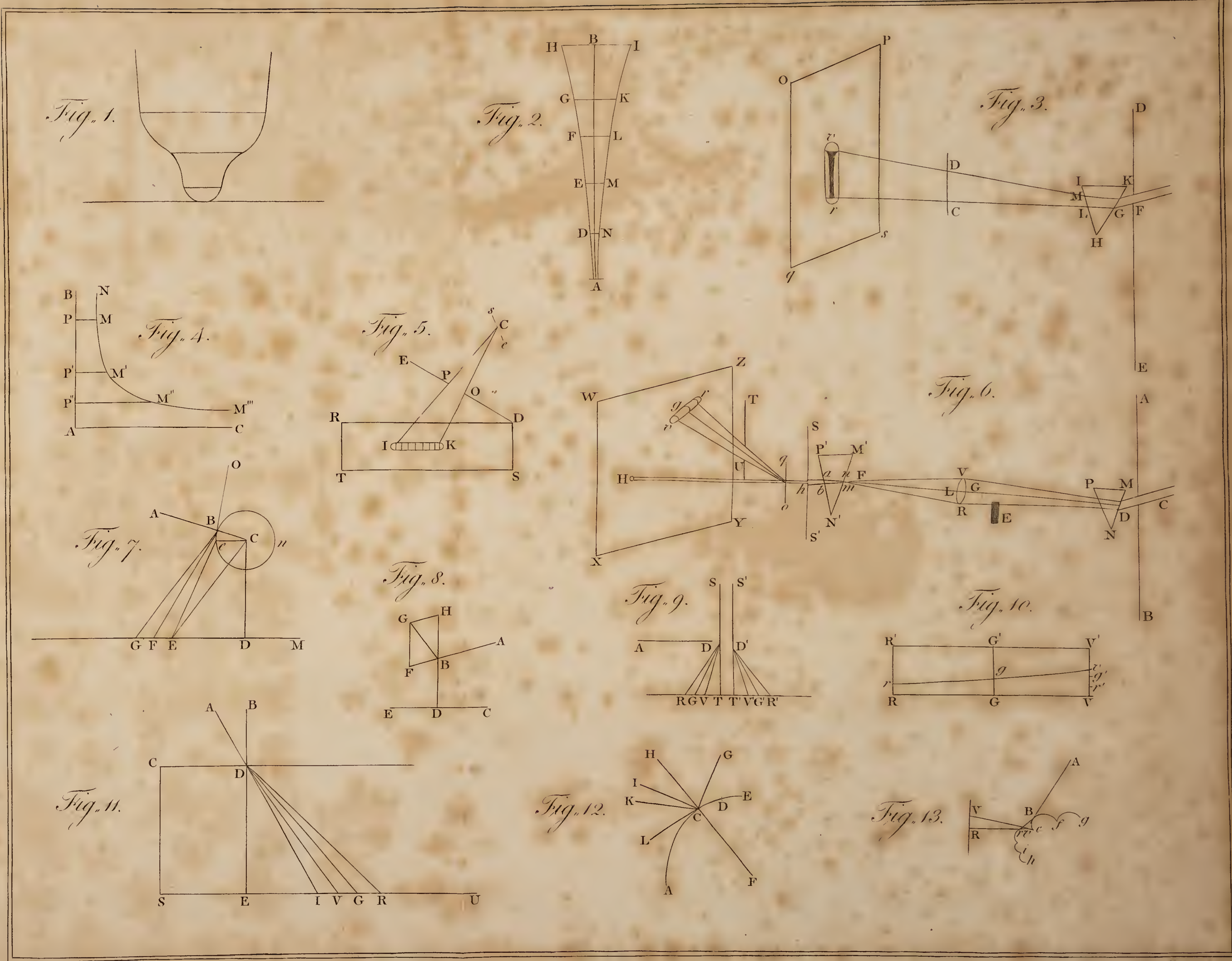
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.

Prop. VIII. The sines of reflection of the different rays are in given ratios to those of incidence (which are determined in the paper.)

Prop. IX. The ratio of the sizes of the different parts of light are found.

Prop. X. The colours of natural bodies are found to depend on the different reflexibilities of the rays, and sometimes on their flexibilities.

Prop. XI. The rays of light are reflected, refracted, inflected, and deflected, by one and the same power, variously exerted in different circumstances.

ERRATA.

Page 52, line 26, *dele* practically.

Page 61, line 5, *for* in general is, *read* is general, being.

Page 65, line 8, *for* point X, *read* horizontal line drawn through the point X parallel to the axis of motion.

Page 77, lines 1 and 9, *for* WGS, *read* UGS; and line 24, *for* WGO, *read* UGO.

Page 78, line 2, *for* VW, *read* VU.

Page 85, line 26, *for* B, *read* R.

Page 91, last line, *for* QA, *read* NF.

Page 96, last line, *for* prop. iii. *read* prop. ii.

Page 97, line 18, *for* GZ, *read* ΓZ.

Page 100, line 12, *for* horizontal line, *read* indefinite horizontal line.

Page 107, line 3 and 4, *dele* HD = HA.

Page 115, line 5, *for* AB — PX, *read* WP — PX, fig. 11. and 28.

Page 124, line 11, *for* is, *read* are.

Note to be added to page 104, last line, to the word "inquiry."

The following remark on the propositions and demonstrations of Apollonius Pergæus, equally, or rather more applicable to those of Archimedes, is extracted from Dr. Wallis's Algebra.

"Et quidem meritò censeri posset ille, magnus geometra, et prodigiosæ, tum phantasiæ tum memoriæ vir, si possibile putemus ut potuerit ille propositiones et demonstrationes perplexas, eo ordine quo ad nos perveniunt invenire, absque cujusmodi aliquâ *inveniendi arte* qualis est quam nos algebram dicimus."

Dr. Wallis's Algebra, cap. LXXVI.

Page 124, line 26, note to the words "first applied."

Pere Pardies and Chevalier Renaud published some partial observations on the theory of naval architecture rather before this period: but the treatise of M. L'Hoste seems to be the first work in which this subject is considered systematically, and at length.

Page 127, line 8, *for* whatever may have been, *read* whatever may be.

Page 135, line 7, insert *the Rev.* before Nevil.

Page 202, lines 28, 30, and 31, *for* w^1, w^2, w^3, *read* ω^1, ω^2, ω^3.

Page 205, line 27, *for* w, *read* ω.

METEOROLOGICAL JOURNAL,

KEPT AT THE APARTMENTS

OF THE

ROYAL SOCIETY,

BY ORDER OF THE

PRESIDENT AND COUNCIL.

METEOROLOGICAL JOURNAL

for January, 1795.

1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Jan. 1	22	8	0	22	47	30,17	77		ENE	1	Foggy.
	26	2	0	26	47	30,18	73		ENE	1	Foggy.
2	20	8	0	21	46	30,23	77		NE	1	Foggy.
	23,5	2	0	23,5	48	30,26	76		NE	1	Foggy.
3	13,5	8	0	14	44	30,35	75		E	1	Cloudy.
	20	2	0	20	44,5	30,37	73		S	1	Hazy.
4	13	8	0	13,5	42	30,47	73		E	1	Cloudy.
	23,5	2	0	23	44	30,44	73		E	1	Foggy.
5	19,5	8	0	22	42	30,31	75		S	1	Cloudy.
	33	2	0	32	45	30,26	76		SW	1	Fair.
6	26,5	8	0	28,5	43	30,26	75		W	1	Cloudy.
	35	2	0	32	47	30,28	77		W	1	Cloudy.
7	31	8	0	33	46	30,36	83		E	1	Cloudy.
	35	2	0	33	46,5	30,31	83		ESE	1	Cloudy.
8	32,5	8	0	33,5	45	30,10	79		E	1	Cloudy.
	36	2	0	36	47	30,01	74		ENE	1	Cloudy.
9	29	8	0	31	46	30,01	79		WNW	1	Cloudy.
	38	2	0	37	48	30,08	80		WNW	1	Cloudy.
10	29	8	0	29,5	46,5	30,41	77		NE	1	Cloudy.
	34,5	2	0	34	49	30,45	73		NE	1	Fine.
11	23	8	0	23,5	47	30,46	77		NE	1	Fair.
	31	2	0	31	48	30,44	75		WNW	1	Hazy.
12	21	8	0	22	46	30,40	75		WNW	1	Cloudy.
	33	2	0	29	48	30,38	74		WNW	1	Cloudy.
13	31	8	0	32	46	30,29	79		E	1	Snow.
	34	2	0	32	49	30,22	72		E	1	Fine.
14	22,5	8	0	23	43	30,18	73		E	1	Fine.
	29	2	0	28	44,5	30,18	66		ENE	1	Cloudy.
15	27	8	0	28	45	30,10	77		NE	2	Cloudy.
	30	2	0	25,5	46	30,14	66		NE	2	Cloudy.
16	21	8	0	22	42	29,91	67		NNE	2	Cloudy.
	29	2	0	29	46	29,78	66		NNE	2	Cloudy.

METEOROLOGICAL JOURNAL
for January, 1795.

1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Jan. 17	22	8	0	24	42	29,74	76		NE	1	Snow.
	28	2	0	26	44,5	29,76	72		NE	1	Cloudy.
18	25	8	0	26	42	29,60	74		NE	1	Snow.
	28,5	2	0	28	45	29,58	72		NNE	2	Cloudy.
19	20,5	8	0	22	41,5	29,65	77		NE	1	Cloudy.
	28,5	2	0	28,5	44	29,69	69		NE	1	Cloudy.
20	14	8	0	17	39	29,72	71		NE	1	Cloudy.
	23,5	2	0	23	40	29,71	70		NE	1	Cloudy.
21	19	8	0	19	38,5	29,77	74		NE	1	Snow.
	23	2	0	23	40	29,81	72		NE	1	Cloudy.
22	17	8	0	18	38,5	29,85	74		NNE	1	Cloudy.
	23	2	0	22,5	40,5	29,82	72		NNE	1	Hazy.
23	13	8	0	16	38	29,69	73		NE	1	Cloudy.
	20	2	0	20	40	29,69	70		NE	1	Fair.
24	14,5	8	0	21,5	38	29,94	73		NE	1	Cloudy.
	25	2	0	25	41	30,00	73		NW	1	Cloudy.
25	7	8	0	8	37	30,07	72		E	1	Foggy.
	21	2	0	21	39	30,08	71		NW	1	Hazy.
26	17,5	8	0	19	36	29,75	72		ESE	2	Cloudy.
	38	2	0	22	38	29,52	74		ESE	2	Cloudy.
27	40	8	0	43	39	29,19	89	0,183	SSW	1	Cloudy.
	46	2	0	46	43	29,18	90		SSW	1	Cloudy.
28	38	8	0	38	42	29,16	92	0,243	NE	1	Rain.
	38	2	0	34	44	29,44	88		NE	2	Cloudy.
29	24	8	0	24	42	30,02	75	0,050	NNE	1	Fair.
	29	2	0	29	43,5	30,01	76		NNE	1	Fine.
30	21	8	0	23	42	30,13	83		NNE	1	Foggy.
	26,5	2	0	26,5	44	30,16	80		N by E	1	Cloudy.
31	19,5	8	0	21	41	30,13	81		ENE	1	Foggy.
	33	2	0	33	44	30,06	72		ESE	1	Cloudy.

METEOROLOGICAL JOURNAL
for February, 1795.

1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
	°	H.	M.	°	°	Inches.		Inches.	Points.	Str.	
Feb. 1	32	7	0	33	42,5	29,65	86		ESE	1	Foggy.
	41	2	0	36	45	29,30	90		ESE	1	Rain.
2	32	7	0	33	43	29,29	73	0,330	W	2	Cloudy.
	35	2	0	34,5	45	29,32	75		WNW	2	Cloudy.
3	30	7	0	26	44	29,22	81		SW	2	Snow.
	35,5	2	0	35	46	29,12	74		WSW	2	Cloudy.
4	27	7	0	27	42	29,08	75		WSW	1	Cloudy.
	35,5	2	0	35,5	47	29,25	64		W	2	Fine.
5	27	7	0	30	44	29,60	75		WNW	1	Cloudy.
	36,5	2	0	36,5	47	29,72	70		WNW	1	Fine.
6	26	7	0	27	44,5	29,98	76		WNW	1	Cloudy.
	34	2	0	34	47	29,95	73		E	1	Cloudy.
7	31	7	0	31,5	44	29,55	75		E	1	Cloudy.
	35,5	2	0	35	48	29,46	73		E	1	Fair.
8	33	7	0	39	46	29,27	88		SE	1	Cloudy.
	44	2	0	43,5	49	29,27	89		S	2	Cloudy.
9	40	7	0	45	48	29,22	91	0,160	S	2	Rain.
	51	2	0	50	50,5	29,17	90		S b. W	2	Rain.
10	48	7	0	50,5	50	29,16	86	0,505	S	2	Rain.
	51	2	0	51	53	29,04	81		S	2	Cloudy.
11	47	7	0	48	53	29,16	82	0,080	SSW	2	Cloudy.
	51	2	0	51	55	29,10	73		SSW	2	Fair.
12	45	7	0	45	54	29,10	78		WSW	2	Cloudy.
	44	2	0	44	54	29,27	75		SW	2	Cloudy.
13	31	7	0	32	51	29,24	88		NW	2	Snow.
	37,5	2	0	37	52	29,42	69		NE	2	Cloudy.
14	30	7	0	30	51	29,97	72	0,060	NE	1	Cloudy.
	37	2	0	37	53	30,10	70		NE	1	Fine.
15	28	7	0	28	50	30,47	77		NE	1	Fair.
	38	2	0	38	51,5	30,51	73		W	1	Hazy.
16	34	7	0	34	50	30,66	84		NNE	1	Cloudy.
	37	2	0	36	50	30,68	79		E	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	o	o	Inches.		Inches.	Points.	Str.	
	o										
Feb. 17	28	7	0	29	49	30,60	82		E	1	Cloudy.
	35	2	0	33	49	30,50	81		E	1	Cloudy.
18	28	7	0	29	47	30,39	67		E	2	Cloudy.
	32	2	0	31	48	30,32	64		E	2	Cloudy.
19	26	7	0	27	44	30,08	66		NE	2	Cloudy.
	30	2	0	28	44,5	29,99	67		NE	2	Snow.
20	24	7	0	25	43	29,90	66		NE	2	Cloudy.
	27	2	0	26	45	29,78	74		NE	2	Snow.
21	24	7	0	25	42	29,73	70		E	1	Cloudy.
	32	2	0	31	45	29,73	70		ESE	1	Fair.
22	32	7	0	32	42,5	29,55	73		E	1	Cloudy.
	37	2	0	37	45,5	29,45	74		E	2	Cloudy.
23	35,5	7	0	39	45	29,52	88	0,035	SSW	2	Cloudy.
	46	2	0	44	48,5	29,57	80		S	2	Cloudy.
24	37	7	0	39	47	29,57	80		SSW	1	Cloudy.
	44	2	0	44	50	29,60	77		WNW	2	Cloudy.
25	38,5	7	0	39	48	29,48	83		SSW	1	Cloudy.
	46	2	0	46	51	29,38	74		SSE	2	Cloudy.
26	38	7	0	38	48	29,18	83		ESE	1	Cloudy.
	43	2	0	43	51	29,05	78		ESE	2	Cloudy.
27	37	7	0	37,5	49	29,06	82	0,085	NE	2	Cloudy.
	41	2	0	41	51	29,18	70		NNE	1	Cloudy.
28	31	7	0	31	49	29,41	78		E	1	Snow.
	34	2	0	34	50	29,23	80		E	1	Snow.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
	°	H.	M.	°	°	Inches.		Inches.	Points.	Str.	
Mar. 1	24	7	0	25	47	29,72	71	0,160	N	1	Fine.
	34	2	0	33,5	50,5	29,90	67		NW	1	Fine.
2	28	7	0	34	48	29,97	73		SW	1	Cloudy.
	38	2	0	36	49	29,86	73		SE	2	Cloudy.
3	34	7	0	34,5	48	30,19	74		E	1	Cloudy.
	37	2	0	36,5	49	30,23	72		W	1	Cloudy.
4	34	7	0	38	48	30,00	85		SSW	1	Cloudy.
	49	2	0	48,5	53	29,92	79		W	1	Cloudy.
5	40,5	7	0	44	50	29,84	85	0,060	S	1	Rain.
	51,5	2	0	50	53	29,68	71		S	2	Cloudy.
6	46	7	0	48	52	29,15	82	0,031	WSW	2	Cloudy.
	50,5	2	0	48	56	29,34	68		W	2	Cloudy.
7	43,5	7	0	43,5	52,5	29,57	72		NW	2	Cloudy.
	45	2	0	44	55	29,81	70		NW	2	Cloudy.
8	33	7	0	35	52	30,28	69		N	2	Fine.
	44	2	0	43,5	55	30,35	61		NNE	2	Fair.
9	37	7	0	38	52,5	30,16	78		S	1	Cloudy.
	46	2	0	43	54	29,95	80		S	1	Rain.
10	38	7	0	39	52	29,59	81	0,268	SW	1	Fair.
	48	2	0	47	55	29,46	68		SSW	2	Cloudy.
11	37	7	0	37	52	29,22	78	0,141	WSW	1	Fair.
	47	2	0	44	55	29,26	65		W	1	Fair.
12	34	7	0	35	52	29,35	76		WSW	1	Cloudy.
	45	2	0	44	55	29,28	72		SE	2	Cloudy.
13	33,5	7	0	35	53	29,14	80	0,295	NE	1	Cloudy.
	38	2	0	37	53	29,25	74		NE	1	Cloudy.
14	32	7	0	33	50	29,64	73		E	1	Cloudy.
	39	2	0	38,5	52	29,60	69		E	2	Cloudy.
15	33	7	0	36	50	29,54	80	0,116	WSW	1	Cloudy.
	45	2	0	44,5	52,5	29,58	63		SW	1	Fair.
16	33	7	0	38	50	29,13	85	0,238	E	1	Rain.
	46	2	0	46	53	29,02	85		SSE	1	Rain.

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1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom. Inches.	Hygrometer.	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
	°			°	°						
Mar. 17	36	7	0	36	51	29,40	74	0,389	NW	1	Cloudy.
	38	2	0	36,5	52	29,40	73		NW	1	Cloudy.
18	34	7	0	35,5	50	29,56	78		NE	2	Cloudy.
	38	2	0	37	50,5	29,74	71		NE	2	Cloudy.
19	30	7	0	31	49,5	30,14	75		NE	1	Cloudy.
	43	2	0	43	51	30,17	65		W	1	Fair.
20	35	7	0	38	50	30,21	73		WNW	1	Fair.
	48,5	2	0	48,5	52	30,27	67		NW	1	Cloudy.
21	34	7	0	36	51	30,22	76		W	1	Fair.
	50	2	0	50	54	30,08	64		WSW	1	Fair.
22	39	7	0	40	52	30,07	76		NW	1	Cloudy.
	49	2	0	49	55,5	30,11	63		NNE	1	Cloudy.
23	34	7	0	35	52	30,15	77		W	1	Hazy.
	45,5	2	0	45	55	30,10	71		W	1	Hazy.
24	38	7	0	39,5	53	30,00	75		WSW	1	Cloudy.
	50	2	0	50	55	29,93	66		SW	1	Cloudy.
25	37	7	0	38	53	29,82	75		S	1	Fair.
	49	2	0	48	55	29,78	63		S	1	Cloudy.
26	37	7	0	39	53	29,81	75		NE	1	Fair.
	50,5	2	0	49	57	29,84	63		NE	1	Cloudy.
27	35	7	0	38	53	29,90	72	0,046	NE	1	Cloudy.
	46	2	0	45	57	29,90	67		NE	1	Fair.
28	34	7	0	36	53	30,00	74		ENE	1	Cloudy.
	48	2	0	48	57	30,03	69		ENE	1	Fair.
29	35	7	0	37	54	30,02	76		E	1	Cloudy.
	50	2	0	49,5	57	30,02	70		E	1	Fine.
30	38	7	0	39	54,5	30,00	75		SSE	1	Cloudy.
	51	2	0	51	58	29,98	65		S by E	2	Fair.
31	38	7	0	41	55	29,95	77		ESE	1	Fine.
	54,5	2	0	54,5	59	29,90	56		ESE	1	Fine.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
	o	H.	M.	o	o	Inches.		Inches.	Points.	Str.	
April 1	36,5	7	0	37,5	54,5	29,93	76		E	1	Cloudy.
	48	2	0	47	59	29,91	74		ESE	1	Fair.
2	36	7	0	37	54	29,87	78		ESE	1	Cloudy.
	48	2	0	45	57	29,86	74		ESE	1	Fair.
3	38	7	0	39	55	29,82	80		ESE	1	Cloudy.
	51,5	2	0	50	58	29,83	70		ESE	1	Fair.
4	37	7	0	38,5	55	30,08	70		NE	1	Cloudy.
	44	2	0	42	56	30,10	70		NE	2	Cloudy.
5	39	7	0	39	54	30,09	74		NE	1	Cloudy.
	44	2	0	42,5	56	30,08	72		NE	1	Cloudy.
6	38,5	7	0	40	55	30,06	75		NE	1	Cloudy.
	43	2	0	43	56	30,04	76		NE	1	Rain.
7	37,5	7	0	39	54	29,95	74	0,033	NE	1	Cloudy.
	43	2	0	43	56	29,89	71		NE	1	Cloudy.
8	37,5	7	0	38	53	29,84	72		NNW	1	Cloudy.
	47,5	2	0	44,5	57	29,80	69		NE	1	Cloudy.
9	36	7	0	38	54	29,84	76		NE	1	Cloudy.
	46	2	0	45	55,5	29,87	75		NE	1	Cloudy.
10	39	7	0	40	53,5	30,00	79		NE	1	Cloudy.
	58	2	0	56	58	30,00	71		E	1	Fine.
11	42	7	0	44	56	30,04	79		NE	1	Cloudy.
	60	2	0	60	59	30,04	71		E	1	Fine
12	47	7	0	47,5	57	30,02	79		E	1	Cloudy.
	56	2	0	55	59	30,02	70		NE	1	Cloudy.
13	42,5	7	0	44	57	30,00	70		NE	1	Cloudy.
	49	2	0	49	58	30,07	65		NE	1	Cloudy.
14	39	7	0	41,5	57	30,22	72		E	1	Fair.
	61	2	0	61	60	30,20	61		S	1	Hazy.
15	50	7	0	51	58	30,15	75		SW	1	Cloudy.
	58,5	2	0	58	59,5	30,08	71		SSW	1	Cloudy.
16	44	7	0	47	57	29,93	72		S b. W	1	Fair.
	59,5	2	0	58,5	60	29,88	65		SSW	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.		Inches.			
Apr. 17	44,5	7	0	46	58	29,88	70		SW	1	Fair.
	56	2	0	56	60,5	29,88	54		WSW	1	Fair.
18	43	7	0	45	57	29,82	70		SSW	2	Fair.
	55	2	0	54	58	29,66	62		S	2	Cloudy.
19	46	7	0	47	57	29,41	71	0,033	SSE	2	Fair.
	56	2	0	56	58	29,38	61		SE	2	Fair.
20	46	7	0	46	57	29,36	74	0,034	E	1	Rain.
	55	2	0	55	59	29,34	66		S	1	Cloudy.
21	43	7	0	45	57,5	29,35	72	0,071	S	2	Fair.
	56	2	0	55	59	29,34	61		S	2	Cloudy.
22	41	7	0	42	58	29,35	73	0,032	SSW	1	Hazy.
	57	2	0	56	59,5	29,35	58		W	1	Fair.
23	39	7	0	41	58	29,49	68		SW	1	Fine.
	56	2	0	49	59	29,49	69		W	1	Cloudy.
24	44	7	0	46	58	29,48	70	0,186	NE	1	Cloudy.
	58	2	0	58	61	29,60	58		W	2	Fair.
25	48	7	0	49	58	29,40	75	0,037	SW	2	Cloudy.
	59	2	0	55	60	29,50	66		SW	2	Cloudy.
26	45	7	0	47	58	29,73	67		WSW	2	Fair.
	58	2	0	57,5	61,5	29,87	60		WSW	2	Fair.
27	47	7	0	48	58	29,91	69		SSW	2	Cloudy.
	57,5	2	0	56	60	29,91	65		SSW	2	Cloudy.
28	48	7	0	48	58,5	29,60	69		S	2	Cloudy. much wind last night.
	55	2	0	52	60	29,38	69		S	2	Cloudy.
29	43	7	0	45,5	58	29,59	70	0,071	SW	2	Cloudy.
	57,5	2	0	56	60	29,64	60		SW	2	Fair.
30	48	7	0	52	58,5	29,48	68		S	2	Cloudy.
	57	2	0	56	59,5	29,53	62		SSW	2	Fair.

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1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer.	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
	°										
May 1	44	7	0	47	58	29,76	68	0,035	SSW	2	Fine.
	61	2	0	61	62	29,81	54		SSW	2	Fine
2	44	7	0	49	59,5	29,99	69		S	2	Hazy.
	60	2	0	60	62	30,04	60		NE	2	Hazy.
3	40	7	0	43	59	30,28	70		ENE	1	Hazy.
	63	2	0	63	63	30,35	60		NE	1	Fine.
4	44	7	0	46	60	30,38	71		NE	1	Hazy.
	64	2	0	63	63	30,36	60		NE	1	Hazy.
5	48	7	0	53	61	30,34	71		SSW	1	Fair.
	72,5	2	0	72	63	30,30	60		NE	1	Fine.
6	55	7	0	56	62,5	30,37	70		NE	1	Cloudy.
	64,5	2	0	63	63	30,37	64		NE	1	Cloudy.
7	46	7	0	49	61	30,33	68		NE	1	Hazy.
	68,5	2	0	68	63	30,25	54		W	1	Fine.
8	54	7	0	55	63	30,23	62		NE	1	Fine.
	56	2	0	56	63	30,38	55		NE	1	Fine.
9	37	7	0	40,5	61	30,48	60		NE	1	Fine.
	59	2	0	58	61	30,37	55		W	1	Fine.
10	47	7	0	51	60,5	30,19	65		W	1	Fine.
	70	2	0	70	63	30,08	56		W	2	Fine.
11	48	7	0	50	61	29,92	63	0,030	NW	2	Fair.
	58	2	0	57,5	62,5	29,98	59		NW	2	Fair.
12	39	7	0	42	59	30,30	62	0,038	NE	2	Fair.
	56	2	0	56	61	30,29	57		N	2	Fair.
13	47	7	0	49	59	30,17	62		NW	2	Cloudy.
	58	2	0	56	59	30,13	61		NNW	1	Cloudy.
14	43	7	0	45	57	30,22	63		NE	1	Cloudy.
	56	2	0	56	57	30,22	60		ESE	1	Cloudy.
15	44	7	0	47	57,5	29,98	68	0,028	W	1	Cloudy.
	58	2	0	57,5	58	29,87	60		WSW	2	Cloudy.
16	41	7	0	43,5	57,5	29,94	67		NW	2	Cloudy.
	55	2	0	55	58	29,97	57		WNW	1	Hazy.

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	°	H.	M.	°	°	Inches.		Inches.	Points.	Str.	
May 17	45	7	0	46	57	29,98	61		E	1	Fair.
	63,5	2	0	62	59	29,91	59		ESE	1	Fair.
18	48	7	0	52	58	29,92	65		S	1	Fine.
	74	2	0	74	61	29,93	56		S	1	Fine.
19	52,5	7	0	54	60	30,07	70		SW	1	Fine.
	71,5	2	0	71	62	30,11	55		WSW	1	Fine.
20	49	7	0	54	61	30,28	67		WSW	1	Hazy.
	69,5	2	0	68,5	63	30,42	56		N	1	Hazy.
21	52	7	0	55	62	30,49	68		SE	1	Fine.
	77	2	0	76	64	30,46	56		SSW	1	Fine.
22	56	7	0	60	64	30,45	62		W	1	Hazy.
	78	2	0	76	68	30,40	56		NW	1	Fine.
23	60	7	0	64	66,5	30,29	64		W	1	Fine.
	81,5	2	0	81	68	30,20	47		NW	1	Fine.
24	50	7	0	53	67	30,16	65		E	1	Cloudy.
	60	2	0	60	67	30,19	56		NE	1	Cloudy.
25	45	7	0	47	65	30,24	59		NE	1	Cloudy.
	56	2	0	55,5	64	30,22	56		E	1	Fine.
26	42	7	0	45	62	30,24	60		NE	1	Cloudy.
	55	2	0	55	61,5	30,24	55		NE	1	Cloudy.
27	36	7	0	43	60	30,37	61		NE	2	Fair.
	53	2	0	53	58	30,37	56		NE	2	Cloudy.
28	38	7	0	43	58	30,42	60		NE	2	Fine.
	56	2	0	56	58	30,30	56		N	2	Fair.
29	46	7	0	48	58	29,85	79	0,145	W	1	Rain.
	61,5	2	0	61	59	29,73	61		NW	2	Fair.
30	45	7	0	46	58	29,90	65		NW	1	Cloudy.
	57	2	0	57	59	29,93	57		NW	1	Cloudy.
31	49	7	0	50	58	29,90	70		SSW	2	Cloudy.
	60	2	0	58	59	29,84	64		S	2	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
	°	H.	M.	°	°	Inches.		Inches.	Points.	Str.	
June 1	50	7	0	53	58	29,68	66		S	2	Cloudy.
	66	2	0	65	59,5	29,60	58		SSE	2	Cloudy.
2	50	7	0	53	59	29,57	68	0,054	S	2	Cloudy.
	67	2	0	66	61	29,64	57		SSW	2	Fair.
3	52	7	0	52	59	29,84	67		S b. W	2	Cloudy.
	68,5	2	0	68	61	29,82	56		ESE	1	Cloudy.
4	55	7	0	58	60,5	29,72	70	0,046	SE	1	Fine.
	77,5	2	0	76	63,5	29,71	56		SE	1	Fair.
5	59	7	0	62	63	29,78	80	0,473	E	1	Hazy.
	77	2	0	76	65,5	29,75	60		WSW	1	Cloudy.
6	59	7	0	61	64	29,64	79		SW	1	Cloudy.
	76	2	0	70	66	29,60	71		SW	1	Rain.
7	55	7	0	55	65	29,69	73	0,060	N	1	Cloudy.
	59	2	0	59	64	29,75	72		N	1	Cloudy.
8	54	7	0	54	64	29,92	87	0,254	N	1	Rain.
	57	2	0	56	63	29,98	83		NE	1	Rain.
9	51	7	0	53	63	30,05	85	0,505	NE	1	Rain.
	63	2	0	62	63,5	30,10	77		NE	1	Cloudy.
10	50	7	0	52	62,5	30,09	83		ENE	1	Cloudy.
	60	2	0	58	62	30,09	78		NE	1	Cloudy.
11	50	7	0	52	62	30,02	84		NE	1	Cloudy.
	66	2	0	65	63	29,93	68		NE	2	Cloudy.
12	51	7	0	51	62	29,97	73		NE	2	Cloudy.
	56	2	0	56	61,5	30,01	68		NE	2	Cloudy.
13	46	7	0	47	60	30,00	72		NE	1	Cloudy.
	58	2	0	54	60	29,98	71		NE	1	Cloudy.
14	45	7	0	47	59,5	29,91	73		NE	1	Cloudy.
	60	2	0	59	60	29,87	66		NE	2	Cloudy.
15	47	7	0	49	59	29,87	71		NE	1	Cloudy.
	66	2	0	63	60,5	29,87	67		NE	1	Cloudy.
16	52	7	0	53	60	29,99	74		NNW	1	Cloudy.
	60	2	0	59	60	30,03	73		WNW	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
June 17	55,5	7	0	57	60	30,05	83		WNW	1	Cloudy.
	59,5	2	0	59	60	30,04	78		E	1	Cloudy.
18	52	7	0	53,5	60	29,96	81	0,043	NE	1	Cloudy.
	54	2	0	53	60	29,90	83		E	1	Rain.
19	41	7	0	41	58	29,95	90	1,490	NE	1	Rain.
	50	2	0	49	57,5	29,94	69		NNE	2	Cloudy.
20	44,5	7	0	46	56	30,02	68	0,032	NE	2	Cloudy.
	52	2	0	51	56	30,03	64		NE	2	Cloudy.
21	42	7	0	44	56	30,14	70		NE	2	Cloudy.
	62	2	0	62	58	30,12	59		NE	1	Fair.
22	49	7	0	52	57	30,08	67		WNW	1	Fair.
	63	2	0	63	58	30,05	58		WNW	1	Cloudy.
23	48	7	0	51	57	29,97	71		SW	1	Cloudy.
	64,5	2	0	61	58	29,98	62		WNW	1	Cloudy.
24	46	7	0	51	58	30,00	70	0,040	W	1	Fair.
	68	2	0	68	59,5	30,00	56		W	1	Cloudy.
25	54	7	0	55	59,5	29,78	73	0,173	WSW	2	Cloudy.
	65	2	0	63	60,5	29,75	73		SW	2	Cloudy.
26	56	7	0	57	59,5	29,50	75	0,065	SW	2	Cloudy.
	69,5	2	0	67	61,5	29,63	63		WNW	1	Cloudy.
27	53	7	0	55	60,5	29,85	75		E	1	Cloudy.
	64	2	0	63	62	29,83	68		E	1	Cloudy.
28	51	7	0	53,5	61	29,71	72	0,062	SSE	2	Cloudy.
	60	2	0	58	61	29,62	70		S	2	Rain.
29	57	7	0	56	61	29,54	72	0,042	S	2	Cloudy.
	68	2	0	67	62	29,57	62		SSW	2	Cloudy.
30	52	7	0	56	61	29,66	68		SW	2	Cloudy.
	62	2	0	56	61	29,54	76		SSE	2	Rain.

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for July, 1795.

1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
	°	H.	M.	°	°	Inches.		Inches.	Points.	Str.	
July 1	48	7	0	52	60	29,54	76	0,276	SW	2	Cloudy.
	63	2	0	61	62	29,57	69		SSW	2	Rain.
2	50	7	0	52	61	29,94	71	0,172	NW	1	Cloudy.
	65	2	0	64,5	61	29,98	66		SE	1	Cloudy.
3	53	7	0	54	61	29,93	74		E	1	Cloudy.
	66	2	0	66	62	29,87	66		ESE	1	Cloudy.
4	55	7	0	56	61	29,81	86	0,263	NE	1	Rain.
	60	2	0	59	62	29,96	72		NE	1	Rain.
5	46	7	0	51	61	30,20	66	0,143	NW	1	Fine.
	67	2	0	66	62	30,16	66		N	1	Fair.
6	48	7	0	52	61	30,22	70		N	1	Fine.
	63	2	0	62	62	30,24	61		NE	1	Cloudy.
7	50	7	0	51	61,5	30,26	74		E	1	Fair.
	65	2	0	63	62	30,26	64		NE	1	Fair.
8	51	7	0	51,5	62	30,26	72		NE	1	Cloudy.
	62	2	0	62	62	30,26	66		NE	1	Cloudy.
9	50,5	7	0	52	62	30,23	69		NE	1	Fair.
	64	2	0	63	62,5	30,17	61		NE	1	Fair.
10	51	7	0	52	61	30,13	75		NE	1	Cloudy.
	60	2	0	60	62,5	30,12	65		NE	1	Cloudy.
11	50	7	0	51	61	30,08	71		NE	1	Cloudy.
	61	2	0	58,5	61,5	30,05	66		NE	1	Cloudy.
12	50	7	0	51	60	30,00	72		NE	1	Cloudy.
	58	2	0	56,5	60,5	29,99	69		NE	1	Cloudy.
13	51,5	7	0	53	60	29,94	68		NE	2	Cloudy.
	56	2	0	56	60	29,96	66		NE	2	Cloudy.
14	49	7	0	52	59	30,12	68		NE	2	Cloudy.
	66	2	0	66	61	30,12	59		NE	1	Fine.
15	57	7	0	57	61	30,14	70		NW	1	Fair.
	64	2	0	58	61	30,11	75		NE	1	Rain.
16	47	7	0	53	60	30,12	71	0,033	NW	1	Cloudy.
	69,5	2	0	68,5	62	30,06	60		WNW	1	Fair.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
July 17	58	7	0	60	61	29,87	71		WNW	1	Cloudy.
	65	2	0	63,5	62	29,87	65		NW	2	Cloudy.
18	52	7	0	53	61	30,00	65		WNW	1	Cloudy.
	66,5	2	0	65	62	29,98	63		WSW	1	Cloudy.
19	58	7	0	58	62	30,00	69		WSW	1	Cloudy.
	73	2	0	71,5	64,5	30,00	58		SW	1	Fair.
20	55	7	0	56,5	63	29,98	70		SW	1	Fine.
	76	2	0	74	67	29,94	60		S	2	Fine.
21	59	7	0	62	64,5	29,81	69		NE	1	Fair.
	76	2	0	75	68	29,72	63		E	1	Fair.
22	58	7	0	60	66	29,67	72	0,050	SW	2	Cloudy.
	69	2	0	67	66	29,69	66		SW	2	Cloudy.
23	53	7	0	54	64	29,78	66		W	2	Cloudy.
	66	2	0	65	64,5	29,76	64		W	2	Fair.
24	51	7	0	54	64	29,76	71		SW	1	Cloudy.
	65,5	2	0	61	64	29,64	64		SW	2	Cloudy.
25	51	7	0	53	64	29,76	70	0,055	S	1	Fair.
	63	2	0	61	64	29,78	70		ESE	1	Fair.
26	52	7	0	53,5	63	30,04	71	0,240	NE	1	Fair.
	69,5	2	0	68	64,5	30,05	59		NW	1	Fine.
27	53	7	0	56	64	30,05	67		SW	1	Hazy.
	66	2	0	63	64	30,05	70		SW	1	Cloudy.
28	61	7	0	62	64	29,94	78	0,060	WSW	1	Cloudy.
	72,5	2	0	70	66	29,98	67		WSW	1	Cloudy.
29	61	7	0	63	65,5	30,04	71		SW	1	Cloudy.
	70	2	0	70	67	29,97	64		SW	2	Cloudy.
30	58	7	0	60	65,5	29,89	76	0,077	SW	1	Cloudy.
	72,5	2	0	72	67	29,89	59		W	1	Fair.
31	55	7	0	58	66	29,84	71	0,031	SW	1	Fine.
	70	2	0	69	66,5	29,82	64		WSW	1	Fair.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Aug. 1	54	7	0	57	65	29,78	75	0,038	S	2	Rain.
	64	2	0	63	65	29,71	72		S	2	Cloudy.
2	57	7	0	59	65	29,64	75	0,225	SSW	2	Cloudy.
	67,5	2	0	67	65	29,65	64		SSW	2	Cloudy.
3	56	7	0	58	64,5	29,88	71	0,163	WSW	2	Fair.
	70,5	2	0	70	66	29,97	59		WSW	2	Fair.
4	57	7	0	58	65,5	30,01	78	0,170	S	1	Rain.
	65	2	0	61	65,5	29,96	75		E	1	Cloudy.
5	52	7	0	54	64,5	30,13	77		SW	1	Fair.
	70	2	0	68	67	30,13	64	0,270	E	1	Fair.
6	57	7	0	59	65	30,12	77		E	1	Cloudy.
	70	2	0	69	68	30,12	62		E	1	Fine.
7	54	7	0	58	65	30,02	73		E	1	Cloudy.
	66	2	0	65	65,5	29,96	70		S	2	Cloudy.
8	56	7	0	58	65	29,96	73		S	2	Fair.
	71	2	0	70	66	29,95	58		S	1	Cloudy.
9	56	7	0	58	65,5	29,97	72	0,053	S	1	Fair.
	71	2	0	70	66	29,97	62		SW	1	Fair.
10	59,5	7	0	61	66	29,99	74		WSW	1	Fair.
	73	2	0	71	68	29,98	63		WSW	1	Fair.
11	64	7	0	64,5	67	30,01	75		WSW	1	Cloudy.
	75	2	0	74,5	68,5	30,04	66		SW	1	Cloudy.
12	66	7	0	68	68	30,01	75		WSW	1	Cloudy.
	79	2	0	78	70	29,96	64		SE	1	Cloudy.
13	62	7	0	64	69	29,86	78		E	1	Hazy.
	79	2	0	78	73	29,78	64		E	1	Fine.
14	64	7	0	65	70	29,75	81	0,265	WSW	1	Cloudy.*
	73	2	0	72,5	71,5	29,80	69		WNW	1	Cloudy.
15	61	7	0	63	70	29,82	76		NW	1	Cloudy.
	72,5	2	0	69	69,5	29,82	70		NW	1	Cloudy.
16	53	7	0	55	69	29,77	75		SSW	1	Hazy.
	71	2	0	71	69	29,79	65		SSW	1	Fine.

* much thunder & lightning last night.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	o	o	Inches.		Inches.	Points.	Str.	
	o										
Aug. 17	56	7	0	58	68	29,72	73		SSW	2	Cloudy.
	68,5	2	0	64,5	68	29,62	75		S	2	Cloudy.
18	52	7	0	55	67	29,79	73	0,167	W	2	Fine.
	69	2	0	68	68	29,81	57		WNW	2	Fair.
19	51	7	0	54	66	30,02	74		SW	1	Fine.
	70,5	2	0	68	67	30,01	67		S	2	Cloudy.
20	63	7	0	64	67	30,05	74		S	2	Fine.
	77	2	0	76	69,5	30,07	65		SSW	1	Fair.
21	61	7	0	63	68	29,94	77		NE	1	Cloudy.
	79	2	0	78	70	29,80	75		SE	1	Cloudy.
22	54,5	7	0	56	68	29,73	76	0,487	SW	1	Fine.
	68	2	0	65	68	29,73	64		SW	1	Fair.
23	51	7	0	53	68,5	29,92	75	0,018	SW	1	Fine.
	72	2	0	71	68	29,97	62		SW	1	Fine.
24	57	7	0	62	68	30,02	73		SSW	1	Cloudy.
	70	2	0	70	68	30,08	67		SSW	1	Cloudy.
25	59	7	0	63	67	30,18	64		S	1	Cloudy.
	73,5	2	0	73	69	30,17	62		E	1	Fair.
26	57,5	7	0	60	68,5	30,09	71		E	1	Fine.
	75	2	0	74,5	70	30,00	63		SE	1	Fine.
27	58	7	0	59	68	29,98	71		WNW	1	Fine.
	70	2	0	68	70	30,04	60		NW	1	Fine.
28	52	7	0	54	66	30,20	69		WNW	1	Fair.
	67	2	0	66	68	30,23	60		WNW	1	Fine.
29	52	7	0	55	67	30,31	67		W	1	Cloudy.
	70	2	0	68	68	30,29	59		SE	1	Fine.
30	51,5	7	0	54	65	30,26	71		SE	1	Fine.
	70	2	0	70	68	30,25	61		SSE	1	Fine.
31	51,5	7	0	54	64	30,25	70		ENE	1	Fine.
	70	2	0	70	67,5	30,25	61		E	1	Fine.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Sept. 1	53	7	0	55	63,5	30,14	79		NE	1	Cloudy.
	73	2	0	73	69	30,07	66		ENE	1	Fine.
2	58,5	7	0	61	66	29,93	81		NE	1	Fine.
	71,5	2	0	71,5	70,5	29,89	69		ENE	1	Fine.
3	58,5	7	0	63,5	67	29,81	76		ENE	1	Fine.
	75	2	0	74,5	71,5	29,80	65		E	2	Fine.
4	64	7	0	65	69	29,64	69		E	2	Cloudy.
	68	2	0	67	69	29,64	70		S	2	Cloudy.
5	60,5	7	0	63	67,5	29,74	76	0,061	S	2	Cloudy.
	73	2	0	72	69,5	29,85	66		S	2	Cloudy.
6	63	7	0	65	69	29,89	71		S b. W	1	Fine.
	77	2	0	76,5	72	29,90	62		S b. W	1	Fine.
7	62	7	0	63,5	70	30,04	74		S b. W	1	Fine.
	78	2	0	77,5	73	30,04	66		E	1	Fine.
8	67	7	0	67,5	71	30,01	75		ESE	1	Cloudy.
	72	2	0	71	71	30,06	70		SSW	2	Cloudy.
9	61	7	0	63	70	30,14	73		SSW	2	Cloudy.
	72	2	0	71	71	30,13	65		W	2	Fine.
10	60	7	0	61,5	69	30,20	75		W	1	Cloudy.
	74	2	0	72,5	70,5	30,20	65		W	1	Hazy.
11	61,5	7	0	62	70	30,20	74		S	1	Cloudy.
	75	2	0	74	71	30,24	64		S	1	Fine.
12	55	7	0	59	68	30,36	68		NE	1	Fine.
	70,5	2	0	70	71	30,35	64		E	1	Hazy.
13	58	7	0	60	68	30,33	74		NE	1	Cloudy.
	70,5	2	0	69,5	70	30,32	64		NE	1	Hazy.
14	55	7	0	58	66	30,37	70		W	1	Cloudy.
	70	2	0	69	69	30,39	65		NE	1	Cloudy.
15	49	7	0	52	64	30,46	71		NE	1	Fine.
	65	2	0	65	68	30,43	64		E	1	Fine.
16	49,5	7	0	52	66	30,35	74		NE	1	Fine.
	65	2	0	63,5	68	30,29	66		E	1	Fine.

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1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer.	Rain. Inches.	Winds. Points.	Winds. Str.	Weather.
	°										
Sept. 17	52	7	0	54	64,5	30,17	77		E	1	Fine.
	69	2	0	69	68	30,10	62		E	1	Fine.
18	52	7	0	53	65,5	29,98	76		ENE	1	Fair.
	73	2	0	73	70	29,96	65		S	1	Fine.
19	58	7	0	61	67	29,93	73		E	1	Fair.
	77	2	0	77	71	29,93	64		ESE	1	Fine.
20	58	7	0	60	68	29,93	74		E	1	Fine.
	75,5	2	0	75	71	29,95	65		E	1	Fine.
21	54	7	0	56,5	67	30,09	78		NE	1	Fine.
	67,5	2	0	67	70	30,11	63		E	1	Fine.
22	50,5	7	0	52	65	30,11	72		NNE	1	Fine.
	68,5	2	0	68,5	68,5	30,09	62		ESE	1	Fine.
23	48	7	0	50	64,5	30,09	70		S	1	Fine.
	70	2	0	70	68	30,10	61		NW	1	Cloudy.
24	50	7	0	53	64	30,23	68		NE	1	Hazy.
	64,5	2	0	64	67	30,22	62		W	1	Hazy.
25	53,5	7	0	56	64	30,18	73		WNW	1	Cloudy.
	72	2	0	71	68,5	30,16	65		WNW	1	Fine.
26	58	7	0	59	66	30,05	72		SSW	1	Cloudy.
	69	2	0	65	67	30,07	73		NW	1	Cloudy.
27	45	7	0	46	64,5	30,34	72	0,020	NE	1	Fair.
	60,5	2	0	60	66	30,31	61		E	1	Fine.
28	46	7	0	48	64	30,19	68		ENE	2	Fine.
	60	2	0	60	65,5	30,06	62		E	2	Fine.
29	52	7	0	53	64	29,84	69		E	2	Cloudy.
	68	2	0	67	64,5	29,79	69		SE	1	Cloudy.
30	58	7	0	60	64	29,91	84		S	1	Cloudy.
	69,5	2	0	68	66,5	29,98	67		S	2	Fair.

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1795	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygro-meter.	Rain.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.		Inches.			
Oct. 1	52	7	0	53	64	30,13	80		NE	1	Cloudy.
	68	2	0	67	66	30,05	74		E	1	Cloudy.
2	59	7	0	59	65	30,00	84	0,071	S	2	Cloudy.
	67	2	0	66	66	30,04	69		SSW	2	Fair.
3	54	7	0	53	65,5	30,09	80		NE	1	Foggy.
	61	2	0	61	65	29,97	75		NNE	1	Cloudy.
4	56	7	0	57	64,5	29,77	87	0,168	NE	1	Cloudy.
	59	2	0	59	65	29,77	85		NE	1	Rain.
5	52	7	0	52,5	63,5	29,84	84	0,286	SW.	1	Fair.
	64	2	0	62	64,5	29,84	71		SSW	1	Cloudy.
6	48	7	0	48	63	29,97	79		WSW	1	Fair.
	58	2	0	56,5	63	30,07	65		WNW	1	Fair.
7	42	7	0	44	62	30,18	74		WSW	1	Cloudy.
	59	2	0	59	63	30,07	64		E	1	Fine.
8	50	7	0	52	62	29,72	78		E	1	Cloudy.
	54,5	2	0	54	62	29,55	83		E	1	Rain.
9	50	7	0	52	61	29,36	83	0,348	S	1	Fair.
	58	2	0	56,5	61	29,16	83		E	2	Rain
10	45,5	7	0	47	60	29,24	77	0,260	SE b. S	2	Fair.
	58	2	0	58	61	29,19	74		S	2	Hazy.
11	47	7	0	48	59	29,16	80	0,210	E	1	Cloudy.
	58,5	2	0	58	60	29,18	69		S	2	Rain.
12	46,5	7	0	47	59	29,50	83	0,128	W	1	Fine.
	58,5	2	0	58	60	29,62	68		W	1	Hazy.
13	53	7	0	55	59	29,68	86	0,230	E	1	Cloudy.
	61	2	0	61	61	29,72	84		S	1	Cloudy.
14	56	7	0	58	61	29,76	85	0,160	SE	1	Cloudy.
	66,5	2	0	65	63	29,68	76		SE	2	Cloudy.
15	57	7	0	57	62	29,51	80	0,118	S	2	Cloudy.* (* much lightning with thunder last night.)
	63,5	2	0	62,5	64,5	29,70	69		SW	2	Fair.
16	55	7	0	58	62,5	29,78	76		SSW	2	Cloudy. (much wind last night.)
	61	2	0	60	64	29,71	76		SSW	2	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Oct. 17	52	7	0	53	62	29,90	79	0,066	SW	1	Fine.
	61	2	0	60,5	64	29,97	70		SSW	1	Hazy.
18	53	7	0	56	63	29,74	84		EbE	1	Cloudy.
	62	2	0	62	64	29,68	76		SSW	2	Cloudy.
19	54	7	0	55	63	29,73	86		E	1	Cloudy.
	66,5	2	0	66	66	29,67	73		S	1	Fair.
20	59,5	7	0	60	64	29,66	82		SE	1	Fair.
	65	2	0	65	66	29,60	74		S	2	Cloudy.
21	55	7	0	55	64	29,54	78	0,028	S	2	Fair.
	63	2	0	61	64	29,43	76		SSW	2	Cloudy.
22	52	7	0	52	63	29,26	86	0,102	E	1	Rain.
	58	2	0	58	65	29,18	73		SSW	2	Fair.
23	44	7	0	45	62	29,23	79	0,156	SW	2	Cloudy.
	58,5	2	0	55	64,5	29,35	65		SW	2	Fair.
24	51	7	0	57	62	29,14	70		SSW	2	Cloudy.
	62	2	0	61,5	64	29,33	65		SW	2	Fair.
25	51,5	7	0	52	62	29,72	72		SW	2	Fine.
	60,5	2	0	59,5	64	29,84	63		W	2	Fair.
26	49	7	0	49	62	29,95	79		W	1	Fair.
	57	2	0	57	64	29,95	68		W	1	Fair.
27	44,5	7	0	47	61	29,82	78		SW	1	Fair.
	58	2	0	58	61,5	29,72	71		SSW	2	Cloudy.
28	54	7	0	55	61	29,50	81		S	2	Cloudy. [much wind last night.
	60	2	0	59	63,5	29,43	69		S	2	Fair.
29	48	7	0	48	60	29,52	71		SSW	2	Fair.
	54	2	0	54	62	29,62	64		SSW	2	Fine.
30	46	7	0	48	59	29,50	76		SSW	2	Cloudy.
	51,5	2	0	51,5	61	29,65	68		SW	2	Cloudy.
31	44	7	0	49	59	29,87	79	0,208	SSW	2	Cloudy.
	58	2	0	56,5	61	29,74	70		S	2	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	o	o	Inches.		Inches.	Points.	Str.	
	o										
Nov. 1	52	7	0	53	59,5	29,44	81	0,476	SE	2	Cloudy.
	55	2	0	53,5	61	29,27	76		SSE	2	Fair.
2	44	7	0	45	58,5	29,31	77	0,250	S	1	Cloudy.
	49	2	0	49	60	29,46	73		NE	1	Cloudy.
3	38	7	0	39	56	30,02	74		NE	1	Fair.
	45	2	0	44,5	58,5	30,09	67		NE	1	Cloudy.
4	33	7	0	34	55	30,22	74		NW	1	Fine.
	44	2	0	42,5	57	30,16	69		SW	1	Fine.
5	41	7	0	50	56	29,79	86	0,118	SW	2	Cloudy.
	53	2	0	51,5	58	29,83	75		WNW	1	Cloudy.
6	46	7	0	46	56	29,84	73		NW	1	Cloudy. much wind last night.
	51	2	0	50,5	58	30,15	67		NW	1	Fine.
7	42	7	0	47	56	30,25	77		SW	1	Cloudy.
	56	2	0	56	58,5	30,15	72		NW	1	Fine.
8	44	7	0	46	57	30,28	74		N	1	Fair.
	52	2	0	51	58,5	30,31	69		NE	1	Cloudy.
9	41	7	0	41	56,5	30,40	82		NE	1	Cloudy.
	49	2	0	47	58	30,45	64		E	1	Fine.
10	39	7	0	40	56	30,48	79		ENE	1	Fine.
	49	2	0	47	57	30,50	67		E	1	Fine.
11	39	7	0	40	56	30,58	77		ENE	1	Fair.
	49	2	0	49	56	30,56	70		NE	1	Cloudy.
12	41	7	0	41	56	30,48	76		NE	1	Cloudy.
	49	2	0	48	56,5	30,34	74		WNW	1	Cloudy.
13	49	7	0	49	57	30,11	78		WNW	1	Cloudy.
	53	2	0	53	58	30,02	73		NW	1	Cloudy.
14	36	7	0	36	57	30,05	74		N	2	Fine.
	43	2	0	42	56,5	30,12	71		N	2	Fair.
15	34	7	0	34	55	30,35	75		NE	1	Fair.
	42	2	0	41,5	58	30,40	71		NE	1	Fine.
16	31	7	0	33	54	30,40	75		WSW	1	Cloudy.
	42,5	2	0	42,5	55	30,30	74		NW	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Nov. 17	40	7	0	40	54	30,14	80		WSW	1	Cloudy.
	48	2	0	47	56	30,03	73		SW	2	Cloudy.
18	45	7	0	46	54	29,55	78		S	3	Cloudy.
	50	2	0	49	55	29,15	80		S	3	Rain.
19	38	7	0	39	54	29,27	79	0,702	SW	1	Cloudy.
	45	2	0	45	55	29,27	73		SSW	1	Fair.
20	34,5	7	0	35	54	29,29	77		SW	1	Fair.
	40	2	0	40	55	29,48	75		NE	1	Cloudy.
21	26	7	0	27	52	29,75	74		WSW	1	Fine.
	39	2	0	38	54	29,80	70		SW	1	Fine.
22	36	7	0	45	53	29,55	85	0,038	S	2	Cloudy.
	51	2	0	50	55	29,44	81		S	2	Cloudy.
23	36	7	0	36	53	29,78	80		SW	1	Fair.
	44	2	0	44	55	29,88	74		WSW	1	Fine.
24	39	7	0	48	53	29,28	85	0,167	S	2	Rain.
	51	2	0	50	56	29,17	81		S	2	Cloudy.
25	41	7	0	42	54	29,00	82		S	1	Cloudy.
	46,5	2	0	43	55	28,94	80		NE	2	Cloudy.
26	28,5	7	0	29	51	29,40	76		W	1	Fine.
	37	2	0	37	53	29,44	71		NW	1	Fine.
27	30	7	0	30	51	29,77	77		WNW	1	Fair.
	39	2	0	38	54	29,74	74		SSE	1	Cloudy.
28	36	7	0	36	51	29,30	87	0,377	NE	1	Cloudy.
	40	2	0	40	52	29,62	80		NE	1	Fair.
29	28	7	0	30	49	30,15	78		WSW	1	Fair.
	40	2	0	40	51	30,14	76		W	1	Cloudy.
30	39	7	0	47	51	29,95	90	0,300	SW	1	Cloudy.
	54	2	0	53	55	29,92	89		SW	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	°	°	Inches.		Inches.	Points.	Str.	
	°										
Dec. 1	48	8	0	48	54	30,09	88		WSW	1	Cloudy.
	54	2	0	54	57	30,07	86		SW	1	Cloudy.
2	50	8	0	50	56	29,93	85	0,025	WNW	1	Rain.
	51	2	0	49	57	29,92	71		WNW	1	Fair.
3	43	8	0	44	56	29,90	75		WNW	1	Cloudy.
	48	2	0	48	57	29,90	73		NW	1	Fair.
4	41	8	0	42	55	30,08	76		NW	1	Cloudy.
	47	2	0	46,5	56,5	30,11	73		NW	1	Cloudy.
5	40	8	0	41	56	30,14	81		SW	1	Cloudy.
	49	2	0	49	57	30,03	85		SW	1	Cloudy.
6	47	8	0	51	56	30,03	88		WSW	1	Cloudy.
	54	2	0	54	58,5	30,08	84		NW	1	Cloudy.
7	47	8	0	47	57	30,13	86		WSW	1	Cloudy.
	49	2	0	49	58	30,16	81		WSW	1	Cloudy.
8	45	8	0	46	57	30,21	86		NNE	1	Cloudy.
	47	2	0	46	58	30,25	86		NE	1	Cloudy.
9	34	8	0	36	56	30,38	80		E	1	Fine.
	42	2	0	40	57	30,39	80		NE	1	Cloudy.
10	39	8	0	39	55	30,35	80		E	1	Cloudy.
	42	2	0	41	56	30,31	80		E	1	Cloudy.
11	40	8	0	41	56	30,23	80		E	1	Cloudy.
	43	2	0	42	56	30,19	80		E	1	Cloudy.
12	41	8	0	41	54	29,97	83		E	2	Cloudy.
	44	2	0	44	57	29,83	76		ESE	2	Cloudy.
13	41	8	0	45	55	29,53	83	0,157	SE	2	Cloudy.
	52	2	0	51	57	29,42	80		SSE	2	Cloudy.
14	47	8	0	48	56	29,56	84	0,060	ESE	1	Cloudy.
	52	2	0	51	57	29,49	84		SSE	1	Cloudy.
15	44	8	0	46	56	29,78	84		SSE	1	Fair.
	51,5	2	0	50,5	59	29,78	80		SSE	1	Fair.
16	45	8	0	46	57	29,70	84		ESE	1	Fair.
	52	2	0	52	59	29,64	82		SE	1	Cloudy.

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1795	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Rain.	Winds.		Weather.
		H.	M.	o	o	Inches.		Inches.	Points.	Str.	
	o										
Dec. 17	47	8	0	50	57	29,58	80		SE	2	Cloudy.
	54	2	0	52,5	59	29,62	77		SSE	2	Cloudy.
18	47	8	0	48	58	29,71	80		SE	1	Fair.
	53	2	0	52	61	29,71	79		SE	1	Cloudy.
19	49	8	0	50	59,5	29,60	84		S	2	Cloudy.
	54	2	0	54	62	29,51	79		S	2	Fair.
20	48	8	0	48	59	29,50	83		S	1	Cloudy.
	52	2	0	52	62	29,56	77		SW	1	Fair.
21	44,5	8	0	45	58	29,92	81	0,056	SE	1	Rain.
	53	2	0	52	60	29,84	85.		S	2	Rain.
22	47	8	0	54	59	29,71	86	0,132	SSW	2	Cloudy.
	56	2	0	54	61	29,71	78		SSW	2	Cloudy.
23	43	8	0	43	57	29,93	78		SW	2	Fine.
	48	2	0	47	59	30,11	72		SW	1	Fine.
24	47	8	0	48	57	30,25	81		SW	1	Cloudy.
	54	2	0	53	60	30,16	84		SW	2	Cloudy.
25	43	8	0	43	60	30,31	75	0,097	SW	1	Fair.
	46	2	0	46	62	30,34	73		WNW	1	Cloudy.
26	38	8	0	38	57	30,22	77		NNE	1	Fine.
	42	2	0	42	60	30,32	74		NE	1	Fine.
27	34	8	0	37	55	30,32	78		SW	1	Cloudy.
	43	2	0	43	59,5	30,22	75		S	2	Fair.
28	38,5	8	0	48	57	29,94	82		S	2	Cloudy.
	50	2	0	47	59	30,06	77		SSW	2	Fair.
29	44	8	0	52	57	29,78	86	0,048	SSW	2	Cloudy.
	55	2	0	55	59,5	29,75	84		SSW	2	Cloudy.
30	39	8	0	40	54	30,13	80	0,398	W	2	Fair.
	43	2	0	43	58	30,32	74		NW	2	Fine.
31	35	8	0	44	55	30,14	80		SW	2	Cloudy.
	44	2	0	44	57,5	29,98	80		SW	2	Cloudy.

1795.	Six's Therm. without.			Thermometer without.			Thermometer within.			Barometer.*			Hygrometer.			Rain.
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Inches.
January	46,0	7	26,0	46	8	26,0	49	36	43,4	30,47	29,16	30,01	92	66	71,2	0,476
February	51	24	35,9	51	25	36,4	55	42	47,9	30,68	29,04	29,60	91	64	77	1,255
March	54,5	24	40,4	54,5	25	41,1	59	47	52,7	30,35	29,02	29,80	85	56	72,6	1,744
April	59,5	36	47,9	58,5	37	48	61,5	53	57,5	30,22	29,34	29,79	80	54	69,7	0,497
May	81,5	36	54,5	81	43	55,8	68	57	61,1	30,49	29,73	30,17	79	47	61,5	0,276
June	77,5	41,0	56,6	76	41	57,2	66	56	61,1	30,14	29,50	29,86	90	56	71,0	3,339
July	76	46	64,0	75	51	59,8	68	59	62,8	30,26	29,54	29,97	86	58	68,0	1,400
August	79	51	63,9	78	53	64,3	73	64	67,4	30,31	29,62	29,97	81	57	69,2	1,856
September	78	45	63,1	77,5	46	63,7	73	63,5	67,8	30,46	29,64	30,08	84	61	69,3	0,081
October	68	42	55,6	67	44	55,8	66	59	62,6	30,18	29,16	29,66	87	64	75,7	2,539
November	56	28	42,4	56	27	43,1	61	49	55,4	30,58	28,94	29,87	90	64	76,1	2,428
December	56	34	46,1	55	36	46,9	62	54	57,5	30,39	29,42	29,97	88	71	80,4	0,973
Whole year			49,7			49,9			58,1			29,90			71,8	16,864

* The quicksilver in the bason of the barometer is 81 feet above the level of low water spring tides at Somerset-house.